Financial Regulation in a Quantitative Model of the Modern Banking System*

Juliane Begenau  Tim Landvoigt
Stanford GSB & NBER  Wharton & NBER
September 2018

Abstract

How does the shadow banking system respond to changes in capital regulation of commercial banks? We propose a tractable quantitative general equilibrium model with regulated and unregulated banks to study the unintended consequences of capital requirements. Tightening the capital requirement from the status quo leads to a safer banking system despite riskier shadow banking activity. A reduction in aggregate liquidity provision decreases the funding costs of all banks, raising profits and investment. Calibrating the model to data on financial institutions in the U.S., the optimal capital requirement is around 17%.

*First draft: December 2015. Email addresses: begenau@stanford.edu, timland@wharton.upenn.edu. We would especially like to thank our discussants Dean Corbae, Mark Gertler, Christian Opp, Goncalo Pino, David Chapman, and Hendrik Hakenes, as well as Hanno Lustig, Arvind Krishnamurthy, Martin Schneider, Monika Piazzesi, and Joao Gomes for many helpful conversations. We also benefited from comments and suggestions from seminar participants at the ASSA 2016 and 2017 meetings, Berkeley-HAAS, Barcelona GSE 2017, Carnegie Mellon, CITE 2015 conference, CREI, Federal Reserve Bank of Boston, Federal Reserve Bank of New York, MFM Winter 2016 meeting, MIT Sloan, NBER SI 2016, Northwestern-Kellogg, NYU Junior Macro-Finance 2016 Meeting, SAFE 2016 Conference on Regulating Financial Markets, SED Toulouse 2016, SFI Lausanne, Stanford GSB, Stanford SITE 2017, Stanford 2016 Junior Faculty Workshop, U Wisconsin-Madison, Texas Finance Festival, University of Texas at Austin, WFA 2016 and Wharton.
1 Introduction

The goal of the regulatory changes for banks in the aftermath of the Great Recession, notably higher capital requirements, was to reduce the fragility of the financial system. Today’s financial system, however, is much more complex and extends beyond the institutions that are under the regulatory umbrella. Nowadays, the so-called shadow banking sector fullfills many functions of regulated banks, such as lending and liquidity provision. Do tighter regulations on regulated commercial banks cause an expansion of shadow banks as they become relatively more profitable? Does a larger shadow banking sector imply an overall more fragile financial system? The example of Regulation Q tells a cautionary tale. Introduced after the Great Depression in the 1930s to curb excessive competition for deposit funds it had little effect on banks as long as interest rates remained low. When interest rates rose in the 1970s, depositors looked for higher yielding alternatives and the competition for their savings generated one: money market mutual funds (Adrian and Ashcraft (2016)). This and numerous other examples\(^1\) highlight the unintended consequences of regulatory policies.

In this paper, we build a tractable general equilibrium model to quantify the costs and benefits of tighter bank regulation in an economy with regulated commercial banks and unregulated shadow banks.\(^2\) The role of banks in this economy is to provide access to assets that require intermediation, such as loans. Banks fund their investment in intermediated assets through short-term debt, which provides liquidity services to households. Commercial and shadow banks differ in their ability to produce liquidity, but not to intermediate assets. All banks have the option to default and therefore may not repay their creditors. However, commercial banks are insured and their depositors are always repaid, while shadow banks are not insured and thus risky for depositors. This government guarantee drives a wedge between the competitive equilibrium and what a social planner would choose. Calibrating the model to aggregate data from the Flow of Funds, banks’ call reports, and NIPA we find that higher capital requirements indeed increase the size of the shadow banking sector. However, instead of becoming more fragile, the aggregate banking system is safer at the optimal level of bank capital requirements. The optimal

---

\(^1\)Asset-backed commercial paper conduits are another example for entities that emerged arguably as a response to regulation, more precisely capital regulation (see Acharya, Schnabl, and Suarez (2013)).

\(^2\)We define shadow banks as financial institutions that share features of depository institutions, either by providing liquidity services such as money market mutual funds or by providing credit directly (e.g. finance companies) or indirectly (e.g. security-broker and dealers). At the same time, they are not subject to the same regulatory supervision as traditional banks. We adopt a consolidated view of the shadow banking sector, e.g. money market mutual funds invest in commercial paper that fund security brokers-dealers that provide credit. We view this intermediation chain as essentially being carried out by one intermediary.
requirement finds the welfare maximizing balance between a reduction in aggregate liquidity provision and an increase in the safety of the financial sector. At the optimum, increased riskiness of shadow banks is more than offset by greater stability of commercial banks. Welfare is maximized at a capital requirement of roughly 17%.

We derive this result in a production economy with households, commercial banks, shadow banks, and a regulator. To capture the value of liquidity services produced by bank deposits, we assume that households derive utility from bank deposits in addition to consumption. The consumption good is produced with two technologies. One technology is directly accessed by households through their ownership of a tree that produces an endowment of the good stochastically. The second technology, a Cobb-Douglas, stochastic production technology, is operated by banks and uses capital owned by intermediaries and labor rented from households. Banks further possess an investment technology and compete over capital shares. Their assets are funded by issuing equity and deposits to households. When either type of bank defaults on its debt, its equity becomes worthless and a fraction of the remaining bank value is destroyed in bankruptcy. Shadow banks are fragile because some households may randomly lose confidence and run on them akin to Allen and Gale (1994). Run-induced shadow bank defaults cause additional deadweight losses.

The model’s equilibrium determines the optimal leverage of each type of bank, and the relative sizes of the regulated and unregulated sectors. Households value shadow bank debt both for its payoff (in consumption goods) and for its liquidity benefits. Greater leverage increases the default risk of shadow banks, reducing the expected payoff of their debt. It also increases the amount of liquidity services shadow banks produce per unit of assets. Shadow banks internalize this trade-off, choosing leverage to equate the marginal liquidity benefit to households with the marginal bankruptcy cost. Commercial bank debt is insured, and as a result the regulatory constraint is the only limit on their leverage. Given these capital structure choices, how does the model pin down the relative size of both sectors? A key force is that the liquidity services produced by deposits of commercial and shadow banks are imperfect substitutes, with diminishing returns in each. The liquidity premium earned by bank debt is the main source of bank equity value, allowing banks to issue debt at interest rates that are lower than the expected return of the debt. This links to the second force in the model. Since banks compete with each other, the to-

\footnote{Since commercial deposits are safe and shadow deposits are risky, risk averse households in our model assign an optimal portfolio share to each type of debt. In theory, this force would determine the relative size of shadow banks to commercial banks even if commercial and shadow deposits were perfect substitutes in terms of liquidity. However, for realistic risk aversion and shadow bank risk, this mechanism fails to provide the relatively stable shadow bank share observed in the data.}
tal quantity of their debt, commercial and shadow, must align such that equity holders – households – are indifferent on the margin between holding each type of equity. Thus, for given leverage, the asset share of each bank type expands or shrinks to align their equity values.

This core feature of the model also drives the response to higher capital requirements. Increasing capital requirements makes commercial banks less profitable, since it forces them to fund each dollar of assets with a smaller share of advantageous deposits. As a result, shadow bank equity becomes relatively more profitable, inducing households to invest in shadow banks and raising shadow bank demand for intermediated assets. As the shadow sector expands, it issues more debt, lowering its marginal liquidity benefit to depositors and thus its marginal profitability. This process continues until both sectors are equally profitable again.

We develop this intuition in a simplified static version of our model, which also allows us to analytically characterize the welfare-maximizing size and leverage of both banking sectors. We show that a capital requirement alone fails to implement the social optimum in the decentralized economy. Competition forces shadow banks to have too much leverage in order to overcome the competitive advantage of commercial banks created by deposit insurance. Our simple model further demonstrates that the welfare consequences of higher capital requirements crucially depend on the responses of liquidity premia for commercial and shadow debt, which in turn depend on liquidity preferences. In particular, the questions by how much the shadow sector expands, and whether shadow banks become riskier, depend on how elastic liquidity premia of both banks are with respect to their debt quantities. In the model, these elasticities are functions of the overall returns to scale in liquidity production, and the degree of substitutability between commercial and shadow liquidity. Thus, in the full quantitative model, we match these elasticities to dynamic properties of liquidity premia in the data, building on empirical work by Krishnamurthy and Vissing-Jorgensen (2012), Greenwood, Hanson, and Stein (2016), and Nagel (2016). We find that our model matches the data elasticities with moderately decreasing returns in overall liquidity provision, implying that tightening capital regulation from 10% to 17% of assets (the welfare optimum) will raise liquidity premia on both commercial and shadow deposits, by 4 basis (11.5%) and 1.2 basis points (5.3%), respectively. We further find a relatively high degree of substitutability, leading to an expansion of the shadow share in debt markets by 2 percentage points (6.2%). We explore the robustness of these results to different preference specifications.

Overall, our carefully calibrated quantitative model confirms the assertion of some
commentators that tighter capital regulation will cause an expansion of shadow banking. However, our results also demonstrate that despite this substitution, there is substantial room for welfare-improving increases in capital requirements, as a greater shadow banking sector does not necessarily imply a riskier financial system.

Finally, we use our calibrated model to evaluate alternative policy measures. Specifically, we consider the effects of time-varying capital requirements and higher deposit insurance fees. We also evaluate proposals to impose a tax on shadow bank debt, at the same time as tightening the capital requirement on traditional banks (the so-called Minneapolis plan\(^4\) outlines such a proposal). We find that this simultaneous taxation of shadow bank debt achieves a larger welfare gain than the other policies. While such a proposal greatly reduces liquidity provision by both kinds of banks in our model, this negative effect is more than offset by the increased stability of the financial system as a whole. This is consistent with the view that the existence of shadow banks limits the scope of regulatory measures that only target traditional banks.

**Related Literature.** Our paper is part of a growing literature at the intersection of macroeconomics and banking that tries to understand optimal regulation of banks in a quantitative general equilibrium framework.\(^5\) Our modeling approach draws on recent work that analyzes the role of financial intermediaries in the macroeconomy.\(^6\) These papers study economies with assets that investors can only access through an intermediary, as in our paper. By introducing limited liability and deposit insurance, and by defining the role of banks as liquidity producers, we bridge the gap to a long-standing microeconomic literature on the function of banks in the economy.\(^7\)

Our goal is to quantify the unintended consequences of regulating commercial banks for financial stability and macroeconomic outcomes. Other papers have addressed closely related questions but not in a quantitative setting.\(^8\) More broadly, the role of shadow banks in the recent financial crisis has motivated a number of papers that propose theories why the shadow banking system emerged and why it can become unstable (e.g.,

---

\(^4\)https://www.minneapolisfed.org/publications/special-studies/endingtbtf


\(^7\)For an overview of standard microeconomic models of banking see Freixas and Rochet (1998). Recent theoretical contributions with a focus on the role of bank capital include Malherbe (2015) and Harris, Opp, and Opp (2017).

Gennaioli, Shleifer, and Vishny (2013) and Moreira and Savov (2017)). Shadow banks are often viewed to emerge in response to tighter financial regulation, for example by providing liquid-yet-fragile securities off-balance sheet (e.g., Plantin (2015); Huang (2016); Xiao (2018)). Or they emerge because they produce financial services using a different technology compared to traditional banks (e.g., Gertler, Kiyotaki, and Prestipino (2016); Ordoñez (2018); Martinez-Miera and Repullo (2017)). Our paper captures both views as shadow banks can exist independently of how tightly the traditional banking sector is regulated. Yet tighter financial regulation can make shadow banking more attractive and lead to an expansion of the shadow banking system.\footnote{The paper by Buchak, Matvos, Piskorski, and Seru (2018) empirically estimates how much of the rise in shadow banking, most notably Fintech firms, is due to a change in the regulatory system or a change in technology.}

A key difference to other quantitative work is that we explicitly model moral hazard arising from deposit insurance (and more generally government guarantees) akin to Bianchi (2016).\footnote{In contrast to Bianchi (2016), our focus is not on whether the government guarantee itself is optimal.} In addition, we account for a key institutional feature of financial intermediaries by modeling them with limited liability. Hence our setup allows to study welfare-improving bank regulation in a quantitative framework. Since our focus is on liquidity provision as a fundamental role of banking, we also relate to the literature on the demand for safe and liquid assets,\footnote{E.g. Bernanke (2005), Caballero and Krishnamurthy (2009), Caballero, Farhi, and Gourinchas (2016), Gorton et al. (2012), Krishnamurthy and Vissing-Jorgensen (2012).} and on the role of financial intermediaries in providing such assets.\footnote{There is a large theoretical literature on this subject with seminal papers by Gorton and Pennacchi (1990) and Diamond and Rajan (2001).} Pozsar, Adrian, Ashcraft, and Boesky (2012), Chernenko and Sunderam (2014), Sunderam (2015), Adrian and Ashcraft (2016), among others, are recent empirical papers documenting the role of shadow banks for liquidity creation.

2 Simple two-period model

Before laying out the dynamic model in Section 3, we build intuition for its main mechanisms in a simplified static version that we can solve by hand. This version keeps the model’s essential features, but abstracts from some of its more complicated details such as shadow bank runs that are necessary to generate a good quantitative fit. The key features of our model is that two different types of banks provide valuable liquidity services to households by issuing debt. Both bank types are risky, and thus can default. But commercial bank debt is insured and thus safe for households, while shadow bank debt is
not. Two equilibrium conditions determine the relative size of the shadow banking sector as a function of shadow banks’ relative profitability vis-a-vis commercial banks and the leverage of shadow banks as a function of the marginal liquidity value of its debt. Increasing the capital requirement for commercial banks increases the relative profitability of shadow banks, which leads to an expansion of the shadow banking sector. The effect on shadow bank leverage, whether positive or negative, does however depend on households’ preference for liquidity. Ultimately, this leads us to the quantitative part of our paper.

2.1 Setup

Production Technology and Preferences. Time is discrete and there are two dates, times 0 and 1. The economy is populated by two types of banks, C-banks and S-banks, and households. Banks can buy capital at time 0 that trades at price \( p \) in a competitive market. Each unit of capital produces one unit of the consumption good at time 1. The total supply of capital is fixed at unity. Banks are financed with equity and uncontingent debt, issued to households.

Households are endowed with 1 unit of the capital good at time 0. We assume that households are less efficient at operating the capital stock than banks, in the sense that when owned by households each unit of capital will only produce \( \zeta << 1/2 \) units of output at time 1. Households have preferences over consumption at time 0, and over consumption and liquidity services at time 1. We assume that liquidity services are derived from holding the liabilities issued by banks

\[
U = C_0 + \beta (C_1 + \psi H(A_S, A_C)),
\]

where \( H(A_S, A_C) \) is the utility\(^{13} \) from liquidity, and \( A_j, j = S, C \) is the quantity of deposits of bank type \( j \) held by households. The parameter \( \psi \) governs the weight of liquidity services relative to numeraire consumption.

Financial Technology. S-banks and C-banks issue debt \( B_j \) and equity shares \( S_j, j = C, S \), in competitive markets to households. Debt trades at prices \( q_j \) and equity at \( p_j \).

\(^{13}\)We model liquidity preferences akin to the classic money literature that elicits a demand for money via a money in the utility function specification (see Poterba and Rotemberg (1986)). Feenstra (1986) showed that the reduced-form preference specification is functionally equivalent to microfounding a demand for money with transaction costs.
Both types of banks have limited liability and make optimal default decisions at time 1. In case of default, bank equity becomes worthless. Debt issued by C-banks is perfectly safe for households. That is, when the C-bank’s equity is insufficient to pay back its depositors in full, the government makes up the shortfall to depositors, and raises the funds through lump-sum taxes on households.

In contrast, debt issued by S-banks is risky. When a S-bank defaults with insufficient equity to pay out its depositors in full, depositors lose all deposits they held with the defaulting bank.

2.2 Bank Problem

S-banks. The problem of an individual S-bank at time 0 is to choose how much capital to buy, $K_S$, and how many deposits to issue, $B_S$. At time 1, the capital produces consumption goods that the bank sells to households. Further, the bank needs to pay out its depositors if it does not default. Individual banks receive idiosyncratic production shocks $\rho_S$ at time 1 that are distributed i.i.d uniform on support [0, 1], such that the total payoff of its capital at time 1 is $\rho_S K_S$. Each S-bank maximizes its expected net present value

$$\max_{K_S \geq 0, B_S \geq 0} q_S(B_S, K_S)B_S - pK_S + \beta \max \{\rho_S K_S - B_S, 0\}.$$ 

At time 0, the bank needs to pay $pK_S$ to purchase its capital and raises $q_S B_S$ in deposits. It needs to raise the difference in initial equity from households. At time 1, the capital produces $\rho_S K_S$ consumption goods that banks sell to households at a price of one. They further pay off their deposits at unit face value. The difference is the dividend they pay to their equity owners, the households. Due to the default option, the dividend cannot be negative. Banks discount time 1 payments using the discount factor of households (their shareholders), which, given household preferences in (1), is simply $\beta$.

The price at which individual S-banks can issue deposits, $q_S(B_S, K_S)$, depends on the portfolio choice of the bank. Households take into account that the probability of default depends on the bank’s leverage. The bank in turn internalizes this effect of its leverage choice on its deposit price.

C-banks. The problem of C-banks is analogous to that of S-banks, with two important differences. First, C-banks issue safe deposits due to the government guarantee (deposit insurance). Hence the price at which they raise deposits, $q_C$, is not sensitive to the port-
folio choice of the individual bank. Secondly, C-banks are subject to a regulatory capital constraint that limits the amount of deposits they can issue to a fraction $1 - \theta$ of the capital they purchase; put differently, C-banks face an equity capital requirement of $\theta$. We formulate the capital requirement in terms of the expected payoff $E(\rho_C K_C)$, which is $\frac{1}{2}K_C$ since $\rho_C \sim \text{Uniform}[0, 1]$. C-banks solve the problem

$$\max_{K_C \geq 0, B_C \geq 0} q_C B_C - p K_C + \beta \max \{\rho_C K_C - B_C, 0\},$$

subject to the equity capital requirement

$$B_C \leq \frac{1}{2}(1 - \theta) K_C.$$ 

**Bank Size and Leverage Choices.** Banks make two choices: (1) how much capital to buy (size), and (2) how much debt to issue (leverage). Defining bank leverage

$$L_j = \frac{B_j}{K_j},$$

for $j = S, C$, we can write the payoff in period 1 as

$$\max \{\rho_j K_j - B_j, 0\} = K_j (1 - F_j(L_j)) \left( E(\rho_j | \rho_j > L_j) - L_j \right).$$

In other words, the dividend is the expected dividend per unit of capital conditional on the bank having survived, scaled by the capital stock. Since $\rho_C \sim \text{Uniform}[0, 1]$, this simplifies to

$$\frac{1}{2} K_j (1 - L_j)^2,$$

and the default rate of banks of type $j$ is given by $L_j$. The output of defaulting banks is destroyed in bankruptcy, i.e., depositors of S-banks recover zero, and the government needs to “bail out” the full face value of defaulting C-banks’ deposits.

We exploit the fact that the problem of banks is homogeneous in capital to separate the problem of each bank into a leverage and a size choice. For C-banks, the leverage problem is

$$v_C = \max_{L_C \in [0, 1]} q_C L_C - p + \beta \frac{1}{2} (1 - L_C)^2$$

subject to

$$L_C \leq \frac{1}{2} (1 - \theta).$$
and for S-banks it is

$$v_S = \max_{L_S \in [0,1]} q_S(L_S)L_S - p + \beta \frac{1}{2} (1 - L_S)^2.$$  \hfill (3)

The capital purchase decision for each bank is then given by

$$\max_{K_j} K_j v_j,$$  \hfill (4)

subject to $K_j \geq 0$.

### 2.3 Households

Households are endowed with one unit of capital. They optimally sell this capital to banks at price $p$. They buy deposits $A_j$ and equity shares $S_j$ of bank type $j$ at time 0, such that their time 0 budget constraint is

$$C_0 = p - q_S A_S - q_C A_C - p_S S_S - p_C S_C.$$  \hfill (5)

The time-1 consumption is therefore

$$C_1 = (1 - L_S) A_S + A_C + \frac{1}{2} S_S K_S (1 - L_S)^2 + \frac{1}{2} S_C K_C (1 - L_C)^2 - T,$$  \hfill (6)

where $T$ denotes government lump-sum taxation to bail out deposits, i.e., $T = L_C B_C, (1 - L_S) A_S$ and $A_C$ are deposit redemptions for S- and C-banks, respectively, and $\frac{1}{2} S_j K_j (1 - L_j)^2$ denotes the expected cash-flow from owning bank type $j$ equity.

Households choose $C_0, C_1, S_j,$ and $A_j, j = C, S$ to maximize utility (1) subject to constraints (5) and (6).
2.4 Equilibrium Definition

Definition. Market clearing requires

\[ S_S = 1 \]
\[ S_C = 1 \]
\[ A_C = B_C \]
\[ A_S = B_S \]
\[ 1 = K_S + K_C. \]

By Walras law, consumption at time 0 is\(^{14}\)

\[ C_0 = 0, \tag{7} \]

and consumption at time 1 is

\[
C_1 = K_C \left( E(\rho_C) - F(L_C)E(\rho_C | \rho_C < L_C) \right) + K_S \left( E(\rho_S) - F(L_S)E(\rho_S | \rho_S < L_S) \right) \\
= \frac{1}{2} \left( 1 - K_C L_C^2 - K_S L_S^2 \right). \tag{8}
\]

The resource constraint for period-1 consumption (8) clarifies the fundamental trade-off of the model. If banks did not issue any debt, then \( L_C = L_S = 0 \) and household consumption of the numeraire good would be maximized at the full payoff of capital, \( E(\rho_j) = 1/2 \). However, in that case banks would produce no liquidity services from which households also derive utility. To produce liquidity services, banks need to issue debt and take on leverage, which causes a fraction \( L_j \) of them to default. In the process, some payoffs of the numeraire good, \( K_j L_j \), are destroyed.

2.5 Equilibrium Characterization

Now, we are ready to describe how the equilibrium in this model works. We are particularly interested in the leverage choice of banks as this determines the equilibrium response to higher capital requirements.

\(^{14}\)The funds households spend on their portfolio of bank securities, \( q_C A_C + p_C S_C + q_S A_S + p_S S_S \), are equal to the market value of the capital they sell to banks in equilibrium, \( p \).
**Household Optimality.** Before taking the first-order conditions for households’ bond purchases, we denote the partial derivatives of the liquidity utility with respect to the two types of liquidity as

\[ H_C(A_S, A_C) = \frac{\partial H(A_S, A_C)}{\partial A_C}, \]
\[ H_S(A_S, A_C) = \frac{\partial H(A_S, A_C)}{\partial A_S}. \]

Using this notation, the household’s first-order condition for C-bank debt is

\[ q_C = \beta(1 + \psi H_C(A_S, A_C)). \] (9)

The FOC for S-bank debt is

\[ q_S = \beta(1 - L_S + \psi H_S(A_S, A_C)). \] (10)

Equation (9) states that the bond price \( q_C \) must equal the expected discounted bond payoff, \( \beta \), plus the discounted marginal liquidity benefit \( \beta \psi H_C(A_S, A_C) \). Since C-bank debt is insured, the expected payoff is unaffected by C-bank default and hence risk-free. In contrast, equation (10) shows that the expected discounted payoff to uninsured S-bank debt is \( \beta(1 - L_S) \), reflecting that in expectation a fraction \( L_S \) of S-banks defaults (with a recovery value of zero).

**S-bank Optimality.** Each individual S-bank recognizes that the price of its debt is a function of its leverage according to households’ valuation in (10). However, S-banks do not internalize the effect of their leverage choice on the aggregate marginal benefit of S-bank liquidity \( \psi H_S(A_S, A_C) \). The following proposition characterizes the leverage choice and debt price of S-banks.

**Proposition 1.** The price of S-bank debt is

\[ q_S = \beta, \] (11)

and S-bank leverage is equal to the aggregate marginal benefit of S-bank liquidity

\[ L_S = \psi H_S(A_S, A_C). \] (12)

**Proof.** See appendix A. \( \square \)
Proposition 1 states that S-bank debt is priced in equilibrium like riskfree debt with no liquidity benefit. This is because S-banks optimally choose leverage such that the marginal benefit of S-bank liquidity to households, on the RHS of (12), is equal to the marginal loss due to defaulting S-banks ($L_S$ on LHS).

**S-bank’s capital valuation.** The constant returns assumption in the scale decisions of banks in (4) implies that in equilibrium, S-banks must have zero expected value, i.e. $K_S v_S = 0$. The condition for an equilibrium with an S-bank sector, in the sense of $K_S > 0$, therefore requires that the per-capital profits in (3) are zero:

$$v_S = 0,$$

which leads to the zero-profit condition

$$p - q_S L_S = \beta \frac{1}{2} (1 - L_S)^2. \tag{13}$$

Condition (13) states that the initial equity required to start an S-bank on the left, per unit of capital acquired by S-banks, must be equal to the expected payoff to equity holders on the right. Since $q_S = \beta$ by proposition 1, we get the following equilibrium restriction on the capital price

$$p = \frac{1}{2} \beta (1 + L_S^2). \tag{14}$$

Equation (14) states that S-bank demand for capital is perfectly elastic at a price $p$, that incorporates both the direct expected payoff of a unit of capital, $\frac{1}{2} \beta$, and the benefit equity owners derive from debt financing, $\frac{1}{2} \beta L_S^2$. This benefit stems from the fact that the marginal benefit of S-bank liquidity for households (see equation (12)) lowers the debt financing costs of S-banks. The liquidity demand of households leads to an increase in the value of capital in equilibrium.

**C-bank Optimality.** The following proposition characterizes C-bank leverage and debt pricing:

**Proposition 2.** If there is a positive marginal benefit of C-bank liquidity, $\psi H_C (A_S, A_C) > 0$, the C-bank leverage constraint is always binding

$$L_C = \frac{1}{2} (1 - \theta), \tag{15}$$
and C-banks debt sells at a higher price than S-bank debt

\[ q_C - q_S > 0. \] (16)

**Proof.** See appendix A.

Since C-banks can issue insured debt that also generates utility for households, there is no interior optimum to their capital structure choice. They issue debt at a price that is strictly greater than its expected payoff \( \beta \) due to the liquidity premium it earns.

Analogous to S-banks, the scale invariance of the C-bank problem requires that \( K_C v_C = 0 \). Any equilibrium with a C-bank sector \( (K_C > 0) \) thus requires \( v_C = 0 \), or

\[ p - q_C L_C = \beta \frac{1}{2} (1 - L_C)^2. \] (17)

Combining this condition with the price of C-bank debt required by HH optimization in (9) gives

\[ p = \frac{1}{2} \beta (1 + L_C^2) + \beta \psi L_C H_C(A_S, A_C). \] (18)

**C-bank’s capital valuation.** Comparing the “zero-profit” condition for C-banks in (18) to that of S-banks in (14) demonstrates how the different debt financing costs for both banks affect their demand for capital. Like S-banks, C-banks value capital at the expected payoff \( \frac{1}{2} \beta \) plus an additional term that represents the advantage of debt funding due to the liquidity benefit households derive from holding debt, \( \frac{1}{2} \beta L_C^2 \). Note that this advantage of debt arises even in absence of deposit insurance. C-banks assign *additional* value to debt funding, since their debt is insured by the government and thus its price is insensitive to C-bank default risk, \( \beta \psi L_C H_C(A_S, A_C) \). This additional debt advantage increases C-bank demand for capital that serves as collateral for debt.

**Equilibrium determination.** To characterize the leverage of both banks and their relative market shares in equilibrium, we assume that the liquidity function has the constant elasticity of substitution (CES) form

\[ H(A_S, A_C) = (\alpha A_S^\epsilon + (1 - \alpha) A_C^\epsilon)^{1/\epsilon}, \] (19)

where \( \alpha \in [0, 1] \) is the weight on S-bank liquidity and \( 1/(1 - \epsilon) \) is the elasticity of substitution between both types of liquidity. Since \( H \) is homogeneous of degree one, the
marginal liquidity benefit functions are homogeneous of degree zero and only depend on
the ratio of S-bank to C-bank debt \( R_S = \frac{A_S}{A_C} \), i.e. \( \mathcal{H}_j(A_S, A_C) = \mathcal{H}_j(R_S) \) for \( j = S, C \). Further, the function \( H \) has diminishing returns in each type of debt, such that \( \mathcal{H}'_S(R_S) < 0 \) and \( \mathcal{H}'_C(R_S) > 0 \) for \( R_S \in (0, \infty) \). We can characterize the equilibrium as a system of two
equations in the two variables \( L_S \) and \( R_S \), which are S-bank leverage and debt ratio. We
obtain the first equation by applying the definition of the debt ratio to the condition for
optimal S-bank leverage, (12):
\[
L_S = \psi \mathcal{H}_S (R_S) . \tag{20}
\]
We obtain the second equation by equating the capital demand conditions for S-banks
(14), and C-banks (18). After simplifying, we get the zero-profit condition
\[
L_S = \left( \frac{1}{4} (1 - \theta)^2 + \psi (1 - \theta) \mathcal{H}_C (R_S) \right)^{1/2} . \tag{21}
\]
Equations 20 and 21 define two functions mapping \( R_S \in [0, \infty) \) into S-bank leverage
\( L_S \in [0, 1] \). The range of values for both functions is bounded below by zero since the
marginal liquidity benefit is weakly positive and \( \theta \leq 1 \). The upper bound is one, since
the idiosyncratic shocks \( \rho_S \) are uniformly distributed over \([0, 1]\), and the probability of
default of the S-bank is equal to its leverage \( L_S \). Denote by \( \hat{R}_S^0 \) the debt ratio such that
the leverage condition implies \( 1 = \psi \mathcal{H}_S (\hat{R}_S^0) \). Further, denote by \( \hat{L}_S^0 \) the leverage ratio
implied by the zero-profit condition at this point, i.e. \( \hat{L}_S^0 = \frac{1}{4} (1 - \theta)^2 + \psi (1 - \theta) \mathcal{H}_C (\hat{R}_S^0) \).
Since equation (20) defines a decreasing function and (21) an increasing function, a unique
equilibrium with S-bank leverage \( L_S \in [0, 1] \) exists if \( \hat{L}_S^0 < 1 \). This condition is satisfied
for a large range of parameter values.

Graphically, the equilibrium is at the intersection of both curves in \((R_S, L_S)\) space, il-
lustrated in Figure 1.

The shape of both curves is a consequence of decreasing returns to scale in each type
of liquidity. The upward-sloping (red) curve is the zero-profit condition in (21). The
condition says that in equilibrium, equity owners must be indifferent between owning
S-banks and C-banks. Thus, S-banks need to generate the same (zero) profit for per unit
of capital as C-banks. Because of diminishing returns, the marginal benefit of C-bank
debt, \( \mathcal{H}_C (R_S) \), is decreasing in the amount of C-bank liquidity provided. This means that
C-bank profitability is increasing in S-bank ratio \( R_S \). Hence, to “keep up” with C-banks,
the zero-profit condition for equity owners requires that S-banks increase their leverage
as their capital share rises along the curve. Basically, S-banks need to lever up to boost
their return to equity.
Figure 1: Equilibrium Existence and Comparative Static in $\theta$

Left panel: The unique equilibrium S-bank leverage and debt ratio satisfy optimal leverage (blue) and zero-profit (red) conditions. Right panel: Higher capital requirement causes a downward shift in the zero-profit condition, implying lower S-bank leverage and greater share.

The downward-sloping (blue) curve is the optimal leverage condition (20). The condition says that S-bank leverage in equilibrium is equal to the marginal benefit of S-bank debt, $\hat{H}_S (R_S)$. For given leverage, this marginal benefit is decreasing in S-bank ratio $R_S$, again because of diminishing returns.

2.6 The effect of C-bank capital requirements

We can now ask how higher capital requirements affect the economy. To this end, we study the comparative static of the equilibrium characterized by equations (20) and (21) with respect to $\theta$, as shown by the right-hand panel of Figure 1 and formalized in proposition 5 in Appendix A. Raising $\theta$ leads to a drop in shadow bank leverage and an expansion in the shadow banking share.

The zero-profit condition (the red curve) shifts down because a rise in $\theta$ decreases the profitability of C-bank equity (for a given level of $K_S$), as C-banks are now further restricted in their ability to take advantage of cheap deposit funding.\(^\text{15}\) In equilibrium, this

\(^{15}\)An increase in $\theta$ reduces the amount of C-bank debt for given $K_S$. With decreasing returns, this causes a rise in the marginal benefit of C-bank debt, which by itself makes issuing debt more attractive for C-banks. However, a higher $\theta$ also directly lowers the fraction of debt funding C-banks can use per dollar of capital. The second effect dominates. This can be seen analytically by inspecting the RHS of equation (21), which represents the benefit of debt funding to C-bank equity owners.
means that S-bank equity also needs to be less profitable. However, after the increase in $\theta$ and at the old S-bank share, S-bank equity is now more profitable than C-bank equity. Hence households (equity holders) invest in new S-banks, which results in a greater quantity of S-bank deposits produced at constant leverage. As the quantity of S-bank debt rises, the marginal liquidity benefit of S-bank debt declines and S-banks reduce leverage. The declining benefit requires S-banks to pay higher interest on their deposits, which in turn lowers the profitability of S-bank equity until equilibrium is restored.

To summarize, higher capital requirement shrink the C-bank sector and expand the S-bank sector. The policy makes the S-bank sector initially relatively more profitable by reducing C-banks’ access to subsidized deposit funding. In that sense, our model’s prediction lines up with the intuitive reason why regulators fear that tighter capital requirements will cause substitution towards shadow banking. However, our simple model also clarifies that shadow bank incentives for risk taking in the form of higher leverage do not necessarily rise with tighter regulation.

**Welfare.** To understand under which conditions higher capital requirements improve welfare, we first solve for the optimal allocation of capital and leverage of each type of bank from the perspective of a social planner that maximizes household welfare. The planner is restricted to the same resource constraint as the decentralized economy, and has to use the same risky intermediation technology to produce liquidity services. Therefore, numeraire consumption in periods 0 and 1 is restricted by the resource constraints of the decentralized economy in equations (7) and (8). The liquidity production technology implies

\[ A_S = L_S K_S \]  
\[ A_C = L_C K_C = L_C (1 - K_S). \]  

Therefore the planner’s optimization problem is

\[
\max_{K_S, L_S, L_C} \frac{1}{2} \left( 1 - (1 - K_S)L_C^2 - K_S L_S^2 \right) + \psi H (L_S K_S, L_C (1 - K_S)).
\]  

Proposition 3 characterizes the solution to this problem.
Proposition 3. The optimal ratio of S-bank and C-bank capital is given by

\[
\frac{K_S}{K_C} = \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1 - \epsilon}}.
\]

Optimal leverage is equalized across bank types and given by

\[L_S = L_C = L^*,\]

where \(L^*\) is a function of parameters and given in the appendix.

Proof. See appendix A. \(\square\)

Since both banks have equally good technologies for producing liquidity of their own type for a given unit of capital, the planner chooses equal leverage for both. Further, this optimal leverage is equal to the marginal utility of liquidity for each type. The allocation of capital reflects the weight each type of liquidity receives in the utility function. A higher elasticity of substitution \(1/(1 - \epsilon)\) “tilts” the optimal allocation towards the bank type that receives a greater weight in the utility function. Two key properties of the decentralized equilibrium highlight the difference to the planner allocation.

Proposition 4. Index competitive equilibria by the factor \(m > -1\), such that

\[L_C = (1 + m)\tilde{H}_C(R_S),\]

and the function \(\theta = f(m)\) that determines the value of \(\theta\) implementing equilibrium \(m\).

(i) There is no \(\theta \in [0, 1]\) that implements the planner allocation from Proposition 3.

(ii) In any equilibrium with \(m \geq 0\), an increase in the capital requirement \(\theta\) is welfare-improving.

Proof. See appendix A. \(\square\)

The factor \(m\) is the wedge between the social marginal benefit of C-bank liquidity \(\psi\tilde{H}_C(R_S)\), and the cost to society of producing this liquidity \(L_C\). The social planner solution requires that \(m = 0\), such that cost and benefit are equal. In the competitive equilibrium, the C-bank constraint is always binding and consequently \(L_C = (1 - \theta)/2\). A high value of \(m\), resulting from a low capital requirement, implies that C-banks overproduce.
liquidity in the sense that \( L_C > \psi \tilde{H}_C(R_S) \). This is caused by the combination of limited liability and deposit guarantees, which lead to excessive C-bank leverage.\(^{16}\)

What if the regulator chooses \( \theta = f(0) \), such that \( L_C = \psi \tilde{H}_C(R_S) \) and C-banks produce liquidity services efficiently? Part (i) of Proposition 4 states that even in such a case, the competitive equilibrium does not achieve overall efficiency. The reason is deposit insurance for C-banks and competition between S- and C-banks, as formally expressed by condition (21). Since C-banks can issue insured deposits while S-banks cannot, C-banks have a competitive advantage. To compensate for this disadvantage, S-banks always operate at higher leverage than C-banks, which is incompatible with the planner solution. Thus, the capital requirement \( \theta \) is sufficient to align C-bank leverage with the social optimum. However, absent additional policy tools to “regulate” S-banks, it cannot achieve overall efficiency.

Nonetheless, part (ii) of Proposition 4 provides a sufficient condition under which an increase in \( \theta \) improves welfare. In any equilibrium with \( m \geq 0 \), C-banks (weakly) overproduce liquidity, and raising \( \theta \) will shrink the wedge \( m \) towards zero. Even at \( m = 0 \), a marginal increase in \( \theta \) is still unambiguously welfare-improving. The reason is once more competition between both types of banks. As a result of C-banks’ competitive advantage, S-banks’ market share is too small relative to the planner allocation in the decentralized equilibrium. The analysis in Figure 1 shows that raising \( \theta \) will cause an expansion in the S-bank share, moving the allocation of capital closer to the planner solution.

**Decreasing returns in liquidity.** In anticipation of our quantitative results, we generalize the liquidity function to allow for decreasing returns in overall liquidity

\[
H(A_S, A_C) = \frac{(\alpha A_S^e + (1 - \alpha) A_C^e)^{1 - \gamma_H}}{1 - \gamma_H},
\]

parameterized by \( \gamma_H \geq 0 \). The marginal liquidity benefit of S-bank and C-bank liquidity are given by

\[
\psi \mathcal{H}_j(A_S, A_C) = \psi \tilde{H}_j(R_S) \left( (\alpha A_S^e + (1 - \alpha) A_C^e)^{1/2} \right)^{-\gamma_H},
\]

\(^{16}\)We take these basic institutional features as given, with the reasons for their existence outside the model. In the quantitative version of the model, we take into account that S-banks face the risk of large withdrawals (banks runs) due to the lack of deposit insurance.
for $j = S, C$, respectively. For $\gamma_H = 0$, we get the CES case with constant returns in (19), whereas $\gamma_H > 0$ yields decreasing marginal returns in total liquidity $(\alpha A_S^e + (1 - \alpha) A_C^e)^{\frac{1}{\epsilon}}$, as can be seen from the rightmost term in conditions (26).

Analytical characterization of equilibrium in leverage $L_S$ and debt ratio $R_S$ is no longer tractable for this case. However, we can still express equilibrium as two nonlinear functions in two unknowns, leverage $L_S$ and S-bank assets $K_S$. Market clearing and a binding C-bank leverage constraint imply that $A_S = L_S K_S$ and $A_C = \frac{1}{2} (1 - \theta) (1 - K_S)$, yielding the two equations

$$L_S = \psi H_S \left( L_S K_S, \frac{1}{2} (1 - \theta) (1 - K_S) \right), \quad (27)$$

$$L_S = \left( \frac{1}{4} (1 - \theta)^2 + \psi (1 - \theta) H_C \left( L_S K_S, \frac{1}{2} (1 - \theta) (1 - K_S) \right) \right)^{1/2}. \quad (28)$$

The interpretation of (27) and (28) is analogous to (20) and (21), with the asset share $K_S$ replacing the debt ratio $R_S$. We solve the system (27) – (28) numerically. For a wide range of parameter values, allowing decreasing returns leaves the basic properties of equilibrium unchanged.\textsuperscript{17} However, the effect of an increase in the capital requirement $\theta$ is now ambiguous: as with $\gamma_H = 0$, a higher capital charge reduces C-bank liquidity, which in turn lowers the marginal benefit of S-bank liquidity. But with $\gamma_H > 0$, the reduction in C-bank liquidity also causes a rise in the marginal benefit of aggregate liquidity $(\alpha A_S^e + (1 - \alpha) A_C^e)^{\frac{1}{\epsilon}}$, effectively raising demand for both types of liquidity. If the second effect is strong enough, it can dominate the first effect and both the leverage and zero-profit schedule in Figure 1 shift upwards. As a result, both equilibrium S-bank leverage and share increase. More generally, the model with $\gamma_H > 0$ can generate any combination of S-bank leverage and asset share changes in response to a higher capital requirement, depending on parameter values. The response of the financial sector to changes in regulation is therefore a quantitative question in the model with more general preferences over liquidity.

3 The quantitative model

Building on the static model, this section presents the quantitative model that we take to the data.

\textsuperscript{17}For very large values of $\gamma$, the zero-profit condition becomes non-monotonic and an equilibrium with both sectors holding a positive amount of capital may not exist.
Time is discrete and infinite. The household sector owns a Lucas tree (non-bank dependent sector) and all claims on banks. Households value consumption and liquidity services according to the utility function

\[
U \left( C_t, H \left( A_t^S, A_t^C \right) \right) = \frac{C_t^{1-\gamma}}{1-\gamma} + \frac{\psi \left[ \alpha (A_t^S)^{\epsilon} + (1-\alpha)(A_t^C)^{\epsilon} \right]^{1-\gamma_H}}{1-\gamma_H},
\]

(29)

where \( \gamma \) is the inverse of the intertemporal elasticity of substitution for consumption. The preferences for liquidity are the same as in the simple model (equation (25)), with the meaning of liquidity parameters \( \psi, \alpha, \gamma_H, \) and \( \epsilon \) explained in section 2.

Two types of banks, C-banks and S-banks, provide liquidity services to households by issuing deposits under limited liability. Banks operate a Cobb-Douglas production technology combining capital and labor. As in the simple model, C-banks have capital requirements and deposit insurance, while S-banks have neither.

To move closer to the data, we introduce capital- and investment adjustment costs, as well as stochastic deposit redemption shocks (bank runs) for S-banks. The main trade-off in the quantitative model is between liquidity provision and financial fragility as discussed in section 2.

### 3.1 Production Technology

There is a continuum of mass one of each type of bank, \( j = C, S \). Each bank owns productive capital \( \hat{K}_t^j \) at the beginning of the period. Even though banks receive idiosyncratic productivity shocks, we can solve the problem of a representative bank of each type. We provide details on banks’ intertemporal optimization problem and aggregation in sections 3.2 and 3.3 below.

Banks hire labor \( N_t^j \) from households at competitive wage \( w_t \) and combine it with their capital to produce

\[
Y_t^j = Z_t \left( \hat{K}_t^j \right)^{1-\eta} \left( N_t^j \right)^{\eta},
\]

where \( \eta \) is the labor share and \( Z_t \) is an aggregate productivity shock common to all banks. After production, capital depreciates at rate \( \delta_K \). Banks can also invest using a standard convex technology. Creating \( I_t^j \) units of the capital good requires

\[
I_t^j + \frac{\phi_t}{2} \left( \frac{I_t^j}{\hat{K}_t^j} - \delta_K \right)^2 \hat{K}_t^j.
\]
units of consumption. Capital trades in a competitive market among all banks at price \( p_t \). Defining the investment rate \( i_t^j = \frac{I_t^j}{K_t^j} \) and the labor-capital ratio \( n_t^j = \frac{N_t^j}{K_t^j} \), the gross payoff per unit of capital is

\[
\Pi_t^j = Z_t(n_t^j)\eta + (1 - \delta_K) p_t - w_t n_t^j + i_t^j (p_t - 1) - \frac{\phi_t}{2} (i_t^j - \delta_K)^2
\]

\[= (1 - \eta) Z_t(n_t^j)\eta + p_t - \delta_K + \frac{(p_t - 1)^2}{2\phi_t}, \tag{30}\]

which already embeds banks’ optimal labor demand and investment decisions. Households can also hold capital and produce directly. However, they have an inferior ability to operate the asset, leading to lower productivity \( Z_t < Z_t^H \) and a higher depreciation rate \( \delta_K > \delta_K^H \). They further do not have access to an investment technology. When households hold capital and produce, the gross payoff per unit is hence

\[
\Pi_t^{H} = (1 - \eta) Z_t(n_t^{H})\eta + p_t (1 - \delta_K). \tag{31}\]

### 3.2 S-banks

In addition to labor input and investment, each period S-banks choose the amount of capital to purchase for next period \( K_{t+1}^S \) and the amount of deposits to issue to households \( B_{t+1}^S \) at price \( q_t^S \).

**Bank Runs and Timing** To capture the fragility of S-banks, we introduce bank runs in the S-bank sector similar to Allen and Gale (1994). A fraction of shadow bank deposits \( q_t \) is withdrawn early within a given period (affecting all shadow banks equally), where \( q_t \in \{0, q^* > 0\} \), following a two-state Markov chain. When deposits are withdrawn, shadow banks need to liquidate a fraction of their assets to pay out depositors. Assets that are liquidated early are sold to households at price \( \Pi_t^{H} \) defined in equation (31) and do not yield any output to the bank. Households sell the assets again in the regular capital market later in the same period.\(^\text{18}\)

The timing of decisions within each period is now as follows:

1. Aggregate productivity shocks \( Z_t, Z_t^H \) and the early withdrawal shock \( q_t \) are realized.

\(^{18}\text{Since both transactions take place within the same period and households are unconstrained, it immediately follows that } \Pi_t^{H} \text{ is the marginal value households attach to capital. The marginal product of capital to households is always lower than that of banks, so households never optimally own any capital at the end of the period.}\)
2. If $\varrho_t = \varrho^*$, S-banks sell capital worth $\varrho^* B^S_t$ to households at price $\Pi^H_t$.

3. Production of all banks and households and investment decisions of banks ensue.

4. Idiosyncratic payoff shocks of banks are realized. Default decisions.

5. Trade occurs in asset markets. Surviving banks pay dividends and new banks are set up to replace liquidated bankrupt banks.

6. Households consume.

To pay out its depositors in case of a withdrawal shock ($\varrho^S_t = \varrho^*$) at stage 2, the fraction of assets that needs to be liquidated is

$\ell^S_t = \frac{\varrho^S_t B^S_t}{K^S_t \Pi^H_t}$.

Thus, the capital available for production at stage 3 is $\hat{K}^S_t = (1 - \ell^S_t) K^S_t$.

**S-banks** In appendix B.1, we show that at the time banks choose their new portfolio (at step 5 of the intraperiod sequence of events), all banks have the same value and face the same optimization problem. The two properties of the bank problem sufficient to obtain this aggregation result are that (i) idiosyncratic profit shocks $\rho^S_t \sim F^S$ are uncorrelated over time, and (ii) the value function of S-banks is homogeneous in capital. We use these properties to write the value of a bank with capital $\hat{K}^S_t$, $V^S(\hat{K}^S_t, Z_t)$, in terms of the value per unit of capital $v^S(Z_t) = V^S(\hat{K}^S_t, Z_t)/\hat{K}^S_t$. The two intertemporal choices are the deposit-capital ratio $b^S_{t+1} = \frac{B^S_{t+1}}{K^S_{t+1}}$ and capital growth $k^S_{t+1} = \frac{K^S_{t+1}}{K^S_t}$, which is subject to quadratic adjustment costs $\phi_K (k^S_{t+1} - 1)^2 / 2$. Defining leverage

$L^S_t = \frac{b^S_t}{\Pi^S_t}$,

we write the bank problem as

$v^S(Z_t) = \max_{k^S_{t+1} \geq 0, b^S_{t+1} \geq 0} k^S_{t+1} \left( q_S(b^S_{t+1}) b^S_{t+1} - p_t \right) - \frac{\phi_K}{2} (k^S_{t+1} - 1)^2 + k^S_{t+1} E_t \left[ M_{t,t+1} \Pi^S_{t+1} \Omega^S(L^S_{t+1}) \right]$, (32)

where $M_{t,t+1}$ is the stochastic discount factor of households. The function $\Omega^S(L^S_t)$, defined in equation (50) in appendix B.1, reflects the bank’s continuation value including
the option to default and possible losses from bank runs and depends on the capital structure choice \( b_{t+1}^S \) through leverage \( L_{t+1}^S \). Intuitively, banks choose asset growth \( k_{t+1}^S \) and capital structure \( b_{t+1}^S \) to maximize shareholder value under limited liability. As in the simple model of section 2, S-banks internalize that the price of their deposits, \( q_S(b_{t+1}^S) \), is a function of their default risk and thus their capital structure. S-banks optimally default at stage 4 in the intraperiod time line when \( \rho_t^S < \hat{\rho}_t^S \), with

\[
\hat{\rho}_t^S = \frac{L_t^S - (1 - \ell_t^S) \frac{v(Z_t)}{\Pi_t^S} - \delta_S}{1 - \ell_t^S (1 - \Pi_t^H / \Pi_t^S)},
\]

(33)

where \( \delta_S \geq 0 \) is a default penalty parameter. The probability of default is thus \( F_{\rho,t}^S \equiv F^S(\hat{\rho}_t^S) \). We derive Euler equations for S-banks in appendix B.3.

### 3.3 C-banks and Government

**C-banks.** The problem of C-banks is analogous to S-banks, but they differ from S-banks in four ways: (i) they issue short-term debt that is insured and risk free from the perspective of creditors, (ii) they do not experience runs (as result of (i)), (iii) they are subject to regulatory capital requirements, and (iv) they pay an insurance fee of \( \kappa \) for each bond they issue. Using the same notation as for S-banks, C-banks solve

\[
v^C(Z_t) = \max_{b_{t+1}^C \geq 0, k_{t+1}^C \geq 0} k_{t+1}^C \left( (q_t^C - \kappa) b_{t+1}^C - p_t \right) - \frac{\Phi_K}{2} \left( k_{t+1}^C - 1 \right)^2
\]

\[
E_t \left[ M_{t,t+1} k_{t+1}^C \Pi_{t+1}^C \max \left\{ \rho_{t+1}^C - L_{t+1}^C + \frac{v^C(Z_{t+1})}{\Pi_{t+1}^C}, -\delta_C \right\} \right],
\]

(34)

subject to the capital requirement

\[
(1 - \theta) p_t \geq b_{t+1}^C.
\]

(35)

C-banks optimally default at stage 4 in the intraperiod time line when \( \rho_t^C < \hat{\rho}_t^C \), with

\[
\hat{\rho}_t^C = L_t^C - \frac{v^C(Z_t)}{\Pi_t^C} - \delta_C,
\]

(36)

where \( \delta_C \geq 0 \) is a default penalty parameter. The probability of default is thus \( F_{\rho,t}^C \equiv F^C(\hat{\rho}_t^C) \). We state the full optimization problem of C-banks including Euler equations in appendix B.4.
Bankruptcy, Bailout and Government Budget Constraint. If a bank declares bankruptcy, its equity (and continuation value) becomes worthless, and creditors seize all of the bank’s assets, which are liquidated. The recovery amount per bond issued is

\[ r_i^j = \frac{\rho_i^{j^-} \left( 1 - \ell_i^j \left( 1 - \frac{\Pi_i^j}{\Pi_i^t} \right) \right)}{L_i^j}, \]

for \( j = S, C \). A fraction \( \xi^j \) of assets is lost in the bankruptcy proceedings, with \( \rho_i^{j^-} \equiv \mathbb{E} \left( \rho_i^j | \rho_i^j < \rho_i^j \right) \) being the average idiosyncratic shock of defaulting banks. Since C-banks do not experience runs, \( \ell_i^C = 0 \ \forall \ t \). Bankruptcy losses \( \xi^j \rho_i^{j^-} \left( \ell_i^j \Pi_i^H + \left( 1 - \ell_i^j \right) \Pi_i^j \right) K_i^j \) are real losses to the economy. They reflect both greater capital depreciation of foreclosed banks, and real resources destroyed in the bankruptcy process that reduce bank profits.

After the bankruptcy proceedings are completed, a new bank is set up to replace the failed one. This bank sells its equity to new owners, and is otherwise identical to a surviving bank after asset payoffs.

If a S-bank defaults, the recovery value per bond is used to pay the claims of bondholders to the extent possible. We further consider the possibility that the government bails out the bond holders of the defaulting S-bank with a probability \( \pi_B \), known to all agents ex-ante. If a C-bank declares bankruptcy, the bank is taken over by the government that uses lump-sum taxes and revenues from deposit insurance, \( \kappa B_{i+1}^C \), to pay out the bank’s creditors in full. Summing over defaulting C-banks and S-banks that are bailed out, we define lump sum taxes as

\[ T_i = F_{\rho_i}^C \left( 1 - r_i^C \right) B_i^C - \kappa B_{i+1}^C + \pi_B F_{\rho_i}^S \left( 1 - r_i^S \right) B_i^S. \]

3.4 Households and Equilibrium

Households. Each period, households receive an endowment from a Lucas tree \( Y_t \) and the payoffs from owning all equity and debt claims on intermediaries, yielding financial wealth \( W_t \). They further inelastically supply their unit labor endowment at wage \( w_t \) and pay lump-sum taxes \( T_t \). Households choose consumption \( C_t \), deposits of both banks for redemption next period, \( A_{i+1}^S \) and \( A_{i+1}^C \), and bank equity purchases \( S_i^S \) and \( S_i^C \), to maximize utility (29) subject to their intertemporal budget constraint

\[ W_t + Y_t + w_t - T_t \geq C_t + \sum_{j=S,C} p_i^j S_i^j + \sum_{j=S,C} q_i^j A_{i+1}^j, \] (37)
where \( p^j_t, j = S, C \), denote the market price of bank equity of type \( j \). As for banks, we state the full optimization problem of household including Euler equations in appendix B.2.

**Equilibrium.** The full definition of competitive equilibrium is provided in appendix B.5. Market clearing requires that households purchase all securities issued by banks, which implies \( B^j_{t+1} = A^j_{t+1} \), for \( j = S, C \), in deposit markets, and \( S^j_t = 1 \) in equity markets. Labor supply by households has to equal labor demand by banks, and by producing households in case of fire sales, implying \( N^S_t + N^C_t + N^H_t = 1 \). The market clearing conditions for capital and consumption are provided in appendix B.5. In the capital market, bank failures lead to endogenous depreciation in addition to production-induced depreciation \( \delta_K \). Similarly, bank failures also cause a loss of resources in the goods market. Appendix B.6 lists the full set of nonlinear equations characterizing the equilibrium.

### 3.5 Stochastic Environment and Solution Method

**Stochastic Processes.** The stochastic process for the Y-tree (not intermediated by banks) is an AR(1) in logs

\[
\log(Y_{t+1}) = (1 - \rho_Y)\log(\mu^Y) + \rho_Y\log(Y_t) + \epsilon^Y_{t+1},
\]

where \( \epsilon^Y_t \) is i.i.d. \( \mathcal{N} \) with mean zero and volatility \( \sigma^Y \). To capture the correlation of asset payoffs with fundamental income shocks, we model the payoff of the intermediated asset as

\[
Z_t = v^Z Y_t \exp(\epsilon^Z_t),
\]

where \( \epsilon^Z_t \) is i.i.d. \( \mathcal{N} \) with mean zero and volatility \( \sigma^Z \), independent of \( \epsilon^Y_t \), and \( v^Z > 0 \) is a parameter. This structure of the shocks implies that \( Z_t \) inherits all stochastic properties of aggregate income \( Y_t \) and is subject to a temporary shock reflecting risks specific to intermediated assets, such as credit risk.

**Solution Method.** We solve the dynamic model using nonlinear methods. To this end, we write the equilibrium of the economy as a system of nonlinear functional equations of the state variables, with the unknown functions being the agents’ choices, the asset prices, and the Lagrange multiplier on the C-bank’s leverage constraint. We parametrize these functions using splines and iterate on the system until convergence. We check the relative Euler equation errors at the solution we obtain to make sure the unknown functions are
well approximated. We then simulate the model for many periods and compute moments
of the simulated series.

The model features three exogenous state variables, the stochastic endowment \( Y_t \), pro-
ductivity \( Z_t \), and the run shock \( \varrho_t \). These shocks are jointly discretized as a first-order
Markov chain with three nodes for \( Y_t \) and three nodes for \( Z_t \). We assume that runs only
occur in low productivity states, yielding a total of 12 different discrete states.

The endogenous state variables are (1) the aggregate capital stock \( K_t = K^C_t + K^S_t \), the
outstanding amount of bank debt of each type (2) \( B^C_t \) and (3) \( B^S_t \), and the share of the
capital stock held by S-banks (4) \( K^S_t / K_t \). Appendix B.7 contains a description of the com-
putational solution method.

4 Mapping the model to the data

In this section, we discuss the parametrization of the model and its fit with the data.

Our calibration relies on various data sources, including the data from bank call reports
collected by the Federal Reserve Board and the Federal Deposit Insurance Corporation,
the Flow of Funds, Compustat and NIPA. We match our model to quarterly data from
1999 Q1 to 2017 Q4. We choose 1999 as the start date because it marked the passage
of the Gramm-Leach-Bliley Act that deregulated the banking sector. For example, this
legislation removed the mandated separation between commercial and investment banks.
For some calibration targets, we use longer time series to reduce measurement noise (for
example, if only annual data are available).

We organize the description of our parametrization into four parts: (1) parameters
governing bank leverage and default, (2) liquidity preference parameters, (3) bank-run
related parameters, and (4) all remaining parameters. This section discusses how we
select the first three sets of parameters (see Table 1), while Appendix C.1 contains the
description of the remaining parameter choices.

Bank leverage and defaults. Banks have the option to default. The default penalties
\( \delta_j \) with \( j \in \{C, S\} \) determine the default threshold. Typically the default threshold is
assumed to be zero with the reasoning that default occurs whenever equity holders are
wiped out. Mapping this concept precisely to the data is difficult because the distressed
firm’s franchise value is often difficult to measure. We therefore choose to set \( \delta_C \) and \( \delta_S \) to
match the default rates of the assets held by commercial and shadow banks, respectively.
Table 1: Parametrization

<table>
<thead>
<tr>
<th>Values</th>
<th>Target Data Model</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bank leverage and default</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>0.300 Quarterly corp. bond default rate</td>
<td>0.36%</td>
<td>0.31%</td>
</tr>
<tr>
<td>$\delta_C$</td>
<td>0.175 Quarterly net loan charge-offs</td>
<td>0.25%</td>
<td>0.26%</td>
</tr>
<tr>
<td>$\xi_C$</td>
<td>0.515 Recovery rate Moody’s</td>
<td>63%</td>
<td>62%</td>
</tr>
<tr>
<td>$\xi_S$</td>
<td>0.415 Recovery rate Moody’s</td>
<td>63%</td>
<td>63%</td>
</tr>
<tr>
<td>$\pi_B$</td>
<td>0.905 Shadow bank leverage</td>
<td>93%</td>
<td>93%</td>
</tr>
</tbody>
</table>

| Liquidity preferences                     |           |            |
| $\beta$     | 0.993 C-bank debt rate              | 0.39%  | 0.39%  |
| $\alpha$    | 0.330 Shadow banking share; Gallin (2015) | 35%  | 34%  |
| $\psi$      | 0.0103 Liquidity premium C-banks; KV2012 | 0.18%  | 0.17%  |
| $\gamma_H$  | 1.700 Corr(GDP, C-bank liquid. premium) | -0.28 | -0.39 |
| $\epsilon$ | 0.420 S-bank liquidity elasticity   | 0.17%  | 0.16%  |

| Runs                               |           |            |
| $\delta_K$ | 0.10 Max. haircut non-subprime; Gorton and Metrick (2009) | 20%  | 19%  |
| $Z$       | 26% × Z Real asset foreclosure discount; Campbell et al. (2011) |           |            |
| $\theta$  | [0, 0.3] Fraction of households run; Covitz et al. (2013) |           |            |
| Prob$_\theta$ | $\begin{bmatrix}0.97 & 0.03 \ 0.33 & 0.67\end{bmatrix}$ | Uncond. run prob. | 3%  | 3%  |

In case of $\delta_C$, we match the charge-off rate on loans held by commercial banks, and for the case of $\delta_S$, we match the default rate on corporate bonds.\textsuperscript{19}

The bankruptcy costs parameters $\xi_j$ with $j \in \{C, S\}$ determine how much of banks’ asset value can be recovered to pay out their creditors in case of default. We choose these parameters to match Moody’s financial sector corporate bond recovery rate of 63%.\textsuperscript{20}

The shadow bank bailout probability $\pi_B$ affects the optimal leverage of S-banks. A high value of $\pi_B$ means that a large fraction of S-bank debt is insured. For this reason, creditors do not fully price the default risk of S-banks, lowering S-banks’ incentives to internalize default costs. S-banks can then increase their equity valuation by increasing their leverage. We choose $\pi_B$ to match the book value leverage of shadow banks in Com-

\textsuperscript{19}Loan charge-offs are from Federal Reserve Board data, and corporate bond defaults are from the S&P 2017 Annual Global Corporate Default Study. We use 1985-2017 data for both targets, the longest available series. Since our model consolidates bank-dependent producers and banks, it does not explicitly include corporate loans or bonds. To capture the fundamental source of risk from the intermediation activity of C- and S-banks, we calibrate the default rates of both types to the risk of the key assets they own.

\textsuperscript{20}We use Moody’s 1984-2004 reports. Exhibit 9 in the report presents the recovery rates of defaulted bonds for financial institutions. We use the average for financial institutions over all bonds and preferred stocks.
To this end, we compute the value weighted leverage of publicly traded shadow banks as debt over assets weighted by the relative market value of each institution and average across time and banks.\textsuperscript{21} This procedure leads to a value-weighted book leverage ratio of 93%.

**Liquidity Preferences.** The parametrization of households’ liquidity preferences is at the core of our quantitative model as they admit a range of relationships between commercial bank- and shadow bank liquidity services.

Our model determines two interest rates, one for C-banks and one for S-banks, which can be understood from the household Euler equations (9) and (10) in the simple model, with their quantitative counterparts (56) and (57) in appendix B.2. Both rates are affected by the representative consumer’s stochastic discount factor, and both contain a liquidity premium. In addition, the S-bank rate reflects the default risk of S-banks.

To compute the liquidity premium in our model, it is useful to define the price of a hypothetical asset

$$\hat{q}_t = E_t \{ M_{t,t+1} \},$$

which is a short-term riskfree bond without any liquidity benefits.

The most direct measure of the liquidity premium in the model is the marginal benefit of C-bank liquidity $q^C_t - \hat{q}_t = E_t \{ M_{t,t+1} \text{MRSC}_t \},$ with $\text{MRSC}_{t+1}$ as defined in equation (55). The weight on liquidity services $\psi$ in the utility function directly scales this liquidity premium. Unfortunately, a measure for $q^C - \hat{q}$ is difficult to obtain in the data, since almost all short-term safe interest rates convey some form of liquidity benefit. Krishnamurthy and Vissing-Jorgensen (2012) estimate a liquidity premium of 73 bps per annum (18 bps quarterly) based on the spread between the yield on commercial paper (CP) and T-bills for a long time series. We choose $\psi$ such that the marginal value of commercial bank liquidity matches this premium (net of the deposit insurance fee).\textsuperscript{22} This calibration strategy relies on the assumption that T-bills and commercial bank deposits are close substitutes in terms of their liquidity benefits, as has been argued by Nagel (2016) among others. It further requires that the CP yield is close to a “pure” riskfree rate without liq-

\textsuperscript{21} We define shadow banks as all institutions with SIC codes 6111-6299, 6798, 6799, 6722, 6726, excluding SIC codes 6200, 6282, 6022, 6199.

\textsuperscript{22} C-banks pay a deposit insurance fee $\kappa_C$ for each bond they issue, with more details on the calibration of $\kappa_C$ below. In equilibrium, they pass on this fee to consumers in the form of lower deposit rates. The fee is what makes bank deposits free of credit-risk and comparable to T-bills. Further, T-bill investors do not pay such a fee (directly or indirectly). This is why we match the premium measured by Krishnamurthy and Vissing-Jorgensen (2015) for T-bills to $\text{MRSC}_{t} - \kappa_C$ in the model.
uidity benefits, which is less likely to be true. To the extent that commercial paper also provides liquidity benefits and is closer in spirit to the S-bank interest rate in our model, the calibration of $\psi$ is a lower bound on the level of the liquidity premium of deposits.

The parameter $\beta$ governs households’ time preferences and the level of the C- and S-bank interest rates for a given liquidity premium. We choose $\beta$ to match the C-bank rate in the model to the deposit rate in the data, calculated as the total interest expense on deposits at the end of period $t$, divided by total amount of deposits at the beginning of period $t$. We use call report data for this calculation.

The parameter $\alpha$ is the weight on shadow bank liquidity services in households’ preferences and therefore governs how much shadow bank debt contributes to aggregate liquidity. This in turn determines the relative size of the shadow banking sector. We calibrate $\alpha$ to the share of shadow bank funding of real production activity, as estimated by Gallin (2015).23

The curvature parameter $\gamma_H$ determines how the marginal value of liquidity moves with the total amount of liquidity. When the marginal value of liquidity decreases in the amount of liquidity provision ($\gamma_H > 0$), households how a downward-sloping demand curve for liquidity. Since good economic times are typically characterized by an abundance of liquidity, we can use the correlation between GDP and the liquidity premium as a target for $\gamma_H$. Consistent with the calibration of $\psi$ discussed above, we define the liquidity premium as the difference between the rate on 3-month AA rated financial commercial paper and the rate on 3-month Tbills.24 Over our sample period, the correlation of real per capita GDP growth with this liquidity premium is -0.28. We use this value as our calibration target. The value of $\gamma_H = 1.7$ minimizes the distance between this data target and its model analogue (the correlation between GDP growth and the liquidity premium on C-bank debt).

The parameter $\epsilon$ determines the elasticity of substitution between S-bank and C-bank debt. To calibrate $\epsilon$, we target the effect of changes in the supply of S-bank debt on the spread between both interest rates. We derive an equation for the S–C debt spread by log-

---

23 Alternatively, we could have used the share of liquid shadow bank debt (i.e., money market mutual fund shares, REPO funding, and short term commercial paper) relative to the sum of liquid shadow bank debt and commercial bank deposits. The average share is 38% over our sample period using Flow of Funds data and therefore close to the 35% estimate by Gallin (2015).

24 Instead of the Tbill rate, we could have directly used the bank deposit rate that we computed as target for the C-bank rate. However, the combination of accounting rules and lower reporting frequency means that bank accounting data lags behind market data. Any sensitivity measure based on deposits is therefore downward biased. To avoid this problem, we follow Nagel (2016), who finds deposits and Tbills to be near perfect substitutes in terms of their liquidity premia, and use the Tbill rate instead of the deposit rate in our definition of the liquidity premium.
linearizing households’ combined first-order conditions for liquidity holdings for both bank types, with details given in Appendix C.2. Based on the equation, imperfect substitutability between both types ($\epsilon < 1$) implies that an increase in S-bank (C-bank) debt supply should widen (compress) the spread, everything else equal. The intuition is simple: if both types of liquidity are imperfect substitutes, then the liquidity benefit provided by each type does not only depend on the total amount of debt, but also on the individual supply of each type. Decreasing returns in each type of liquidity then imply that a greater supply of S-bank debt will lower the liquidity premium earned by this debt, such that the spread will widen, and vice versa for C-banks. If both types were perfect substitutes, then the spread would not depend on the supply of each type of bank, since both banks earn a premium that only depends on the total supply.

Hence, controlling for S-bank default risk, in a regression of the spread on the supply of each type, the coefficient on each type is informative about $\epsilon$. We run such a regression in both real and model-generated data, and set $\epsilon$ to match the coefficient on S-bank debt supply in the model regression to the corresponding coefficient in the data regression. In the data, we regress the CP–Tbill spread described above on the logarithms of Tbill supply divided by GDP and a measure of short-term money market debt divided by GDP, while including the VIX to control for default risk. In the model, we run analogous regressions, directly controlling for the S-bank default rate.25

Consistent with our theory, the CP–Tbill spread in the data reacts positively to an increase in money market debt supply, with a coefficient of 20bp. The model matches this coefficient with a value of $\epsilon = 0.42$, implying a relatively high degree of substitutability. The coefficient means that a one percent rise in the supply of S-bank debt leads to an increase of the spread by 20bp. Regression results are available in appendix C.2.

**Bank runs.** The introduction of bank runs to the dynamic model is a significant departure from the simple two-period model outlined in Section 2. Bank runs make shadow banks more risky compared to commercial banks.

During the run-state the capital stock of affected shadow banks is transferred to households. We assume that the depreciation rate of the capital stock held by households is four times higher than when capital is held by banks, implying a value of run-state depreci-

25Neither the Tbill nor the CP yield are direct measures of C-bank and S-bank interest rates in the sense of our model. However, we already argued above that Tbills are likely close enough substitutes to deposits, and commercial paper is an important intermediate asset in shadow bank liquidity production. Thus we believe that the sensitivity of the data spread to the respective supply of each security is a good approximation of the sensitivities in our model (even though the level of the CP–Tbill spread is not a good measure of the level of the S–C model spread).
This figure presents the impulse response functions to a productivity shock (in black) and a productivity shock together with a run shock (red). The x-axis denotes quarters. The shocks occur in the first quarter. The y-axis denotes percentage deviations from the stationary equilibrium.

ation of $\delta_K = 0.10$. When households hold the financial assets of run-impaired shadow banks the productivity of financial assets falls to 26%, consistent with the foreclosure discounts documented by Campbell, Giglio, and Pathak (2011). Together this implies a maximum haircut of 20% for non-subprime assets as documented by Gorton and Metrick (2009). We set the fraction of households that run in the run-state to 30% (Covitz, Liang, and Suarez (2013)). The probability of entering a bank run state from a non-bank run state is set to 3% and the probability of staying in a bank run state is 67%, the latter implies that the average crisis lasts for about three quarters.

How bad are bank runs for the economy? Figure 2 compares the impulse response functions of key model variables to a typical productivity crisis (in black) with the impulse response functions to a productivity crisis coupled with a bank run (in red). They show that a shadow bank run significantly worsens recessions, leading to higher losses in output, consumption, and investment. This is summarized by 10 percentage points.

See the haircut for non-subprime bonds during the crisis in Figure 2 in Gorton and Metrick (2009).
higher deadweight losses. A bank run forces shadow banks to delever, resulting in a liquidity crunch. The lower productivity of physical capital during a run reduces the value of the intermediated assets, making investments less attractive.

5 Effect of Bank Capital Requirements

In this section, we answer two questions. (1) How do static bank capital requirements affect the economy, and (2) what is the value of the optimal capital requirement? We later discuss a range of alternative regulatory policies in Section 6.

The optimal static capital requirement. How do higher capital requirements affect liquidity provision and do they improve overall financial stability? A safer financial system is naturally the desired outcome of tighter bank regulation since the 2008 financial crisis. But what if tighter bank regulation shifts activity to the shadow banking sector? We answer this question by solving our model numerically and simulating the economy for 5,000 periods under different levels of commercial bank capital requirements. All other parameters stay at their benchmark level. The results are in Table 2 and Figure 3.

Figure 3: Welfare in consumption equivalent units

![Figure 3: Welfare in consumption equivalent units](image)

This figure plots the welfare gain relative to the benchmark ($\theta = 10\%$) in consumption equivalent units over a range of different values for $\theta$ capital requirements.

We calculate welfare based on households’ value function. Figure 3 shows that the optimal capital requirement level is around 17%. This level trades off an increase in con-
sumption against a reduction in liquidity services (see Table 2). For capital requirement levels exceeding 17%, the loss in liquidity services exceeds 5%, which outweights the increase in consumption. This is why welfare declines after 17%. The increase in consumption is driven by (i) a reduction in commercial bank defaults that lowers deadweight losses and (ii) an increase in GDP that is caused by an increase in the capital stock. As in the simple model, the capital requirement is binding (see equation 15 in Section 2), because deposits are insured and enjoy a liquidity premium. A higher capital requirement tightens this constraint and consequently forces commercial banks to lower their leverage. While lower leverage makes commercial banks safer, it also restricts their ability to produce liquidity. The presence of shadow banks means that, in principle, aggregate liquidity would not need to fall when commercial banks lower their supply of liquidity services. Indeed, shadow banks expand their liquidity production by 6.2% when the capital requirement is increased to 17%. This is driven by an increase in leverage (+.40%) and in the size of S-banks’ balance sheet (+1.2%). However, this increase in S-bank liquidity production does not compensate for the loss in C-bank liquidity services. Thus, aggregate liquidity services fall.

Do higher capital requirements make the system overall safer? This question is of great concern to policy makers who worry about the unintended consequences of tighter regulation. Undoubtedly, regulated C-banks get safer with tighter regulation as highlighted by the reduction in deadweight losses and leverage. The question is rather how shadow banks are going to react. If S-banks are more fragile, an expansion of the shadow banking sector could undo the gains in financial stability caused by higher restrictions on C-banks’ leverage. Table 2 shows that shadow banks indeed partially fill the void by providing more liquidity. They do this by both expanding their balance sheet (both aggregate capital and the S-bank capital share increase) and increasing leverage. Higher leverage makes S-banks riskier. The deadweight losses caused by S-banks increase by 11.7%, which however does not offset the 94.6% reduction in C-bank deadweight losses. Table 2 shows that even though S-banks take up a higher share of financial intermediation activity and become riskier themselves, the net effect of higher capital requirements still improves overall financial stability.

To get a better sense for what happens on the transition to the new capital requirement regime, we plot the transition paths in Figures 4 and 5 of key model variables from the benchmark capital requirement of 10% to the optimal capital requirement of 17%. Liquidity drops sharply the moment the new capital requirement is introduced and slowly increases in the following quarters until it reaches its new stationary equilibrium level that is 5.2% below the benchmark level. The S-bank debt share transition mirrors aggre-
<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>13%</th>
<th>15%</th>
<th>17%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capital and Debt</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>4.005</td>
<td>+0.2%</td>
<td>+0.4%</td>
<td>+0.6%</td>
<td>+0.9%</td>
<td>+1.4%</td>
<td>+2.2%</td>
</tr>
<tr>
<td>Debt share S</td>
<td>0.349</td>
<td>+3.4%</td>
<td>+4.9%</td>
<td>+6.2%</td>
<td>+8.1%</td>
<td>+11.7%</td>
<td>+16.0%</td>
</tr>
<tr>
<td>Capital share S</td>
<td>0.342</td>
<td>+1.1%</td>
<td>+1.0%</td>
<td>+0.6%</td>
<td>-0.1%</td>
<td>-1.0%</td>
<td>-1.7%</td>
</tr>
<tr>
<td>Leverage S</td>
<td>0.933</td>
<td>+0.1%</td>
<td>+0.3%</td>
<td>+0.4%</td>
<td>+0.6%</td>
<td>+0.9%</td>
<td>+1.0%</td>
</tr>
<tr>
<td>Leverage C</td>
<td>0.900</td>
<td>-3.3%</td>
<td>-5.6%</td>
<td>-7.8%</td>
<td>-11.1%</td>
<td>-16.7%</td>
<td>-22.4%</td>
</tr>
<tr>
<td>Early Liquidation (runs)</td>
<td>0.004</td>
<td>+0.3%</td>
<td>+0.5%</td>
<td>+0.8%</td>
<td>+1.2%</td>
<td>+1.8%</td>
<td>+2.3%</td>
</tr>
<tr>
<td><strong>Prices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deposit rate S</td>
<td>0.49%</td>
<td>-0.7%</td>
<td>-1.3%</td>
<td>-2.0%</td>
<td>-3.0%</td>
<td>-4.8%</td>
<td>-6.5%</td>
</tr>
<tr>
<td>Deposit rate C</td>
<td>0.39%</td>
<td>-4.1%</td>
<td>-6.6%</td>
<td>-9.1%</td>
<td>-13.0%</td>
<td>-20.3%</td>
<td>-28.6%</td>
</tr>
<tr>
<td>Convenience Yield S</td>
<td>0.23%</td>
<td>+2.0%</td>
<td>+3.6%</td>
<td>+5.3%</td>
<td>+8.0%</td>
<td>+12.7%</td>
<td>+17.8%</td>
</tr>
<tr>
<td>Convenience Yield C</td>
<td>0.31%</td>
<td>+5.2%</td>
<td>+8.4%</td>
<td>+11.5%</td>
<td>+16.4%</td>
<td>+25.4%</td>
<td>+36.2%</td>
</tr>
<tr>
<td><strong>Welfare</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DWL S</td>
<td>0.001</td>
<td>+5.2%</td>
<td>+8.5%</td>
<td>+11.7%</td>
<td>+16.7%</td>
<td>+25.1%</td>
<td>+31.8%</td>
</tr>
<tr>
<td>DWL C</td>
<td>0.003</td>
<td>-68.6%</td>
<td>-86.5%</td>
<td>-94.6%</td>
<td>-98.8%</td>
<td>-99.9%</td>
<td>-100.0%</td>
</tr>
<tr>
<td>GDP</td>
<td>1.365</td>
<td>+0.0%</td>
<td>+0.0%</td>
<td>+0.1%</td>
<td>+0.1%</td>
<td>+0.1%</td>
<td>+0.2%</td>
</tr>
<tr>
<td>Liquidity Services</td>
<td>1.969</td>
<td>-2.3%</td>
<td>-3.8%</td>
<td>-5.2%</td>
<td>-7.3%</td>
<td>-10.8%</td>
<td>-14.4%</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.261</td>
<td>+0.13%</td>
<td>+0.17%</td>
<td>+0.19%</td>
<td>+0.20%</td>
<td>+0.21%</td>
<td>+0.24%</td>
</tr>
<tr>
<td>Vol(Liquidity Services)</td>
<td>0.068</td>
<td>-2.5%</td>
<td>-4.3%</td>
<td>-6.1%</td>
<td>-8.9%</td>
<td>-13.6%</td>
<td>-18.7%</td>
</tr>
<tr>
<td>Vol(Consumption)</td>
<td>0.005</td>
<td>+0.4%</td>
<td>+0.5%</td>
<td>+0.6%</td>
<td>+0.7%</td>
<td>+2.1%</td>
<td>+10.5%</td>
</tr>
<tr>
<td>Welfare</td>
<td>+0.106%</td>
<td>+0.127%</td>
<td>+0.129%</td>
<td>+0.116%</td>
<td>+0.074%</td>
<td>+0.048%</td>
<td></td>
</tr>
</tbody>
</table>
This figure plots the transition paths from the benchmark capital requirement to the optimal capital requirement of 17% over 20 quarters (x-axis). The y-axis marks the percentage change relative to benchmark economy. We plot consumption, aggregate liquidity, the aggregate capital stock, the S-bank capital share (S Cap Share), the leverage of S-banks, and the share of S-bank debt in total debt.

gate liquidity. It increases on impact by almost 6%. This is at first driven by an increase in S-bank leverage and then subsequently by an expansion in the balance sheet of S-banks.

What moves the leverage of S-banks? As in the simple model, S-bank leverage is a function (see equation 12 in Section 2) of the marginal value of S-bank liquidity services. With $\gamma_H > 0$, the marginal value of liquidity is decreasing in total liquidity services. Thus, when total liquidity falls enough in response to higher capital requirements and when households’ preference for liquidity provision is sufficiently inelastic, the marginal value of shadow bank liquidity services also increases. We can see this in the top-right panel of Figure 5 that shows S-banks’ deposit rate. The higher marginal value of S-bank debt lowers S-banks’ debt financing costs. We also see from the bottom-middle panel that the reduction in the debt financing costs reduces S-banks’ overall funding costs: their weighted-average-cost-of-capital (WACC) falls by 0.4% the instant the new capital requirement is introduced and keeps on falling subsequently. This funding cost reduction
makes S-banks more profitable and leads to more entry of S-banks, hence increasing their capital share and the aggregate balance sheet size of S-banks. It also increases the value of the collateral for liquidity production: the physical capital stock. Since the value of liquidity services increases, the collateral value of capital also increases, leading to a higher capital stock in equilibrium.

Figure 5: Transition path from the benchmark economy to the economy with the optimal capital requirement: default rates and prices

This figure plots the transition paths from the benchmark capital requirement to the optimal capital requirement of 17% over 20 quarters (x-axis). The y-axis marks the percentage change relative to benchmark economy. We plot the default rate of shadow banks and commercial banks, the deposit rate of shadow banks and commercial banks, and the weighted average cost of capital (funding costs of assets) for both shadow- and commercial banks.

Our general equilibrium model shows that changes in the capital requirement also affect financial institutions that were not targeted by this policy. These effects were caused by a reduction in C-banks’ liquidity services. But why do commercial bank liquidity services fall when the capital requirement constraint is tightened? To answer this question it is useful to visualize the balance sheet. A tighter constraint means that the ratio of bank equity to bank assets has to increase. It does not necessarily follow that bank debt has to fall. Instead, C-banks could increase their equity-asset ratio while keeping bank debt constant by funding an asset expansion with equity. Banks would only find it optimal to
do this if (1) the marginal benefit of assets exceeded their marginal funding costs, and (2) at the margin equity was cheaper than debt. The marginal benefit of assets is not affected by changes in the bank capital requirement. Thus, to incentivize banks to expand their balance sheet their funding costs have to fall substantially. The bottom-right panel of Figure 5 shows that indeed C-banks’ overall funding costs \((\text{WACC}_C)\) fall in response to higher capital requirements. This is a general equilibrium effect that is also present in Begenau (2018).\footnote{The reason for the reduction in overall funding costs is simple. Households like commercial bank deposits. Their preferences (see 29) imply that the marginal value of commercial bank liquidity services is positive and decreasing in the amount of commercial bank debt. To put it simply, households have a downward-sloping demand curve for commercial bank liquidity. The increase in the convenience yield in response to a higher capital requirement in Table 2 reflects this demand curve.} This effect indeed motivates commercial banks to increase their assets and fund new investments with equity. However, commercial banks merely increase their assets by .30%, which is not large enough to offset the higher equity ratio.

How liquidity preferences shape our welfare results. Households’ liquidity preferences are at the heart of our model and critical for the response of shadow banks to higher capital requirements on commercial banks. Higher shadow bank leverage can counteract the benefits of tighter requirements for commercial banks. In section 2, we show that with constant returns in overall liquidity \((\gamma_H = 0)\), shadow banks’ leverage decreases unambiguously with higher C-bank capital requirements. However, our benchmark calibration has moderately diminishing returns \((\gamma_H = 1.7)\), and S-bank leverage rises as we raise \(\theta\). Another take-away from the simple model is that the expansion of the shadow bank share in response to higher \(\theta\) depends on the elasticity of substitution \(1/(1-\epsilon)\). The larger \(\epsilon\), the better are S-banks in replacing reduced C-bank liquidity provision by expanding their asset share.

To explore the robustness of our main conclusions, we analyze the effect of increasing \(\theta\) under two alternative specifications: a calibration in which \(\epsilon = 0\) (i.e., low substitutability between S-bank and C-bank debt), and one in which \(\gamma_H = 0\) (i.e., no decreasing returns to scale in aggregate liquidity). We present detailed results in Table 7 of Appendix C.3. The main take-away is that these variations in preferences affect the results consistent with the intuition developed in the simple model of Section 2.6. A lower degree of substitutability \((\epsilon = 0)\) leads to a smaller expansion in shadow banking, and thus to a greater reduction in overall bankruptcy losses. The net effect is a slightly higher welfare gain. Constant returns in total liquidity \((\gamma_H = 0)\) cause a larger drop both in liquidity provision and deadweight losses. However, they also lead to a smaller rise in the aggregate liquidity premium, which in general equilibrium causes an increase in both banks’ cost of capital,
resulting in less investment and output. Thus, the net effect is a smaller welfare gain despite a more stable financial system. In both cases, we see qualitatively clear differences in the S-bank response. However, the magnitudes of the differences in welfare are relatively small, leading us to conclude that our quantitative results are robust to reasonable variations of our liquidity preference calibration.

6 Other Regulatory Policies

So far we analyzed the effects of a simple static capital requirement. Since the financial crisis, policy-makers and academics have proposed several alternatives, including time varying capital requirements, fairly priced deposit insurance premia, and more recently a proposal that has become known as the Minneapolis plan.28

Table 3 shows how the economies under these alternative policies differ from the benchmark economy. The first column presents the results for the optimal static capital requirement that we discussed in the previous section. In the second column, we present the effects of a fair insurance premium. This means that we set the deposit insurance premium $\kappa$ on C-bank debt to the expected loss of the deposit insurance fund. In other words, $\kappa$ is set such that the deposit insurance fund breaks even in expectations. This does little to reduce C-banks’ leverage and deadweight losses, as they pass on most the fee to depositors in the form of lower rates. Therefore, this policy does not lift consumption like higher capital requirements. The reduction in leverage, albeit tiny, is sufficient to cause a reduction in aggregate liquidity provision. Without gains in consumption, the net effect on welfare of this policy is negative.

In the third column, we experiment with a countercyclical capital requirement. More concretly, we set the average of this time varying capital requirement to the optimal static capital requirement value of 17%, and increase it to 22% during bad aggregate states and decrease it to 12% in good aggregate states. The fourth column experiments with a procyclical capital requirement that is on average 17%, and 12% in bad states and 22% in good states. Interestingly, both types of capital requirements have similar effects on the economy as a static requirement and do not significantly differ from each other. Relative to the static capital requirement they increase the volatility of liquidity provision and lead to a smaller expansion of the shadow banking system.

The last column presents the results from the Minneapolis Plan. In contrast to all other

---

Table 3: Effects of Alternative Bank Regulation Policies

<table>
<thead>
<tr>
<th></th>
<th>$\theta=17%$</th>
<th>fair $\kappa$</th>
<th>$\text{Corr}(\theta_t, Y_t)$ $&lt; 0$</th>
<th>$\text{Corr}(\theta_t, Y_t)$ $&gt; 0$</th>
<th>Minn. plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>+0.56%</td>
<td>-0.30%</td>
<td>+0.59%</td>
<td>+0.58%</td>
<td>-0.40%</td>
</tr>
<tr>
<td>Debt share S</td>
<td>+6.18%</td>
<td>+9.04%</td>
<td>+5.95%</td>
<td>+6.12%</td>
<td>-63.88%</td>
</tr>
<tr>
<td>Capital share S</td>
<td>+0.60%</td>
<td>+9.34%</td>
<td>+0.26%</td>
<td>+0.43%</td>
<td>-68.86%</td>
</tr>
<tr>
<td>Leverage S</td>
<td>+0.39%</td>
<td>-0.22%</td>
<td>+0.42%</td>
<td>+0.37%</td>
<td>+0.02%</td>
</tr>
<tr>
<td>Leverage C</td>
<td>-7.78%</td>
<td>-0.00%</td>
<td>-7.77%</td>
<td>-7.79%</td>
<td>-14.45%</td>
</tr>
<tr>
<td>Deposit rate S</td>
<td>-1.97%</td>
<td>+1.15%</td>
<td>-2.04%</td>
<td>-2.03%</td>
<td>-59.99%</td>
</tr>
<tr>
<td>Deposit rate C</td>
<td>-9.11%</td>
<td>-3.92%</td>
<td>-9.33%</td>
<td>-9.23%</td>
<td>-7.40%</td>
</tr>
<tr>
<td>Convenience Yield S</td>
<td>+5.34%</td>
<td>-3.06%</td>
<td>+5.49%</td>
<td>+5.53%</td>
<td>+130.87%</td>
</tr>
<tr>
<td>Convenience Yield C</td>
<td>+11.51%</td>
<td>+4.78%</td>
<td>+11.74%</td>
<td>+11.67%</td>
<td>+9.42%</td>
</tr>
<tr>
<td>DWL S</td>
<td>+11.70%</td>
<td>+2.57%</td>
<td>+11.83%</td>
<td>+11.99%</td>
<td>-70.70%</td>
</tr>
<tr>
<td>DWL C</td>
<td>-94.63%</td>
<td>-5.49%</td>
<td>-80.47%</td>
<td>-83.73%</td>
<td>-99.72%</td>
</tr>
<tr>
<td>GDP</td>
<td>+0.05%</td>
<td>-0.03%</td>
<td>+0.05%</td>
<td>+0.05%</td>
<td>-0.03%</td>
</tr>
<tr>
<td>Liquidity Services</td>
<td>-5.16%</td>
<td>-1.43%</td>
<td>-5.08%</td>
<td>-5.16%</td>
<td>-14.02%</td>
</tr>
<tr>
<td>Consumption</td>
<td>+0.19%</td>
<td>-0.00%</td>
<td>+0.16%</td>
<td>+0.17%</td>
<td>+0.28%</td>
</tr>
<tr>
<td>Vol(Liquidity Services)</td>
<td>-6.15%</td>
<td>-0.10%</td>
<td>+45.19%</td>
<td>+20.89%</td>
<td>-47.54%</td>
</tr>
<tr>
<td>Vol(Consumption)</td>
<td>+0.59%</td>
<td>+1.37%</td>
<td>+0.23%</td>
<td>+0.87%</td>
<td>-1.29%</td>
</tr>
<tr>
<td>Welfare</td>
<td>+0.129%</td>
<td>-0.011%</td>
<td>+0.115%</td>
<td>+0.118%</td>
<td>+0.144%</td>
</tr>
</tbody>
</table>

This table shows how alternative policies change the behavior of the economy relative to the benchmark calibration. The first column shows the results for the optimal static capital requirement, the second for a fair deposit insurance premium, the third for a countercyclical capital requirement, the fourth for a procyclical capital requirement, the last column for the Minneapolis Plan. Each value represents the percentage change relative to the benchmark economy in the average of the variable of interest.
plans, this proposal also targets the shadow banking sector leading to pronounced differences in the results. Specifically, this plan combines higher capital requirements for commercial banks (23%) with a 30 bps quarterly tax on shadow bank borrowings. Despite tougher capital requirements on commercial banks, fewer shadow banks find financial intermediation and liquidity provision profitable due to the tax. Instead of filling the gap, S-banks’ debt and capital share falls by 64% and 69%, respectively. Thus, the result of this experiment provides an insight into an economy with a much smaller role of shadow banks. Given the reduction in C-bank leverage and lower S-bank liquidity provision it is not surprising that liquidity production takes a large hit relative to the other proposals, a fall by 14% in contrast to only 5% with the static requirement. A lower supply of shadow bank debt drives up its value and thus lowers S-banks’ debt funding costs (gross of the tax). S-bank leverage remains roughly unchanged. Because S-banks’ become much smaller, the aggregate amount of deadweight losses from shadow banks is drastically reduced. In this world with fewer shadow banks, liquidity services and consumption are much less volatile. Together with an overall larger reduction in deadweight losses it leads to the highest welfare gain of all policies considered.

7 Conclusion

As the relative market share of traditional depositary institutions has been declining, both in terms of loan originations and safe asset provision, the question of optimal bank regulation requires modeling both these traditional banks and less regulated shadow banks. We propose a quantitative framework that views shadow banks not just as regulatory avoidance schemes of traditional banks, but as alternative providers of intermediation services and safe and liquid assets. Our approach draws the main distinction between both types of intermediaries on the funding side: while traditional banks issue insured deposits, shadow banks fund their balance sheets through unsecured short-term debt. Our theory is compatible with many underlying reasons why investors want to hold both traditional deposit accounts and other forms of (uninsured) money-like assets.

The model allows us to analyze how higher capital requirements on commercial banks change the relative profitability of shadow banking and the relative attractiveness of shadow bank debt, affecting the size and riskiness of the shadow banking sector. The effect on financial stability depends on investor demand for liquidity provision, which we capture through household preferences for safe and liquid assets. If household demand is sufficiently inelastic, the withdrawal of liquidity produced by traditional banks
following tighter capital regulation causes shadow bank leverage and risk to increase. We let the data guide our specification of preferences for liquidity to quantify the effect of higher capital requirements. At the optimal static capital requirement of 17% of (risky) assets, aggregate liquidity provision is lower compared to the benchmark capital requirement of 10%. As liquidity is more valuable, shadow banking becomes more profitable, leading to an expansion in the asset share and leverage of shadow banks. However, even though shadow banks become riskier, traditional banks become sufficiently safer, such that the net effect for financial stability is positive.

Our analysis shows that the substitution towards shadow banking in response to tighter regulation is indeed a valid concern. However, it also clarifies that a financial system with more shadow banking is not necessarily riskier. It further highlights a positive effect of the shift to shadow banks: the increased liquidity provision by shadow banks can offset some of the reduction in traditional bank liquidity production, causing a smaller overall decline in liquid assets. Furthermore, since production of safe assets in our framework is costly to society, both types of banks may overproduce liquidity relative to the social optimum. In particular, if shadow banks also enjoy implicit government guarantees, policies aimed at reducing both traditional and shadow banks’ leverage and risk, such as a tax on unsecured short-term debt funding, lead to higher welfare gains than policies targeted at traditional banks alone.

While the focus on differences in funding of traditional and shadow banks sharpens our conclusions, our framework abstracts away from differences in technology or expertise on the asset side. Further, we only indirectly account for the government’s role in safe asset provision. We leave these important questions as extensions for future work.
References


A Simple Model

Proof of proposition 1.

Proof. To obtain the S-bank FOC for leverage, we differentiate the S-bank objective in (3) to get

\[ q_S + q_S'(L_S)L_S = \beta(1 - L_S). \]

Differentiating the HH FOC (10) with respect to \( L_S \), and under the restriction that individual S-banks do not internalize their effect on aggregate S-bank liquidity \( A_S \), gives

\[ q_S'(L_S) = -\beta. \]

Combining the two yields \( q_S = \beta \). Substituting this result back into the HH FOC (10) results in equation (12). \( \square \)

Proof of proposition 2.

Proof. Differentiating the C-bank objective in (2) with respect to \( L_C \) gives

\[ q_C = \mu_C + \beta(1 - L_C), \]  

where \( \mu_C \) is the Lagrange multiplier on the leverage constraint. Combining equations (9) and (38) yields

\[ \mu_C = \beta(\psi_HC + L_C) > 0. \]  

The \( L_C \) term on the RHS of equation (39) denotes the probability of default \( FC(L_C) \) and is therefore \( \geq 0 \). Under the assumption that \( \psi_HC > 0 \), this implies that the multiplier is positive and the constraint is binding, with leverage given by equation (15). Combining equations (11) and (9) under the same assumption implies the result in (16). \( \square \)

Proposition 5. An increase in the capital requirement \( \theta \) causes lower S-bank leverage and greater S-bank debt and capital shares, i.e., \( dL_S/d\theta < 0, dR_S/d\theta > 0 \) and \( dK_S/d\theta > 0 \).

Proof. Differentiation of the leverage condition (20) with respect to \( \theta \) gives

\[ \frac{dL_S}{d\theta} = \psi'H_S(R_S)\frac{dR_S}{d\theta}, \]  

and differentiation of the scale condition (21) yields

\[ 2L_S \frac{dL_S}{d\theta} = -\frac{1}{2}(1 - \theta) - \psi'H_C(R_S) + \psi(1 - \theta)H'_C(R_S)\frac{dR_S}{d\theta}. \]  

Combining (40) and (41) to eliminate \( dR_S/d\theta \) and solving for \( dL_S/d\theta \) gives

\[ \frac{dL_S}{d\theta} = \frac{\psi'H'_S(R_S)\left(\frac{1}{2}(1 - \theta) + \psi'H_C(R_S)\right)}{(1 - \theta)H'_C(R_S) - 2L_SH'_S(R_S)}, \]  

46
which is negative since $\mathcal{H}_S'(R_S) < 0$ and $\mathcal{H}_C'(R_S) > 0$. By (40),

$$\frac{dR_S}{d\theta} = \frac{1}{\psi \mathcal{H}_S'(R_S)} \frac{dL_S}{d\theta},$$

which is positive given $dL_S/d\theta < 0$. Finally, the equilibrium S-bank debt ratio can be expressed in terms of $L_S$ and $K_S$

$$R_S = \frac{L_S K_S}{\frac{1}{2}(1 - \theta)(1 - K_S)}.$$ 

Again differentiation with respect to $\theta$ and solving for $dK_S/d\theta$ yields

$$\frac{dK_S}{d\theta} = \frac{dR_S}{d\theta} - \frac{dL_S}{d\theta} \frac{K_S}{\frac{1}{2}(1 - \theta)(1 - K_S)} + \frac{1}{\frac{1}{2}(1 - \theta)(1 - K_S)^2},$$

which given $dR_S/d\theta > 0$ and $dL_S/d\theta < 0$ is positive.

\[ \square \]

**Proof of proposition 3.** The first-order conditions for S- and C-bank leverage are given by

$$L_j = \psi \mathcal{H}_j(L_S K_S, L_C (1 - K_S)) = \psi \tilde{\mathcal{H}}_j(R_S), \quad (42)$$

for $j = S, C$ respectively, i.e. the leverage ratio of each bank is equal to the marginal benefit of liquidity to households. The first-order condition for the S-bank capital share is

$$\frac{1}{2} L_S^2 - L_S \psi \mathcal{H}_S(L_S K_S, L_C (1 - K_S)) = \frac{1}{2} L_C^2 - L_C \psi \mathcal{H}_C(L_S K_S, L_C (1 - K_S)),$$

which combined with the FOCs in (42) implies

$$L_S = L_C.$$

Setting equal the conditions in (42) and using the functional form for utility in (25) yields

$$\alpha \left[ \alpha + (1 - \alpha) \frac{1}{R_S^\epsilon} \right]^{\frac{1}{1-\epsilon}} = (1 - \alpha) \left[ \alpha R_S^\epsilon + 1 - \alpha \right]^{\frac{1}{1-\epsilon}},$$

which can be rearranged to

$$\alpha (1 - \alpha) R_S^\epsilon - (1 - \alpha) \alpha R_S^\epsilon = 0.$$ 

This is an exponential polynomial of the form

$$A \exp(2\epsilon x) + B \exp(\epsilon x) + C = 0,$$
with \( x = \log(R_S) \) and
\[
A = \alpha(1 - \alpha)^{\frac{1}{1+\tau}} \\
B = \left( (1 - \alpha)^{\frac{1}{1+\tau}} - a^{\frac{1}{1+\tau}} \right) \\
C = -(1 - a)a^{\frac{1}{1+\tau}}.
\]

Given \( \alpha \in [0, 1] \), the unique real root is
\[
x = \frac{1}{e} \log \left[ \frac{\alpha^{\frac{1}{1+\tau}} (1 - \alpha)^{-\frac{1}{1+\tau}} - (1 - \alpha) + \sqrt{(1 - \alpha)^2 \left( 1 + \alpha^{\frac{1}{1+\tau}} (1 - \alpha)^{-\frac{1}{1+\tau}} \right)^2}}{2\alpha} \right] \\
= \frac{1}{e} \log \left[ \frac{(1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\tau}} - (1 - \alpha) + (1 - \alpha) + (1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\tau}}}{2\alpha} \right] \\
= \frac{1}{e} \log \left[ \frac{(1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\tau}}}{\alpha} \right],
\]
which simplifies to
\[
R_S = \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\tau}}.
\]

Since
\[
R_S = \frac{L_SM_K}{L_CK_C} = \frac{K_S}{K_C},
\]
we obtain the solution for the capital shares in the proposition. Plugging this solution back into either condition (42) and setting \( L_S = L_C \) gives
\[
L^* \equiv \psi(1 - \alpha) \left[ a \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\tau}} + 1 - \alpha \right]^{\frac{1}{1+\tau}}.
\]

**Proof of proposition 4.** Applying the definition of the wedge \( m \) to the zero-profit condition (21) gives
\[
L_S^2 = L_C^2 + 2(1 + m)(\psi h_C(R_S))^2,
\]
which when combined with the condition for S-bank leverage (20) implies
\[
L_S = \sqrt{\frac{m + 3}{m + 1}L_C},
\]
or equivalently
\[
h_S(R_S) = \sqrt{(m + 1)(m + 3)h_C(R_S)}.
\]
Defining the wedge factor $\mathcal{M} = \sqrt{(m + 1)(m + 3)}$ and using the functional form for utility in (25) yields

$$a \left[ a + (1 - a) \frac{1}{R_S^e} \right]^{\frac{1}{1 - \varepsilon}} = \mathcal{M} (1 - a) [a R_S^e + 1 - a]^{\frac{1}{1 - \varepsilon}},$$

which can be rearranged to

$$a(1 - a)^{\frac{1}{1 - \varepsilon}} \mathcal{M}^{\frac{1}{1 - \varepsilon}} R_C^e + \left( (1 - a)^{\frac{1}{1 - \varepsilon}} \mathcal{M}^{\frac{1}{1 - \varepsilon}} - a^{\frac{1}{1 - \varepsilon}} \right) R_S^e - (1 - a)a^{\frac{1}{1 - \varepsilon}} = 0.$$

This is an exponential polynomial of the form

$$A \exp(2e x) + B \exp(e x) + C = 0,$$

with $x = \log(R_S)$ and

$$A = a(1 - a)^{\frac{1}{1 - \varepsilon}} \mathcal{M}^{\frac{1}{1 - \varepsilon}},$$

$$B = \left( (1 - a)^{\frac{1}{1 - \varepsilon}} \mathcal{M}^{\frac{1}{1 - \varepsilon}} - a^{\frac{1}{1 - \varepsilon}} \right),$$

$$C = -(1 - a)a^{\frac{1}{1 - \varepsilon}}.$$

Given $a \in [0, 1]$, the unique real root is

$$x = \frac{1}{e} \log \left[ \frac{(1 - a) \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1 - \varepsilon}}}{a \mathcal{M}^{\frac{1}{1 - \varepsilon}}} \right],$$

which simplifies to

$$R_S = \left( \frac{1}{\mathcal{M}} \right)^{\frac{1}{1 - \varepsilon}} \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1 - \varepsilon}}.$$

To prove part (i), note that at $m = 0$, which leads to efficient C-bank leverage, we get $\mathcal{M} = \sqrt{3}$, which implies a smaller S-bank ratio $R_S$ by factor $1/\sqrt{3}^{\frac{1}{1 - \varepsilon}}$ compared to the planner solution in proposition 3.

To prove part (ii), we differentiate the household objective given by (24) in the decentralized equilibrium with respect to $\theta$. After collecting terms, the derivative is

$$\frac{dU(\theta)}{d\theta} = \frac{dK_C}{d\theta} \left( \psi \mathcal{H}_C(R_S) L_C - \frac{1}{2} L_C^2 \right) + \frac{dK_S}{d\theta} \left( \psi \mathcal{H}_S(R_S) L_S - \frac{1}{2} L_S^2 \right) + \frac{dL_C}{d\theta} \left( \psi \mathcal{H}_C(R_S) K_C - K_C L_C \right) + \frac{dL_S}{d\theta} \left( \psi \mathcal{H}_S(R_S) K_S - K_S L_S \right).$$

Since in equilibrium $L_S = \psi \mathcal{H}_S(R_S)$ and $L_C = (1 + m) \psi \mathcal{H}_C(R_S)$, this expression becomes

$$\frac{dU(\theta)}{d\theta} = \left( \frac{1}{1 + m} - \frac{1}{2} \right) L_C^2 \frac{dK_C}{d\theta} + \frac{1}{2} L_S^2 \frac{dK_S}{d\theta} + \left( \frac{1}{1 + m} - 1 \right) K_C L_C \frac{dL_C}{d\theta}.$$

49
Further noting that \( L_S = \sqrt{\frac{m+3}{m+1}} L_C \) and \( dK_C / d\theta = -dK_S / d\theta \) (since \( K_C = 1 - K_S \)), this becomes

\[
\frac{dU(\theta)}{d\theta} = L_C^2 \frac{dK_S}{d\theta} - \left( 1 - \frac{1}{1 + m} \right) K_C L_C \frac{dL_C}{d\theta}.
\]

Since \( dK_S / d\theta > 0 \) by Proposition 5, and \( dL_C / d\theta < 0 \), this expression is positive for any \( m \geq 0 \).

B Quantitative Model

B.1 Bank Optimization and Aggregation

This section describes details of the bank optimization problem resulting in equations (32) and (34) in the main text. Anticipating the result that all banks are equal due to i.i.d. shocks and a value function that is homogeneous in capital, we suppress individual bank subscripts throughout.

Production. After aggregate productivity \( Z_t \) and the run shock \( \varrho_t \) are realized, all banks produce and invest. Denote by \( \hat{K}_j^i = (1 - \ell_j^i) K_j^i \) the capital banks retain after a possible fire sale due to runs. The profits generated by the two lines of bank business (production and investment) are

\[
\hat{D}_j^i = Z_t \left( \hat{K}_j^i \right)^{1-\eta} \left( N_j^i \right)^{\eta} + (1 - \delta_K) p_t \hat{K}_j^i - w_t N_j^i + I_j^i (p_t - 1) - \frac{\phi_I}{2} \left( \frac{I_j^i}{\hat{K}_j^i} - \delta_K \right)^2 \hat{K}_j^i,
\]

for \( j = S, C \). Banks choose labor input \( N_j^i \) and investment \( I_j^i \) to maximize (44). Note that the profit also includes the proceeds from selling depreciated capital after production, \( (1 - \delta_K) p_t \hat{K}_j^i \).

The first-order condition for labor demand is the usual intratemporal condition equating the wage to the marginal product of labor

\[
w_t = Z_t \eta \left( \frac{N_j^i}{\hat{K}_j^i} \right)^{\eta-1} = Z_t \eta \left( \frac{n_j^i}{\hat{K}_j^i} \right)^{\eta-1}.
\]

Similarly, the first-order condition for investment yields the usual relationship between the capital price and the marginal value of a unit of capital

\[
p_t = 1 + \phi_I \left( \frac{I_j^i}{\hat{K}_j^i} - \delta_K \right) = 1 + \phi_I \left( i_j^i - \delta_K \right).
\]

We can substitute both conditions back into the definition of profit in (44) to eliminate the wage and investment and define the gross payoff per unit of capital in equation (30) to get

\[
\hat{D}_j^i = \Pi_j^i \hat{K}_j^i.
\]
The total dividend banks pay to shareholders is given by

\[ D_t^j = \rho_t^j \hat{D}_t^j - B_t^j + (q_t^j - \kappa_t^j)B_{t+1}^j - p_t^j K_{t+1}^j - \frac{\Phi_K}{2} \left( \frac{K_{t+1}^j}{\hat{K}_t^j} - 1 \right)^2 \hat{K}_t^j. \]

It scales the profit banks receive from their real business, \( \hat{D}_t^j \), by the idiosyncratic shock, \( \rho_t^j \), and also includes redemptions of last period’s deposits, \( B_t^j \), and the equity cost of the portfolio for next period, \( (q_t^j - \kappa_t^j)B_{t+1}^j - p_t^j K_{t+1}^j - \frac{\Phi_K}{2} \left( \frac{K_{t+1}^j}{\hat{K}_t^j} - 1 \right)^2 \hat{K}_t^j \), where the deposit insurance fee \( \kappa_j = 0 \) for S-banks in the benchmark model.

**Bank value function.** We define the value function of a bank that did not default, at the time it chooses its portfolio for next period as

\[
\hat{V}^j(\hat{K}_t^j, Z_t, \rho_t^j) = \max_{K_{t+1}^j, B_{t+1}^j} D_t^j \\
+ \mathbb{E}_t \left[ M_{t,t+1} \max \left\{ \hat{V}^j((1 - \ell_{t+1}^j)K_t^j, Z_{t+1}, \rho_{t+1}^j) + \ell_{t+1}^j \rho_{t+1}^j \Pi_{t+1}^H K_{t+1}^j, -\delta_{j} \Pi_{t+1}^H K_{t+1}^j \right\} \right].
\]

We assume that the default penalty \( -\delta_{j} \Pi_{t+1}^H K_{t+1}^j \) in (47) is proportional to the asset value of the bank with parameter \( \delta_{j} \). This is reasonable and also retains the homogeneity of the problem in capital \( K_t^j \). Further note that capital sold to households in a run-induced fire sale yields \( \Pi_{t+1}^H \) per unit, as defined in equation (31).

To simplify the problem, we recognize that profits from real business and deposits redemptions \( \rho_t^j \hat{D}_t^j - B_t^j \) are irrelevant for the bank’s choice after the default decision. After the bankruptcy decision, non-bankrupt banks choose their portfolio for next period, and households set up new banks to replace those banks who defaulted. With respect to the portfolio choice for period \( t + 1 \), the optimization problem of all banks is identical conditional on having the same capital \( \hat{K}_t^j \). Thus we define the value function

\[
V^j(\hat{K}_t^j, Z_t) = \hat{V}^j(\hat{K}_t^j, Z_t, \rho_t^j) - \rho_t^j \hat{D}_t^j + B_t^j,
\]

such that we can write the problem in (47) equivalently as

\[
V^j(\hat{K}_t^j, Z_t) = \max_{K_{t+1}^j, B_{t+1}^j} (q_t^j - \kappa_t^j)B_{t+1}^j - p_t^j K_{t+1}^j - \frac{\Phi_K}{2} \left( \frac{K_{t+1}^j}{\hat{K}_t^j} - 1 \right)^2 \hat{K}_t^j \\
+ \mathbb{E}_t \left[ M_{t,t+1} \max \left\{ \rho_{t+1}^j \left( \Pi_{t+1}^H \hat{K}_t^j + \ell_{t+1}^j \Pi_{t+1}^H K_{t+1}^j \right) - B_{t+1}^j + V^j(\hat{K}_{t+1}^j, Z_{t+1}), -\delta_{j} \Pi_{t+1}^H K_{t+1}^j \right\} \right].
\]

(48)

**Aggregation.** Next, we conjecture that \( V^j(\hat{K}_t^j, Z_t) \) is homogeneous in capital \( \hat{K}_t^j \) of degree one. This allows us to define the scaled value function \( \bar{v}^j(Z_t) = \frac{V^j(\hat{K}_t^j, Z_t)}{\hat{K}_t^j} \). Further defining the fire sale
discount

\[ x^j_t = \frac{\Pi^H_t}{\Pi^j_t}, \]

we can write \( v^j(Z_t) \) as

\[
v^j(Z_t) = \max_{k^j_{t+1}, b^j_{t+1}} \left( (q^j_t - \kappa^j) b^j_{t+1} - p_t \right) k^j_{t+1} - \frac{\phi K}{2} \left( k^j_{t+1} - 1 \right)^2
+ E_t \left[ M_{t,t+1} \Pi^j_{t+1} k^j_{t+1} \max \left\{ \rho^j_{t+1} \left( 1 - \ell^j_t \left( 1 - x^j_t \right) \right) - L^j_t + (1 - \ell^j_t) \frac{v^j(Z_{t+1})}{\Pi^j_{t+1}}, -\delta \right\} \right].
\]

(49)

where the choice variables have been redefined as asset growth \( k^j_{t+1} = \frac{k^j_{t+1}}{K^j_t} \) and capital structure

\[ b^j_{t+1} = \frac{b^j_{t+1}}{K^j_t}. \]

We use the definition of the default threshold \( \hat{\rho}^j_t \) in the main text in equation (33) and take the expectation with respect to \( \rho^j_t \) to rewrite the max operator in equation (49). This term represents the leverage-adjusted payoff of banks’ portfolio including the default option and is

\[
\Omega^j(L^j_t) = (1 - F^j_{\rho,t}) \left( \rho^j_t \left( 1 - \ell^j_t \left( 1 - x^j_t \right) \right) - L^j_t + (1 - \ell^j_t) \frac{v^j(Z_t)}{\Pi^j_t} \right) - F^j_{\rho,t} \delta,
\]

(50)

where \( F^j_{\rho,t} = F(\hat{\rho}^j_t) \) is the probability of default and \( \rho^j_t > \hat{\rho}^j_t \) is the expected value of the idiosyncratic shock conditional on not defaulting.

We can thus rewrite (49) more compactly as

\[
v^j(Z_t) = \max_{k^j_{t+1}, b^j_{t+1}} \left( (q^j_t - \kappa^j) b^j_{t+1} - p_t \right) k^j_{t+1} - \frac{\phi K}{2} \left( k^j_{t+1} - 1 \right)^2
+ k^j_{t+1} E_t \left[ M_{t,t+1} \Pi^j_{t+1} \Omega^j(L^j_{t+1}) \right].
\]

(51)

Equation (51) corresponds to equations (32) and (34) in the main text. It shows that the expectation only depends on the capital structure choice through leverage \( L^j_{t+1} = b^j_{t+1}/\Pi^j_{t+1} \). The first-order condition for asset growth \( k^j_{t+1} \) is

\[
p_t - (q^j_t - \kappa^j) b^j_{t+1} + \phi K \left( k^j_{t+1} - 1 \right) = E_t \left[ M_{t,t+1} \Pi^j_{t+1} \Omega^j(L^j_{t+1}) \right].
\]

(52)

Substituting (52) into (51) yields

\[
v^j(Z_t) = k^j_{t+1} \phi K (k^j_{t+1} - 1) - \frac{\phi K}{2} \left( k^j_{t+1} - 1 \right)^2 = \frac{\phi K}{2} \left( (k^j_{t+1})^2 - 1 \right).
\]

The solution for \( v^j(Z_t) \) confirms the conjecture that

\[ V^j(\hat{R}^j_t, Z_t) = \hat{R}^j_t v^S(Z_t), \]

52
and we can thus solve the problem of a representative bank of each type. Note that the scaled value function \( v^i(Z_t) \) only depends on asset growth \( k^j_{t+1} \), which is a choice variable. Intuitively, if the bank expects high future capital growth, then the continuation value is large.

### B.2 Household Problem

Denoting household wealth at the beginning of the period by \( W_t \), the complete intertemporal problem of households is

\[
V^H(A^S_t, A^C_t, W_t, Y_t) = \max_{C_t, A^S_{t+1}, A^C_{t+1}, S^S_t, S^C_t} U(C_t, H(A^S_t, A^C_t)) + \beta E_t \left[ V(A^S_{t+1}, A^C_{t+1}, W_{t+1}, Y_{t+1}) \right]
\]

subject to the budget constraint in (37).

The transition law for household financial wealth \( W_t \) is

\[
W_{t+1} = \sum_{j=S,C} (1 - F^j_{\rho,t+1}) \left( D^{j,+}_{t+1} + p^j_{t+1} \right) S^j_t
\]

\[
+ A^S_{t+1} \left[ 1 - F^S_{\rho,t+1} + F^S_{\rho,t+1} \left( \pi_B + (1 - \pi_B) r^S_{t+1} \right) \right]
\]

\[
+ A^C_{t+1}.
\]

The beginning-of-period dividend paid by banks to households, conditional on survival, for S-banks is

\[
D^{S,+}_{t} = \rho^S_t K^S_t \left( e^S_t \Pi^H_t + (1 - e^S_t) \Pi^S_t \right) - B^S_t + k^S_t \left( q^S_t b^S_{t+1} - p_t \right) - \frac{\phi K}{2} \left( k^S_{t+1} - 1 \right)^2,
\]

and for C-banks

\[
D^{C,+}_{t} = \rho^C_t K^C_t \Pi^C_t - B^C_t + k^C_t \left( (q^C_t - \kappa) b^C_{t+1} - p_t \right) - \frac{\phi K}{2} \left( k^C_{t+1} - 1 \right)^2.
\]

Households' first-order conditions for purchases of bank equity are, for \( j = S, C \),

\[
p^j_t = E_t \left[ M_{t,t+1} \left( 1 - F^j_{\rho,t+1} \right) \left( D^{j,+}_{t+1} + p^j_{t+1} \right) \right],
\]

where we have defined the stochastic discount factor

\[
M_{t,t+1} = \beta \frac{U_C(C_{t+1}, H_{t+1})}{U_C(C_t, H_t)} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}.
\]

The marginal rate of substitution between consumption and liquidity services of bank type \( j \) is defined as

\[
MRS_{j,t} = \frac{U_H(C_t, H_t) \partial H(A^S_t, A^C_t)}{U_C(C_t, H_t) \partial A^j_t},
\]

53
and given by

\[
MRS_{S,t} = \alpha \psi C_t^\gamma H_t^{1-\gamma} \left( \frac{H_t}{A_t^S} \right)^{1-\epsilon},
\]

(54)

\[
MRS_{C,t} = (1 - \alpha) \psi C_t^\gamma H_t^{1-\gamma} \left( \frac{H_t}{A_t^C} \right)^{1-\epsilon},
\]

(55)

for S- and C-bank debt, respectively.

Then the first-order conditions for purchases of bonds of either type of bank are

\[
q_t^S = E_t \left\{ M_{t+1} \left[ (1 - F_{\rho,t+1}^S + F_{\rho,t+1}^S (\pi_B + (1 - \pi_B)r_t^S) + MRS_{S,t+1} \right] \right\},
\]

(56)

\[
q_t^C = E_t \left\{ M_{t+1} \left[ (1 + MRS_{C,t+1}) \right] \right\},
\]

(57)

The payoff of commercial bank bonds is 1, whereas the payoff of shadow bank bonds depends on their default probability, recovery value, and the probability of a government bailout \(\pi_B\). The last term in each expectation represents the marginal benefit of liquidity services to households, as defined in (54) and (55).

### B.3 S-bank Optimality Conditions

Each period, S-banks choose investment \(I_t^S\), labor input \(N_t^S\), capital growth \(k_{t+1}^S\) and capital structure \(b_{t+1}^S\). The first-order conditions for investment and labor are given by (46) and (45), respectively. They are incorporated into the gross payoff of capital \(\Pi_t^S\) in (30). The first-order condition for capital growth is given by (52).

It remains to derive the first-order condition for capital structure. Before doing so, we first recognize that individual S-banks take into account the effect of their capital structure choice on the price of their debt. Thus, they optimally respond to households’ valuation of idiosyncratic S-bank risk, such that we replace \(q_t^S\) in (51) by the function

\[
q_t^S(b_{t+1}^S) = E_t \left\{ M_{t+1} \left[ (1 - F_{\rho,t+1}^S + F_{\rho,t+1}^S (\pi_B + (1 - \pi_B)r_t^S) + MRS_{S,t+1} \right] \right\},
\]

which is households’ first-order condition for S-bank debt purchases in (56). Differentiating equation (51) with respect to \(b_{t+1}^S\) after this substitution yields

\[
q_t^S + b_{t+1}^S q_t^S(b_{t+1}^S) = -E_t \left[ M_{t+1} \Pi_{t+1}^S \frac{\partial \Pi_{t+1}^S}{\partial b_{t+1}^S} \Omega' (L_{t+1}^S) \right].
\]

Using \(\frac{\partial \Pi_{t+1}^S}{\partial b_{t+1}^S} = \frac{1}{\Pi_{t+1}^S}\), this reduces to

\[
q_t^S(b_{t+1}^S) + b_{t+1}^S q_t^S(b_{t+1}^S) = -E_t \left[ M_{t+1} \Omega' (L_{t+1}^S) \right].
\]
In section B.3.1 we calculate Ω′(LS+1), such that the first-order condition becomes

\[q^S_t + b^S_{t+1} q^S(b^S_{t+1}) = E_t \left[ M_{t+1} (1 - F^S_t) \left( 1 + \frac{\ell_{t+1}}{L^S_{t+1}} \left( 1 - x_{t+1} \right) f^S_{t+1} + \frac{v^S_{t+1}}{\Pi^S_{t+1}} \right) \right], \quad (58)\]

To obtain the partial derivative \(q^S_t(b^S_{t+1})\), we differentiate equation (56) to get

\[q^S_t(b^S_{t+1}) = -(1 - \pi_B) E_t \left[ M_{t+1} \frac{\partial L^S_{t+1}}{\partial b^S_{t+1}} \left( \frac{\partial F^S_{t+1} r^S_{t+1}}{\partial L^S_{t+1}} - \frac{\partial F^S_{t+1}}{\partial L^S_{t+1}} \right) \right]. \quad (59)\]

In section B.3.2 we calculate \(\frac{\partial F^S_{t+1} r^S_{t+1}}{\partial L^S_{t+1}} - \frac{\partial F^S_{t+1}}{\partial L^S_{t+1}}\), such that the derivative becomes

\[q^S_t(b^S_{t+1}) = -\frac{1 - \pi_B}{b^S_{t+1}} E_t \left\{ M_{t,t+1} \left[ \frac{F^S_{t+1} r^S_{t+1}}{1 - \ell_{t+1} (1 - x_{t+1})} \right. \right. \]
\[\left. \left. + f^S_{t+1} L^S_t \left( L^S_{t+1} \right) \left( 1 - \xi^S \right) \left( \delta^S + (1 - \ell_{t+1}) \frac{v^S_{t+1}}{\Pi^S_{t+1}} \right) + \xi^S L^S_{t+1} \right) \right\}, \quad (59)\]

where

\[L^S_t = \frac{\partial \delta^S}{\partial L^S_t}\]

is the derivative of the default threshold with respect to leverage (equation (60)).

The full first-order condition for the S-bank capital structure choice is obtained by substituting (59) into (58).

It is useful to examine the FOC for the case of no run \(\ell_{t+1} = 0\), zero default penalty \(\delta^S = 0\), no capital adjustment cost \(\phi^S = 0\) (implying \(v^S_{t+1} = 0\)), and zero bailout probability \(\pi_B = 0\). In that case, the derivative in (59) reduces to

\[q^S_t(b^S_{t+1}) = -\frac{1}{b^S_{t+1}} E_t \left\{ M_{t,t+1} \left[ \frac{F^S_{t+1} r^S_{t+1}}{1 - x_{t+1}} \right. \right. \]
\[\left. \left. + f^S_{t+1} L^S_t \left( L^S_{t+1} \right) \left( 1 - \xi^S \right) \left( \delta^S \right) + \xi^S L^S_{t+1} \right) \right\} \]

and the first-order condition (58) becomes

\[q^S_t + b^S_{t+1} q^S(b^S_{t+1}) = E_t \left[ M_{t+1} (1 - F^S_t) \right]. \]

Combining these two equations, we get

\[q^S_t = E_t \left[ M_{t+1} \left( 1 - F^S_t + F^S_{t+1} r^S_{t+1} + \xi^S f^S_{t+1} L^S_t \left( L^S_{t+1} \right) L^S_{t+1} \right) \right]. \]

We can equate this with the household first-order condition (56) and collect terms to get

\[E_t \left[ M_{t+1} \xi^S f^S_{t+1} L^S_t \left( L^S_{t+1} \right) L^S_{t+1} \right] = E_t \left[ M_{t+1} MRS_{S,t+1} \right]. \]

This equation is the analogue to equation (12) in the simple model. The S-bank chooses leverage to equalize the expected marginal liquidity benefit to households on the RHS with the expected marginal losses due to bankruptcy on the LHS.
B.3.1 Computing $\Omega'(L_t^S)$

Rewrite the function $\Omega^S(L_t^S)$ as

$$\Omega^S(L_t^S) = (1 - F_t^S) \left( \rho_t^{S,+} (1 - \ell_t(1 - x_t)) - L_t^S \right) - F_t^S \delta_S$$

$$= (1 - \ell_t(1 - x_t)) \int_{\rho_t^S}^{\infty} \rho f_t^S(\rho) d\rho - (1 - F_t^S) \left( L_t^S - (1 - \ell_t) \frac{v_t^S}{\Pi_t^S} \right) - F_t^S \delta_S.$$

Recall the default threshold for S-banks is given by

$$\hat{\rho}_t^S = \frac{L_t^S - (1 - \ell_t^t) \frac{v(t)}{\Pi_t^S} - \delta_S}{1 - \ell_t^t(1 - \Pi_t^S / \Pi_t^S)}.$$

We denote the derivative with respect to $L_t^S$ as

$$L_t^S(L_t^S) = \frac{\partial \hat{\rho}_t^S}{\partial L_t^S} = \frac{1 - \frac{L_t^S - (1 - \ell_t^t)^2 \Pi_t^S}{(1 - \ell_t^t(1 - \Pi_t^S / \Pi_t^S)^2}}.$$  \hspace{1cm} (60)

First computing

$$\frac{\partial (1 - \ell_t(1 - x_t))}{\partial L_t^S} = -\rho_t^S \frac{1 - x_t}{x_t} = -\ell_t \frac{1 - x_t}{L_t^S},$$

we can further calculate (using Leibniz’s rule)

$$\frac{\partial \Omega^S(L_t^S)}{\partial L_t^S} = -\ell_t \frac{1 - x_t}{L_t^S} (1 - F_t^S) \rho_t^{S,+} - f_t^S \mathcal{L}_t^S(L_t^S) \left( L_t^S - \delta_S - (1 - \ell_t) \frac{v_t^S}{\Pi_t^S} \right)$$

$$- (1 - F_t^S) \left( 1 + \ell_t \frac{v_t^S}{L_t^S \Pi_t^S} \right) + f_t^S \mathcal{L}_t^S(L_t^S) \left( L_t^S - (1 - \ell_t) \frac{v_t^S}{\Pi_t^S} \right) - f_t^S \mathcal{L}_t^S(L_t^S) \delta_S$$

$$= -(1 - F_t^S) \left( 1 + \ell_t \frac{v_t^S}{L_t^S \Pi_t^S} \right) \left( (1 - x_t) \rho_t^{S,+} + \frac{v_t^S}{L_t^S \Pi_t^S} \right).$$

B.3.2 Computing $q^t_S(b_{t+1}^S)$

Computing $\frac{\partial r_t^S}{\partial L_t^S}$. Recall the definition of the recovery value for S-banks as

$$r_t^S(L_t^S) = (1 - s_t^S) \frac{\rho_t^{S,-} (1 - \ell_t(1 - x_t))}{L_t^S},$$

with the conditional expectation $\rho_t^{S,-} = \mathbb{E} [\rho | \rho < \hat{\rho}_t^S].$

First compute

$$\frac{\partial}{\partial L_t^S} \frac{1 - \ell_t(1 - x_t)}{L_t^S} = -\rho_t^S \frac{1 - x_t}{x_t} \frac{L_t^S - (1 - \ell_t(1 - x_t))}{(L_t^S)^2} = -\frac{1}{(L_t^S)^2}.$$
We can rewrite the recovery value times the probability of default as
\[ F_i^S r_i^S = \frac{(1 - \xi^S) (1 - \ell_t (1 - x_t))}{L_i^S} \int_{-\infty}^{\delta^S_i} \rho \, dF^S(\rho). \]

Differentiating this expression with respect to \( L_i^S \) gives
\[
\frac{\partial F_i^S r_i^S}{\partial L_i^S} = -\frac{(1 - \xi^S) F_i^S r_i^S - \ell_t (1 - \xi^S) L_i^S (L_i^S - \delta_S - (1 - \ell_t) \frac{v_i^S}{\Pi_t^S})}{L_i^S (1 - \ell_t (1 - x_t))}.
\]

Combining. Using that \( \frac{\partial F_i^S}{\partial L_i^S} = f_i^S L_i^S (L_i^S) \),
we get
\[
\frac{\partial F_i^S r_i^S}{\partial L_{i+1}^S} - \frac{\partial F_i^S}{\partial L_{i+1}^S} = -\frac{F_i^S r_i^S}{L_i^S (1 - \ell_t (1 - x_t))} - \frac{f_i^S L_i^S (L_i^S) (1 - \xi^S) \left( \delta_S + (1 - \ell_t) \frac{v_i^S}{\Pi_t^S} \right)}{L_i^S L_i^S}.
\]

B.4 C-bank Optimality Conditions

Like S-banks, C-banks choose investment \( I_t^C \), labor input \( N_t^C \), capital growth \( k_{t+1}^C \) and capital structure \( b_{t+1}^C \). The first-order conditions for investment and labor are given by (46) and (45), respectively. They are incorporated into the gross payoff of capital \( \Pi_t^C \) in (30). The first-order condition for capital growth is given by (52).

To derive the C-bank first-order condition for capital structure, first note that C-banks are subject to the regulatory constraint in (35). Denote the Lagrange multiplier associated with the constraint by \( \lambda_t^C \). Differentiating (51) subject to the constraint with respect to \( b_{t+1}^C \) gives
\[
q_t^C - \kappa_C = \lambda_t^C - E_t \left[ M_{t+1} \Pi_{t+1}^C \frac{\partial L_{t+1}^C}{\partial b_{t+1}^C} \Omega'(L_{t+1}^C) \right].
\]

Using \( \frac{\partial L_{t+1}^C}{\partial b_{t+1}^C} = \frac{1}{\Pi_{t+1}} \), this reduces to
\[
q_t^C - \kappa_C = \lambda_t^C - E_t \left[ M_{t+1} \Omega'(L_{t+1}^C) \right].
\]
In section B.4.1 we calculate $\Omega'(L_{t+1}^C)$, such that the first-order condition becomes
\[ q_t^C - \kappa_t^C = \lambda_t^C + E_t \left[ M_{t+1}(1 - F^C_{t+1}) \right]. \tag{61} \]

### B.4.1 Computing $\Omega'(L_{t+1}^C)$

Rewrite the function $\Omega^C(L_t^C)$ as
\[ \Omega^C(L_t^C) = \left( \frac{1}{1 - \delta^C} \right) \left( L_t^C - v_t^C \rho_t^C \right) + (1 - \delta^C) \left( L_t^C - v_t^C \right) - F_t^C \delta^C. \]

Recall the default threshold for C-banks is given by
\[ \hat{\rho}_t^C = L_t^C - v_t^C \left( Z_t \right) \Pi_t^C - \delta^C, \]

implying that $\frac{\partial \hat{\rho}_t^C}{\partial L_t^C} = 1$.

We calculate (using Leibniz’s rule)
\[ \frac{\partial \Omega^C(L_t^C)}{\partial L_t^C} = -f_t^C \left( L_t^C - \delta^C - \frac{v_t^C}{\Pi_t^C} \right) - (1 - F_t^C) + f_t^C \left( L_t^C - \frac{v_t^C}{\Pi_t^C} \right) - f_t^C \delta^C \]
\[ = - (1 - F_t^C). \]

### B.5 Equilibrium Definition

Given a sequence of aggregate \( \{Y_t, Z_t, \epsilon_t\} \) and idiosyncratic shocks \( \{\rho^S_t, \rho^C_t\} \), a competitive equilibrium consists of a sequence of prices \( \{w_t, p_t, q^S_t, q^C_t, p^S_t, p^C_t\} \), household choices \( \{C_t, A_{t+1}^S, A_{t+1}^C, S_t^S, S_t^C, N_t^H\} \), S-bank choices \( \{I_t^S, N_t^S, B_{t+1}^S, K_{t+1}^S\} \), and C-bank choices \( \{I_t^C, N_t^C, B_{t+1}^C, K_{t+1}^C\} \) such that households and banks optimize given prices, and markets clear.

There is market clearing for capital
\[ K_{t+1}^S + K_{t+1}^C = I_t^S + I_t^C + (1 - \delta^K) \sum_{j=S,C} \left( 1 - \xi^j F_t^j \rho^j_t \right) K_t^j \left( 1 - \ell_t^j \right) + (1 - \delta^K) K_t^S \epsilon_t^S, \tag{62} \]

securities issued by banks
\[ B_{t+1}^S = A_{t+1}^S, \]
\[ B_{t+1}^C = A_{t+1}^C, \]
\[ S_t^S = 1, \]
\[ S_t^C = 1, \]

58
the goods market

\[ C_t + \sum_{j=S,C} \left( I^j_t + \Phi(I^j_t, K^j_t) \right) + \sum_{j=S,C} DWL^j_t \]

\[ = Y_t + Z_t \sum_{j=S,C} (N^j_t)^\eta \left( (1 - \ell^j_t) K^j_t \right)^{1-\eta} + Z_t (N^H_t)^\eta \left( \ell^H_t K^H_t \right)^{1-\eta}, \tag{63} \]

and the labor market

\[ N^S_t + N^C_t + N^H_t = 1. \tag{64} \]

The deadweight losses for each bank type are

\[ DWL^j_t = \xi^j_t \rho^j_t \rho_t^{-} \left( \ell^j_t \Pi^H_t + \left( 1 - \ell^j_t \right) \left( \Pi^j_t - (1 - \delta_K) p_t \right) \right) K^j_t, \]

and the adjustment costs for capital and investment amount to

\[ \Phi(I^j_t, K^j_t) = K^j_t \left( 1 - \ell^j_t \right) \left( \frac{\phi_I}{2} \left( i^j_t - \delta_K \right)^2 + \frac{\phi_K}{2} \left( k^j_t - 1 \right)^2 \right). \]

Note that commercial banks are isolated from bank runs, and so \( \ell^C_t = 0 \ \forall t \). The market clearing condition for capital in Equation (62) is also the transition law for the aggregate capital stock.

**B.6 Equilibrium Conditions**

The solution of the model can be written as a system of 17 nonlinear functional equations in equally many unknown functions of the state variables. The model’s state variables are \( S_t = (Y_t, Z_t, \omega_t, K^C_t, K^S_t, A^C_t, A^S_t) \).

The functions are aggregate consumption \( C(S_t) \), prices of C-bank and S-bank equity \( (p^C(S_t), p^S(S_t)) \), prices of C-bank and S-bank deposits \( (q^C(S_t), q^S(S_t)) \), C-bank and S-bank deposit issuance per unit of capital \( (b^C(S_{t+1}), b^S(S_{t+1})) \), the Lagrange multiplier on C-bank leverage \( \lambda^C(S_t) \), C-bank and S-bank capital purchases \( (K^C(S_{t+1}), K^S(S_{t+1})) \), the capital price \( p(S_t) \), C-bank and S-bank investment \( (I^C(S_t), I^S(S_t)) \), labor demand of C-bank, S-bank and households \( (N^C(S_t), N^S(S_t), N^H(S_t)) \), and the wage \( w(S_t) \). For the equations, we will use time subscripts und suppress the dependence on state variables. All variables can be expressed as functions of current \( (S_t) \) or one-period ahead \( (S_{t+1}) \) state variables.

The equations are
\[
\begin{align*}
\dot{p}_t^C &= E_t \left[ M_{t,t+1} F_{\rho_t}^C \left( D_{t+1}^{C,+} + p_{t+1}^C \right) \right] \\
\dot{p}_t^S &= E_t \left[ M_{t,t+1} F_{\rho_t}^S \left( D_{t+1}^{S,+} + p_{t+1}^S \right) \right] \\
\dot{q}_t^C &= E_t \left[ M_{t,t+1} \left( 1 + MRS_{t+1}^C \right) \right] \\
\dot{q}_t^S &= E_t \left[ M_{t,t+1} \left( 1 - F_{\rho_t}^S \left( 1 - (\pi_B + (1 - \pi_B) r_{t+1}^S) \right) + MRS_{t+1}^S \right) \right] \\
\lambda_t^C + I_t^C + I_t^S + \Phi(\ell_t^C, K_t^C) + \Phi(\ell_t^S, (1 - \ell_t^S) K_t^S) &= Y_t + Y_t^C + Y_t^S + Y_t^H \\
&- \xi_t^C F_t^C \rho_t^C \left( \Pi_t^C - (1 - \delta_t) p_t \right) K_t^C \\
&- \xi_t^S F_t^S \rho_t^S \left( \ell_t^S \Pi_t^H + (1 - \ell_t^S) (\Pi_t^S - (1 - \delta_t) p_t) \right) K_t^S \\
\dot{q}_t^S + b_{t+1}^S q_t^S &= E_t \left[ M_{t,t+1} \left( 1 - F_{\rho_t}^S \left( 1 - (1 - x_{t+1}) \rho_{t+1}^S \right) \right) \right] \\
\dot{q}_t^C - \kappa &= \lambda_t^C \left( p_t - (1 - \theta) b_{t+1}^C \right) = 0 \\
\dot{p}_t - \dot{q}_t^b b_{t+1}^b + \phi_K \left( k_t^S - 1 \right) &= E_t \left[ M_{t,t+1} \Pi_t^S \Omega^S \left( L_t^S \right) \right], \\
\dot{p}_t - \dot{q}_t^b b_{t+1}^b + \phi_K \left( k_t^S - 1 \right) &= E_t \left[ M_{t,t+1} \Pi_t^S \Omega^C \left( L_t^C \right) \right] \\
K_t^C + K_t^S &= I_t^C + I_t^S + (1 - \delta_t) \left( 1 - \xi_t^C F_t^C \rho_t^C \right) K_t^C \\
&+ (1 - \delta_t) \left( 1 - \xi_t^S F_t^S \rho_t^S \right) (1 - \ell_t^S) K_t^S + (1 - \ell_t^S) \ell_t^S K_t^S \\
I_t^C &= \left( \frac{p_t - 1}{\phi_t} + \delta_t \right) K_t^C \\
I_t^S &= \left( \frac{p_t - 1}{\phi_t} + \delta_t \right) (1 - \ell_t^S) K_t^S \\
w_t &= \eta Z_t (n_t^C)^{\eta-1} \\
N_t^C &= \frac{K_t^C}{K_t^C + (1 - \ell_t^S) K_t^S + \ell_t^S K_t^S (Z_t / Z_t)^{1/(1-\eta)}} \quad \text{(E15)} \\
N_t^S &= \frac{(1 - \ell_t^S) K_t^S}{K_t^C n_t^C} \quad \text{(E16)} \\
N_t^H &= 1 - N_t^C - N_t^S \quad \text{(E17)}
\end{align*}
\]

(E1) – (E4) are the household Euler equations for bank equity and debt from equations (53) applied to \( j = C, S \), (56), and (57). (E5) is the resource constraint from (63). (E6) is the S-bank condition for leverage from (58). (E7) is the C-bank condition for leverage (61), with (E8) being the complementary slackness condition for the leverage constraint (35). (E9) and (E10) are the S-bank and C-bank conditions for capital growth from (52), applied to either bank type. (E11) is the market clearing condition for capital (62), and (E12) – (E13) are the first-order conditions for
investment by banks from (46), applied to \( j = C, S \). \( (E14) \) – \( (E16) \) are the first-order conditions for labor demand by banks and households, from (45) applied to \( j = C, S, H \), and \( (E17) \) is the market clearing condition for labor.

### B.7 Computational Solution Method

The equilibrium of dynamic stochastic general equilibrium models is usually characterized recursively. If a stationary Markov equilibrium exists, there is a minimal set of state variables that summarizes the economy at any given point in time. Equilibrium can then be characterized using two types of functions: transition functions map today’s state into probability distributions of tomorrow’s state, and policy functions determine agents’ decisions and prices given the current state. Brumm et al. (2018) analyze theoretical existence properties in this class of models and discuss the literature. Perturbation-based solution methods find local approximations to these functions around the “deterministic steady-state”. For applications in finance, there are often several problems with local solution methods. First, portfolio restrictions such as leverage constraints may be occasionally binding in the true stochastic equilibrium. Generally, a local approximation around the steady state (with a binding or slack constraint) will therefore inaccurately capture nonlinear dynamics when constraints go from slack to binding. Further, local methods have difficulties in dealing with highly nonlinear functions within the model such as probability distributions or option-like payoffs, as is the case for the quantitative model in this paper. Finally, in models with rarely occurring bad shocks (such as the runs in our model), the steady state used by local methods may not properly capture the ergodic distribution of the true dynamic equilibrium.

Global projection methods (Judd (1998)) avoid these problems by not relying on the deterministic steady state. Rather, they directly approximate the transition and policy functions in the relevant area of the state space.

### B.8 Solution Procedure

The projection-based solution approach used in this paper has three main steps.

**Step 1. Define approximating basis for the policy and transition functions.** To approximate these unknown functions, we discretize the state space and use multivariate linear interpolation. Our general solution framework provides an object-oriented MATLAB library that allows approximation of arbitrary multivariate functions using linear interpolation, splines, or polynomials. For the model in this paper, splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation.

**Step 2. Iteratively solve for the unknown functions.** Given an initial guess for policy and transition functions, at each point in the discretized state space compute the current-period optimal policies. Using the solutions, compute the next iterate of the transition functions. Repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. This step is completely parallelized across points in the state space within each iterate.

**Step 3. Simulate the model for many periods using approximated functions.** Verify that the simulated time path stays within the bounds of the state space for which policy and transition
functions were computed. Calculate relative Euler equation errors to assess the computational accuracy of the solution. If the simulated time path leaves the state space boundaries or errors are too large, the solution procedure may have to be repeated with optimized grid bounds or positioning of grid points.

We will now provide a more detailed description for each step.

Step 1 The state space consists of
- two exogenous state variables \([Y_t, \varrho_t]\), and
- four endogenous state variables \([K_t, K^S_t, B^S_t, B^C_t]\).

The banking sector specific shock \(Z_t\) does not contain any persistent shocks in addition to \(Y_t\) and is therefore not an additional state variable. We first discretize \(Y_t\) into a \(N^Y\)-state Markov chain using the Rouwenhorst (1995) method, where \(N^Y\) is an odd number. The procedure chooses the productivity grid points \(\{Y_j\}_{j=1}^{N^Y}\) and the \(N^Y \times N^Y\) Markov transition matrix \(\Pi_Y\) between them to match the volatility and persistence of GDP growth of the bank independent sector. The run shock \(\varrho_t\) can take on two realizations \(\{0, \varrho^*\}\) as described in the calibration section. The 2 x 2 Markov transition matrix between these states is given by \(\Pi_\varrho\). We assume that run shocks only occur in states with negative GDP growth. Denote the set of the \(N^x = N^Y + (N^Y - 1)/2\) values the exogenous state variables can take on as \(S_x\), and the associated Markov transition matrix \(\Pi_x\).

Our solution algorithm requires approximation of continuous functions of the endogenous state variables. Define the “true” endogenous state space of the model as follows: if each endogenous state variable \(S_t \in \{K_t, K^S_t, B^S_t, B^C_t\}\) can take on values in a continuous and convex subset of the reals, characterized by constant state boundaries, \([S_l, S_u]\), then the endogenous state space \(S_n = [K_l, K_u] \times [K^S_l, K^S_u] \times [B^S_l, B^S_u] \times [B^C_l, B^C_u]\). The total state space is the set \(S = S_x \times S_n\).

To approximate any function \(f: S \rightarrow \mathcal{R}\), we form an univariate grid of (not necessarily equidistant) strictly increasing points for each endogenous state variables, i.e., we choose \(\{K_j\}_{j=1}^{N^K}, \{K^S_j\}_{j=1}^{N^K^S}, \{B^S_j\}_{j=1}^{N^B^S}, \{B^C_j\}_{j=1}^{N^B^C}\). These grid points are chosen to ensure that each grid covers the ergodic distribution of the economy in its dimension, and to minimize computational errors, with more details on the choice provided below. Denote the set of all endogenous-state grid points as \(\hat{S}_n = \{K_j\}_{j=1}^{N^K} \times \{K^S_j\}_{j=1}^{N^K^S} \times \{B^S_j\}_{j=1}^{N^B^S} \times \{B^C_j\}_{j=1}^{N^B^C}\), and the total discretized state space as \(\hat{S} = S_x \times \hat{S}_n\). This discretized state space has \(N^\hat{S} = N^x \cdot N^K \cdot N^K^S \cdot N^B^S \cdot N^B^C\) total points, where each point is a 5 x 1 vector as there are 5 distinct state variables (counting the exogenous state as one). We can now approximate the smooth function \(f\) if we know its values \(\{f_j\}_{j=1}^{N^\hat{S}}\) at each point \(\hat{s} \in \hat{S}\), i.e. \(f_j = f(\hat{s}_j)\) by multivariate linear interpolation.

Our solution method requires approximation of of three sets of functions defined on the domain of the state variables. The first set of unknown functions \(C_P: S \rightarrow \mathcal{P} \subseteq \mathcal{R}^{N^C}\), with \(N^C\) being the number of policy variables, determines the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the prices, agents’ choice variables, and the Lagrange multipliers on the portfolio constraints. Specifically, the 8 policy functions are bond prices \(q^S(S), q^C(S)\), capital price \(p(S)\), debt issued by banks in the current period \(B^S(S), B^C(S)\), the capital purchased by S-banks \(K^S(S)\), labor demand of S-banks \(n^S(S)\), and the Lagrange multiplier for the C-bank leverage constraint \(\lambda^C(S)\). There is an equal number of these unknown functions and nonlinear functional equations, to be listed under step 2 below.
The second set of functions \( C_T : S \times S_x \to S_n \) determine the next-period endogenous state variable realizations as a function of the state in the current period and the next-period realization of exogenous shocks. There is one transition function for each endogenous state variable, corresponding to the transition law for each state variable, also to be listed below in step 2.

The third set are forecasting functions \( C_F : S \to F \subseteq R^{N_F} \), where \( N_F \) is the number of forecasting variables. They map the state into the set of variables sufficient to compute expectations terms in the nonlinear functional equations that characterize equilibrium. They partially coincide with the policy functions. In particular, the forecasting functions for our model are the capital price \( p(S), \) S-bank labor input \( n^S(S), \) capital growth of both types of banks \( k^S(S), k^C(S), \) and the value function of households \( V^H(S) \) (to compute welfare).

**Step 2** Given an initial guess \( C^0 = \{C^0_P, C^0_T, C^0_F\} \), the algorithm to compute the equilibrium takes the following steps.

A. Initialize the algorithm by setting the current iterate \( C^m = \{C^m_P, C^m_T, C^m_F\} = \{C^0_P, C^0_T, C^0_F\} \).

B. Compute forecasting values. For each point in the discretized state space, \( s_j \in \hat{S}, j = 1, \ldots, N^S \), perform the steps:

   i. Evaluate the transition functions at \( s_j \) combined with each possible realization of the exogenous shocks \( x_i \in S_x \) to get \( s'_j(x_i) = C^m_T(s_j, x_i) \) for \( i = 1, \ldots, N^x \), which are the values of the endogenous state variables given the current state \( s_j \) and for each possible future realization of the exogenous state.

   ii. Evaluate the forecasting functions at these future state variable realizations to get \( f^m_{ij} = C^m_F(s'_j(x_i), x_i) \).

   The end result is a \( N^x \times N^S \) matrix \( F^m \), with each entry being a vector

\[
[ p_{ij}, n^S_{ij}, n^C_{ij}, k^S_{ij}, k^C_{ij}, V^H_{ij} ]
\]

of the next-period realization of the forecasting functions for current state \( s_j \) and future exogenous state \( x_i \).

C. Solve system of nonlinear equations. At each point in the discretized state space, \( s_j \in \hat{S}, j = 1, \ldots, N^S \), solve the system of nonlinear equations that characterize equilibrium in the equally many “policy” variables, given the forecasting matrix \( F^m \) from step B. This amounts to solving a system of 12 equations in 12 unknowns

\[
[ \hat{p}_j, \hat{q}_j^S, \hat{q}_j^C, \hat{\beta}_j^S, \hat{\beta}_j^C, \hat{K}_j^S, \hat{K}_j^C, \hat{n}_j^S, \hat{\lambda}_j^S, \hat{\lambda}_j^C ]
\]
at each $s_j$. The equations are

\begin{align}
\hat{q}_j^C &= E_{s_j | s_j} \left[ \hat{M}_{ij} \left( 1 + \text{MRS}_{ij}^C \right) \right] \\
\hat{q}_j^S &= E_{s_j | s_j} \left[ \hat{M}_{ij} \left( 1 - F_{ij}^S \left( 1 - (\pi_B + (1 - \pi_B) r_{ij}) \right) + \text{MRS}_{ij}^S \right) \right] \\
\hat{q}_j^S + b_j^S q_j'(b_j^S) &= E_{s_j | s_j} \left[ \hat{M}_{ij} (1 - F_{ij}^S) \left( 1 + \frac{\ell_{ij}}{\ell_{ij}} \left( 1 - x_{ij} \right) \rho_{ij}^S + \frac{v_{ij}^S}{\Pi_{ij}^S} \right) \right] \\
p_j - \hat{q}_j^S b_j^S + \phi_{K} (\hat{k}_j^S - 1) &= E_{s_j | s_j} \left[ \hat{M}_{ij} \Pi_{ij}^S \Omega^S \left( L_{ij}^S \right) \right] \\
\hat{q}_j^C - \kappa &= \hat{\lambda}_j^C + E_{s_j | s_j} \left[ \hat{M}_{ij} (1 - F_{ij}^C) \right] \\
p_j - (\hat{q}_j^C - \kappa) b_j^C + \phi_{K} (\hat{k}_j^C - 1) &= E_{s_j | s_j} \left[ \hat{M}_{ij} \Pi_{ij}^C \Omega^C \left( L_{ij}^C \right) \right] \\
\hat{\lambda}_j^C \left( p_j - (1 - \theta) b_j^C \right) &= 0 \\
1 &= \hat{N}_j^H + \hat{N}_j^C + \hat{N}_j^S.
\end{align}

(C1) and (C2) are the household Euler equations for purchases of deposits. (C3) and (C4) are the intertemporal optimality conditions for S-banks, and (C5) and (C6) are those for C-banks. (C7) is the leverage constraint for C-banks. Finally, (C8) is the market clearing condition labor.

Expectations are computed as weighted sums, with the weights being the probabilities of transitioning to exogenous state $x_{ij}$, conditional on state $s_j$. Hats (‘) in (C1) – E(C8) indicate variables that are direct functions of the vector of unknowns (P). These are effectively the choice variables for the nonlinear equation solver that finds the solution to the system (C1) – (C8) at each point $s_j$. All variables in the expectation terms with subscript $i,j$ are direct functions of the forecasting variables (F).

The latter values are fixed numbers when the system is solved, as they we pre-computed in step B. For example, the stochastic discount factor $\hat{M}_{ij}$ depends on both the solution and the forecasting vector, i.e.

$$\hat{M}_{ij} = \beta \left( \frac{C_{ij}}{C_{j}} \right)^{-\gamma},$$

since it depends on future and current consumption. To compute the expectation of the right-hand side of equation (C1) at point $s_j$, we first look up the corresponding column $j$ in the matrix containing the forecasting values that we computed in step B, $\mathcal{F}^m$. This column contains the $N_x$ vectors, one for each possible realization of the exogenous state, of the forecasting values defined in (F). From these vectors, we need consumption $C_{ij}$. Further, we need current consumption $\hat{C}_j$, which is a policy variable chosen by the nonlinear equation solver. $\text{MRS}_{ij}^C$ is a function of future consumption $C_{ij}$, and the future state variables $B_{ij}^S$ and $B_{ij}^C$ (since market clearing implies $A_{ij}^j = B_{ij}^j$ for $j = S, C$). Denoting the probability of moving from current exogenous state $x_j$ to state $x_i$ as $\pi_{ij,j}$, we compute the expectation of the RHS of (C1)

$$E_{s_j | s_j} \left[ \hat{M}_{ij} \left( 1 + \text{MRS}_{ij}^C \right) \right] = \sum_{x_j | x_j} \pi_{ij,j} \hat{M}_{ij} \left( 1 + \text{MRS}_{ij}^C \right).$$

64
The mapping of solution and forecasting vectors \( P \) and \( F \) into the other expressions in equations (C1) – (C8) follows the same principles and is based on the equations in model appendix B. In particular, the system (C1) – (C8) implicitly uses the budget constraints of all agents, and the market clearing conditions for capital and debt of both banks.

Note that we could exploit the linearity of the market clearing condition in (C8) to eliminate one more policy variable, \( \tilde{n}^S \), from the system analytically. However, in our experience the algorithm is more robust when we explicitly include labor demand of all agents as policy variables, and ensure that these variables stay strictly positive (as required with CD production functions) when solving the system. To solve the system in practice, we use a nonlinear equation solver that relies on a variant of Newton’s method, using policy functions \( C^m_p \) as initial guess. More on these issues in subsection B.9 below.

The final output of this step is a \( N^S \times 12 \) matrix \( \mathcal{P}^{m+1} \), where each row is the solution vector \( \hat{P}_j \) that solves the system (C1) – (C8) at point \( s_j \).

D. **Update forecasting, transition and policy functions.** Given the policy matrix \( \mathcal{P}^{m+1} \) from step B, update the policy function directly to get \( C^{m+1}_p \). All forecasting functions with the exception of the value functions are also equivalent to policy functions. The household value function is updated based on the recursive definition

\[
\hat{V}_j^H = U(\hat{C}_j, H_{i,j}) + \beta \hat{E}_{s_j|s_j} V^H_{i,j}
\]

using the same notation as defined above under step C. Note that the value function combines current solutions from \( \mathcal{P}^{m+1} \) (step C) for consumption with forecasting values from \( \mathcal{F}^m \) (step B). Using these updated value functions, we get \( \hat{C}^{m+1} \).

Finally, update transition functions for the endogenous state variables using the following laws of motion, for current state \( s_j \) and future exogenous state \( x_i \) as defined above:

\[
K^{m+1}_{ij} = \tilde{K}^C_j + \tilde{K}^S_j + (1 - \delta_K) \left( 1 - \tilde{\xi}_F^C \tilde{F}_{ij} \tilde{\rho}^C_{ij} \right) K^C_{ij} \\
+ (1 - \delta_K) \left( 1 - \tilde{\xi}_S^S \tilde{F}_{ij} \tilde{\rho}^S_{ij} \right) (1 - \tilde{\rho}^S_{ij}) K^S_{ij} + (1 - \tilde{\delta}_K) \tilde{\delta}^S_{ij} K^S_{ij}
\]

\[
(K^{S}_{ij})^{m+1} = \hat{K}^S_j K^S_{ij}
\]

\[
(B^{C}_{ij})^{m+1} = \hat{B}^C_j
\]

\[
(B^{S}_{ij})^{m+1} = \hat{B}^S_j
\]

(T1) is simply the law of motion for aggregate capital, and (T2) is the definition of capital growth \( k^S \). (T3) and (T4) follow directly from the direct mapping of policy into state variable for bank debt. Updating according to (T1) – (T4) gives the next set of functions \( \hat{C}^{m+1}_T \).

E. **Check convergence.** Compute distance measures \( \Delta_F = \| C^{m+1}_F - C^m_F \| \) and \( \Delta_T = \| C^{m+1}_T - C^m_T F^m \| \). If \( \Delta_F < \text{Tol}_F \) and \( \Delta_T < \text{Tol}_T \), stop and use \( C^{m+1} \) as approximate solution. Otherwise reset policy functions to the next iterate i.e. \( \mathcal{P}^m \rightarrow \mathcal{P}^{m+1} \) and reset forecasting and transition functions to a convex combination of their previous and updated values i.e. \( C^m \rightarrow C^{m+1} = D \times C^m + (1 - D) \times \hat{C}^{m+1} \), where \( D \) is a dampening parameter set to a value between 0 and 1 to reduce oscillation in function values in successive iterations. Next, go to step B.
Step 3 Using the numerical solution $C^* = C^{m+1}$ from step 2, we simulate the economy for $\bar{T} = T_{ini} + T$ period. Since the exogenous shocks follow a discrete-time Markov chain with transition matrix $\Pi_x$, we can simulate the chain given any initial state $x_0$ using $\bar{T} - 1$ uniform random numbers based on standard techniques (we fix the seed of the random number generator to preserve comparability across experiments). Using the simulated path $\{x_t\}_{t=1}^{\bar{T}}$, we can simulate the associated path of the endogenous state variables given initial state $s_0 = [x_0, K_0, K^S_0, B^C_0, B^C_S]$ by evaluating the transition functions

$$[K_{t+1}, K^S_{t+1}, B^C_{t+1}, B^C_{t+1}] = C^*_T(s_t, x_{t+1}),$$

to obtain a complete simulated path of model state variables $\{s_t\}_{t=1}^{\bar{T}}$. To remove any effect of the initial conditions, we discard the first $T_{ini}$ points. We then also evaluate the policy and forecasting functions along the simulated sample path to obtain a complete sample path $\{s_t, P_t, f_t\}_{t=1}^{\bar{T}}$.

To assess the quality and accuracy of the solution, we perform two types of checks. First, we verify that all state variable realizations along the simulated path are within the bounds of the state variable grids defined in step 1. If the simulation exceeds the grid boundaries, we expand the grid bounds in the violated dimensions, and restart the procedure at step 1. Secondly, we compute relative errors for all equations of the system (C1) – (C8) and the transition functions (T1) – (T4) along the simulated path. For equations involving expectations (such as (C1)), this requires evaluating the transition and forecasting function as in step 2 at the current state $s_t$. For each equation, we divide both sides by a sensibly chosen endogenous quantity to yield “relative” errors; e.g., for (C1) we compute

$$1 = \frac{1}{q^C_{ij}} \mathbb{E}_{s_{ij}|s_j}[\hat{M}_{ij} (1 + \text{MRS}^C_{ij})],$$

using the same notation as in step 2B. These errors are small by construction when calculated at the points of the discretized state grid $\hat{S}$, since the algorithm under step 2 solved the system exactly at those points. However, the simulated path will likely visit many points that are between grid points, at which the functions $C^*$ are approximated by interpolation. Therefore, the relative errors indicate the quality of the approximation in the relevant area of the state space. We report average, median, and tail errors for all equations. If errors are too large during simulation, we investigate in which part of the state space these high errors occur. We then add additional points to the state variable grids in those areas and repeat the procedure.

B.9 Implementation

Solving the system of equations. We solve system of nonlinear equations at each point in the state space using a standard nonlinear equation solver (MATLAB’s fsolve). This nonlinear equation solver uses a variant of Newton’s method to find a “zero” of the system. We employ several simple modifications of the system (C1) – (C8) to avoid common pitfalls at this step of the solution procedure. Nonlinear equation solver are notoriously bad at dealing with complementary slackness conditions associated with a constraint. Judd, Kubler, and Schmedders (2002) discuss the reasons for this and also show how Kuhn-Tucker conditions can be rewritten as additive equations for this purpose. Consider the C-bank’s Euler Equation for risk-free bonds and the
the levels of debt. For the benchmark case, the grid points in each state dimension are as follows:

Grid configuration. We choose to include the relative capital share of S-banks \( \bar{K}_t^S = K_t^S / K_t \) as state variable instead of borrower debt \( K_t^S \) such that the total set of endogenous state variables is \( \{K_t, \bar{K}_t^S, B_t^C, B_t^S\} \). The reason is that the capital share is more stable in the dynamics of the model than the level, since total capital and S-bank capital are strongly correlated. For similar reasons, we choose to include S-bank and C-bank book leverage \( b_t^S = B_t^S / K_t^S \) and \( b_t^C = B_t^C / K_t^C \) instead of the levels of debt. For the benchmark case, the grid points in each state dimension are as follows:

- \( Y \): We discretize \( Y \) and \( Z \) jointly into a 9-state Markov chain (with three possible realizations for each) using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points \( \{Y\}_j \) and \( \{Z\}_j \) and the 9 \times 9 Markov transition matrix \( \Pi_{Y,Z} \) between them to match the volatility and persistence of GDP growth. This yields the possible realizations for \( Y \): \([0.9869, 1.0000, 1.0132]\), and for \( Z \): \([0.9698, 1.0000, 1.0312]\).

- \( q \): \([0.0, 0.3]\) (see calibration)
- \( K \): \([3.8, 3.87, 3.94, 4.01, 4.08, 4.15]\)
- \( \bar{K}_t^S \): \([0.26, 0.28, 0.30, 0.32, 0.34, 0.36, 0.38]\)
- \( b_t^S \): \([0.10, 0.22, 0.34, 0.46, 0.58, 0.71, 0.83, 0.95]\)
- \( b_t^C \): \([0.87, 0.882, 0.894, 0.906, 0.918, 0.93]\)
The total state space grid has 24,192 points. The grid boundaries and the placement of points have to be readjusted for each experiment, since the ergodic distribution of the state variables depends on parameters. Finding the right values for the boundaries is a matter of experimentation.

**Generating an initial guess and iteration scheme.** To find a good initial guess for the policy, forecasting, and transition functions, we solve the deterministic “steady-state” of the model under the assumption that the bank leverage constraint is binding and no runs are occurring. We then initialize all functions to their steady-state values, for all points in the state space. Note that the only role of the steady-state calculation is to generate an initial guess that enables the nonlinear equation solver to find solutions at (almost) all points during the first iteration of the solution algorithm. In our experience, this steady state delivers a good enough initial guess.

In case the solver cannot find solutions for some points during the initial iterations, we revisit such points at the end of each iteration. We try to solve the system at these “failed” points using as initial guess the solution of the closest neighboring point at which the solver was successful. This method works well to speed up convergence and eventually finds solutions at all points.

To determine convergence, we check absolute errors in the value function of households, \( V \). Out of all functions we approximate during the solution procedure, it exhibits the slowest convergence. We stop the solution algorithm when the maximum absolute difference between two iterations, and for all points in the state space, falls below 1e-3 and the mean distance falls below 1e-4. For appropriately chosen grid boundaries, the algorithm converges within 120 iterations.

We implement the algorithm in MATLAB and run the code on a high-performance computing (HPC) cluster. As mentioned above, the nonlinear system of equations can be solved in parallel at each point. We parallelize across 28 CPU cores of a single HPC node. From computing the initial guess and analytic Jacobian to simulating the solved model, the total running time for the benchmark calibration is about 2 hours and 40 minutes. Calibrations that exhibit more financial fragility and/or macro volatility converge up to 15% slower.

**Simulation.** To obtain the quantitative results, we simulate the model for 5,000 periods after a “burn-in” phase of 500 periods. The starting point of the simulation is the ergodic mean of the state variables. As described in detail above, we verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed. We fix the seed of the random number generator so that we use the same sequence of exogenous shock realizations for each parameter combination.

To produce impulse response function (IRF) graphs in Figure 2, we simulate 10,000 different paths of 25 periods each. In the initial period, we set the endogenous state variables to several different values that reflect the ergodic distribution of the states. We use a clustering algorithm to represent the ergodic distribution non-parametrically. We fix the initial exogenous shock realization to mean productivity \( \overline{Y} = \overline{Z} = 1 \) and no run \( \rho = 0 \). The “impulse” in the second period is either only a bad productivity shock, or both low productivity and a run shock \( \rho = 0.3 \). For the remaining 23 periods, the simulation evolves according to the stochastic law of motion of the shocks. In the IRF graphs, we plot the median path across the 10,000 paths given the initial condition. The transition dynamics in Figures 4 and 5 are constructed similarly, with the difference that the economy does not experience a productivity or run shock, but rather an unanticipated change in \( \theta \).
Table 4: Computational Errors

<table>
<thead>
<tr>
<th>Equation</th>
<th>50th</th>
<th>75th</th>
<th>95th</th>
<th>99th</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.60797E-05</td>
<td>1.98285E-05</td>
<td>2.33846E-05</td>
<td>0.00011435</td>
<td>0.000609577</td>
</tr>
<tr>
<td>C2</td>
<td>1.64198E-05</td>
<td>2.02402E-05</td>
<td>2.38312E-05</td>
<td>0.000114068</td>
<td>0.000600429</td>
</tr>
<tr>
<td>C3</td>
<td>1.81838E-05</td>
<td>2.22974E-05</td>
<td>2.62468E-05</td>
<td>0.000115962</td>
<td>0.000590462</td>
</tr>
<tr>
<td>C4</td>
<td>3.51337E-06</td>
<td>6.07882E-06</td>
<td>1.18653E-05</td>
<td>0.000108445</td>
<td>0.000234119</td>
</tr>
<tr>
<td>C5</td>
<td>3.89366E-06</td>
<td>6.20534E-06</td>
<td>1.15129E-05</td>
<td>0.000122232</td>
<td>0.000844679</td>
</tr>
<tr>
<td>C6</td>
<td>1.03779E-05</td>
<td>1.3081E-05</td>
<td>1.92823E-05</td>
<td>6.36015E-05</td>
<td>0.000318513</td>
</tr>
<tr>
<td>C7</td>
<td>0.000161026</td>
<td>0.000196303</td>
<td>0.000229379</td>
<td>0.000250181</td>
<td>0.003953529</td>
</tr>
<tr>
<td>C8</td>
<td>2.9076E-05</td>
<td>3.57655E-05</td>
<td>3.86194E-05</td>
<td>4.05661E-05</td>
<td>0.004224141</td>
</tr>
</tbody>
</table>

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 5,000 period simulation of the benchmark model. Each row contains errors for the respective equation of the nonlinear system (C1) – (C8) listed in step 2 of the solution procedure.

**Evaluating the solution.** Our main measure to assess the accuracy of the solution are relative equation errors calculated as described in step 3 of the solution procedure. Table 4 reports the median error, the 95th percentile of the error distribution, the 99th, and 100th percentiles during the 5,000 period simulation of the model. Median errors are very small for all equations, with even maximum errors only causing small approximation mistakes. Errors are comparably small for all experiments we report.

**C Calibration Appendix**

**C.1 Bank Technology and Other Parameters**

This section describes the remaining parameter choices not yet covered in the main text, listed in table 5. First, we can normalize the average output of the bank-independent sector $\mu_Y$ to one. We also set the average idiosyncratic shock received by banks $\mu_{j\rho}$, for $j = S, C$, to one, which means that absent default these shocks would aggregate away.

**Bank technology and adjustment costs.** We need to take a stand on the share of firms that depend on banks. In the U.S. a large part of the economy is in fact not directly dependent on banks as a large fraction of firms (in a value-weighted sense) can issue debt and equity in public markets. Yet, a sizeable share of the economy does depend on bank funding. We follow the definition in Kashyap, Lamont, and Stein (1994), where bank-dependent firms are those without a S&P long-term credit rating. Because mortgages make up the largest share of the bank loan portfolio, we also add construction and real estate firms as identified by SIC codes 6500-6599 (real estate), 1500-1599 (construction), and 1700-1799 (construction contractors, special trades) to the set of bank dependent firms. We operationalize this definition on Compustat data with the aggregate sales
data of bank dependent firms.\footnote{We use data from Compustat quarterly fundamentals (compm/fundq/) as well as Compustat’s credit rating database (compm/rating/).} We use this definition to parametrize the relative size of bank dependent productivity $\nu^Z$ and the investment technology parameters. We set $\nu^Z$ such that we match the ratio of bank-dependent firms’ value-added to total GDP. The GDP of bank dependent firms is measured as the aggregate revenue of bank-dependent firms. Accordingly, we set $\sigma^Z$ to match the volatility of this series (for a given $\sigma^Y$).

The labor share $\eta$ in bank-dependent production is set to standard value of 67%. Similarly, we set the depreciation rate $\delta_K$ to 2.5% quarterly.

Our model features two types of adjustment costs that are governed by the parameters $\phi_I$ and $\phi_K$, respectively. The value of $\phi_I$ determines the marginal cost of investment. We calibrate it to match the investment-asset ratio volatility of the bank dependent sector. The investment-asset ratio is mapped to the capital expenditure-asset ratio of bank-dependent firms in Compustat. Capital growth is also subject to adjustment costs that are governed by the parameter $\phi_K$. This introduces frictions into the capital flow between shadow- and commercial banks. We do not assume bank-type specific capital growth adjustment costs and thus could target either the asset growth volatility of shadow banks or commercial banks. We have much better data on the latter and use asset growth of commercial banks as a target that is measured as total financial assets of depository institutions from the Flow of Funds.

Table 5: Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^Y = \mu^C = \mu^S$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Normalization</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu^Z$</td>
<td>0.23</td>
<td>Value-added of bank-dependent sector</td>
<td>26%</td>
<td>27%</td>
</tr>
<tr>
<td>$\sigma^Z$</td>
<td>0.0217</td>
<td>Bank dependent output vol</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.67</td>
<td>NIPA labor share</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_K$</td>
<td>0.025</td>
<td>10% annual depreciation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_I$</td>
<td>1.3</td>
<td>Vol(Investment)</td>
<td>0.30%</td>
<td>0.31%</td>
</tr>
<tr>
<td>$\phi_K$</td>
<td>0.0185</td>
<td>Vol(C-bank asset growth)</td>
<td>0.50%</td>
<td>0.54%</td>
</tr>
<tr>
<td>$\sigma^C$</td>
<td>0.11</td>
<td>SD(Dividend per share)</td>
<td>10.6%</td>
<td>11%</td>
</tr>
<tr>
<td>$\sigma^S$</td>
<td>0.15</td>
<td>SD(Dividend per share)</td>
<td>14.9%</td>
<td>15%</td>
</tr>
<tr>
<td><strong>Bank regulation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>10%</td>
<td>Basel capital requirement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.142%</td>
<td>FDIC insurance fee</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bank-independent sector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^Y$</td>
<td>0.0093</td>
<td>Bank-independent sector vol</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho^Y$</td>
<td>0.366</td>
<td>Bank-independent sector AC</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>Risk aversion</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
cross-sectional dispersion in dividends per share for publicly traded commercial and “shadow banks” in the data. As for the calculation of shadow bank leverage, we define shadow banks as all institutions with SIC codes 6111-6299, 6798, 6799, 6722, 6726, excluding SIC codes 6200, 6282, 6022, 6199. The definition of commercial banks in the data is simpler, since here can use publicly traded depository institutions and bank holding companies. We compute the cross-sectional standard deviation of the dividend/share ratio for each quarter from 1999 to 2017 and take the time-series average of these standard deviations, which gives 10.6% for banks and 15% for shadow banks. These ratios directly correspond to the parameters $\sigma_{\rho C}$ and $\sigma_{\rho S}$ in the model.

In the model, we parameterize the idiosyncratic $\rho$ shocks as gamma distributions. Let the gamma cumulative distribution function be given by $\Gamma(\rho; \chi_0, \chi_1)$ with parameters $(\chi_0, \chi_1)$. These parameters map into means $\mu_{\rho}$ and variances $\sigma_{\rho}^2$ as follows:

$$\chi_1 = \sigma_{\rho}^2 / \mu_{\rho},$$
$$\chi_0 = \mu_{\rho} / \chi_1.$$

A standard result in statistics states that the conditional expectations are

$$E(\rho | \rho < x) = \mu_{\rho} \frac{\Gamma(x; \chi_0 + 1, \chi_1)}{\Gamma(x; \chi_0, \chi_1)},$$
$$E(\rho | \rho > x) = \mu_{\rho} \frac{1 - \Gamma(x; \chi_0 + 1, \chi_1)}{1 - \Gamma(x; \chi_0, \chi_1)},$$

which we use to compute the conditional expectations $\rho^{h-}$ and $\rho^{h+}$ used in bank payoffs to shareholders and recovery values for creditors.

**Bank regulation.** We set the regulatory capital ratio $\theta$ in the baseline of the model to 10%. To calibrate the deposit insurance fee $\kappa_C$, we use data from the FDIC on banks’ insurance contributions. According to the 2016 FDIC report, banks paid $10 billion in FDIC insurance fees on an insurance fund balance of $83.162 billion, which represents 1.18% of insured deposits. This means that banks paid 14.2 bps per dollar of insured deposits (see also: https://www.fdic.gov/about/strategic/report/2016annualreport/ar16section3.pdf), which is the value we assign to $\kappa_C$.

**Bank-independent sector and preferences.** We calibrate the stochastic process for the endowment income $Y_t$ that households receive directly to the volatility and autocorrelation of real GDP per capita, net of the contribution of the bank-dependent sector. We further set risk aversion $\gamma$ to the standard value of 2.

### C.2 Details on Calibration of $\epsilon$

To motivate our regression design for calibrating $\epsilon$, we derive an equation for the model spread between the rate on S-bank and C-bank debt. The starting point are the household first-order conditions for holdings of the two types of debt, (56) and (57), under the simplifying assumptions that $\pi_B = 0$ and $\zeta_S = 1$ (these assumptions do not affect the fundamental conclusions from the derivation). Since we are looking for a simple empirical relationship, we further suppress the
expectations operators. Under these assumptions, the equations are

\[ q^C_t = M_{t,t+1} \left( 1 + \text{MRS}^C_{t+1} \right) \]
\[ q^S_t = M_{t,t+1} \left( 1 - F^S_{t,t+1} + \text{MRS}^S_{t+1} \right), \]

where

\[ \text{MRS}_{S,t} = \alpha \psi C^j_t H_t^{-\gamma_H} \left( \frac{H_t}{A^S_t} \right)^{1-\epsilon}, \quad (65) \]
\[ \text{MRS}_{C,t} = (1 - \alpha) \psi C^j_t H_t^{-\gamma_H} \left( \frac{H_t}{A^C_t} \right)^{1-\epsilon}, \quad (66) \]

and

\[ M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \]

We perform a first-order log-linear expansion of both conditions around the deterministic steady state of the model. Variables without time subscript denote steady state values, and hatted (\( \hat{\cdot} \)) variables denote log-deviations from steady state. The usual log-linearization techniques give

\[ \hat{q}^C_t = -\gamma \Delta \hat{C}_{t+1} + \frac{\beta \text{MRS}^C}{q^C_t} \hat{\text{MRS}}^C_{t+1}, \]

and

\[ \hat{q}^S_t = -\gamma \Delta \hat{C}_{t+1} + \frac{\beta \text{MRS}^S}{q^S_t} \hat{\text{MRS}}^S_{t+1} - \frac{\beta F^S}{q^S_t} \hat{F}^S_{t+1}. \]

Further expanding

\[ \hat{\text{MRS}}^j_t = \gamma \hat{C}_t + (1 - \epsilon - \gamma_H) \hat{H}_t - (1 - \epsilon) \hat{A}^j_t, \]

and

\[ \hat{H}_t = \alpha \left( \frac{A^S_t}{H^e} \right)^\epsilon \hat{A}^S_t + (1 - \alpha) \left( \frac{A^C_t}{H^e} \right)^\epsilon \hat{A}^C_t, \]

we can compute the spread \( \hat{q}^C_t - \hat{q}^S_t \) and collect terms to get

\[ \hat{q}^C_t - \hat{q}^S_t = \beta \left( \frac{\text{MRS}^C}{q^C_t} - \frac{\text{MRS}^S}{q^S_t} \right) \gamma \hat{C}_{t+1} + \frac{\beta F^S}{q^S_t} \hat{F}^S_{t+1} \]
\[ + \beta \left( (1 - \epsilon - \gamma_H) \left( \frac{\text{MRS}^C}{q^C_t} - \frac{\text{MRS}^S}{q^S_t} \right) \right) \alpha \left( \frac{A^S_t}{H^e} \right)^\epsilon + (1 - \epsilon) \frac{\text{MRS}^S}{q^S_t} \hat{A}^S_{t+1} \]
\[ + \beta \left( (1 - \epsilon - \gamma_H) \left( \frac{\text{MRS}^C}{q^C_t} - \frac{\text{MRS}^S}{q^S_t} \right) (1 - \alpha) \left( \frac{A^C_t}{H^e} \right)^\epsilon - (1 - \epsilon) \frac{\text{MRS}^C}{q^C_t} \right) \hat{A}^C_{t+1}. \quad (67) \]

The coefficients before \( \hat{A}^S_{t+1} \) and \( \hat{A}^C_{t+1} \) in equation (67) reveal the role of \( \gamma_H \) and \( \epsilon \) for the effect of debt quantities on the spread. Clearly, if \( \epsilon = 1 \) (perfect substitutes) and \( \gamma_H = 0 \) (constant returns...
in total liquidity), the quantity terms vanish. If $\epsilon = 1$ and $\gamma_H > 0$, the equation becomes

$$
\hat{q}_t^C - \hat{q}_t^S = \beta \left( \frac{\text{MRS}^C}{q^C} - \frac{\text{MRS}^S}{q^S} \right) \gamma \hat{C}_{t+1} + \frac{\beta F^S}{q^S} \hat{F}^S_{t+1} \\
- \gamma_H \beta \left( \frac{\text{MRS}^C}{q^C} - \frac{\text{MRS}^S}{q^S} \right) \left( \alpha \frac{A^S}{H} \hat{A}^S_{t+1} + (1 - \alpha) \frac{A^C}{H} \hat{A}^C_{t+1} \right),
$$

(68)

or equivalently

$$
\hat{q}_t^C - \hat{q}_t^S = \beta \left( \frac{\text{MRS}^C}{q^C} - \frac{\text{MRS}^S}{q^S} \right) \gamma \hat{C}_{t+1} + \frac{\beta F^S}{q^S} \hat{F}^S_{t+1} - \gamma_H \beta \left( \frac{\text{MRS}^C}{q^C} - \frac{\text{MRS}^S}{q^S} \right) \hat{H}_{t+1},
$$

i.e., in that case only total liquidity $\hat{H}_t$ matters. Further, as we can see from equation (68), if both quantities are included separately in the equation, they should enter with a coefficient of the same sign.

### Table 6: Elasticity of Substitution Regression

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef</td>
<td>t-stat</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0006</td>
<td>-0.34</td>
</tr>
<tr>
<td>log(S-bank liquidity/GDP)</td>
<td>0.0019</td>
<td>1.44</td>
</tr>
<tr>
<td>log(Tbills/GDP)</td>
<td>-0.0013</td>
<td>-2.31</td>
</tr>
<tr>
<td>log(C-bank liquidity/GDP)</td>
<td>-0.0019</td>
<td>-1.80</td>
</tr>
<tr>
<td>GDP growth</td>
<td>-0.0002</td>
<td>-0.22</td>
</tr>
<tr>
<td>VIX</td>
<td>-0.0080</td>
<td>-0.47</td>
</tr>
<tr>
<td>S-bank defaults</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.29</td>
<td></td>
</tr>
</tbody>
</table>

However, the empirically relevant case is a combination of imperfect substitutes ($\epsilon < 1$) and decreasing returns ($\gamma_H > 0$). If further the difference between liquidity premia, $\frac{\text{MRS}^C}{q^C} - \frac{\text{MRS}^S}{q^S}$, is small relative to the level $\frac{\text{MRS}^j}{q^j}$, then we would expect the second term in each quantity coefficient in (67), $(1 - \epsilon) \frac{\text{MRS}^j}{q^j}$, to dominate. In that case, we would expect a positive coefficient on the S-bank quantity, and a negative coefficient on the C-bank quantity. This is exactly what we find in the data, and we choose $\epsilon$ in the model (for a given $\gamma_H$) to match this pattern.

In particular, for the sample from 1999:Q1 to 2017:Q4, we regress the CP/Tbill spread on log of the TBill/GDP ratio, the log of “shadow bank” debt relative to GDP, the VIX, and log GDP growth. The quantity of shadow bank liabilities is the sum of REPO claims held by shadow banks (Flow of Funds table L.207), total money market mutual fund assets (table L.206), and commercial paper held by the domestic financial sector less depository institutions (table L.209). The TBill supply series is directly computed from Treasury department reports, as in Greenwood, Hanson, and Stein (2016) and Nagel (2016).

Table 6 shows regression results for data and model in the baseline calibration (with $\gamma_H = 1.7$). By setting $\epsilon = 0.42$, the model matches the coefficient on S-bank debt to GDP in the data. Note
that both in data and model, the coefficient on the other source of liquidity (Tbills in the data and C-bank debt in the model) has the expected sign.

C.3 Results Appendix

Table 7: Effect of Liquidity Preference Parametrization on Capital Requirement Results

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>$\epsilon = 0$</th>
<th>$\gamma_H = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 15%$</td>
<td>$\theta = 20%$</td>
<td>$\theta = 15%$</td>
</tr>
<tr>
<td>1. Capital</td>
<td>+0.4%</td>
<td>+0.9%</td>
</tr>
<tr>
<td>2. Debt share S</td>
<td>+4.9%</td>
<td>+8.1%</td>
</tr>
<tr>
<td>3. Capital share S</td>
<td>+1.0%</td>
<td>-0.1%</td>
</tr>
<tr>
<td>4. Leverage S</td>
<td>+0.3%</td>
<td>+0.6%</td>
</tr>
<tr>
<td>5. Leverage C</td>
<td>-5.6%</td>
<td>-11.1%</td>
</tr>
<tr>
<td>6. Deposit rate S</td>
<td>-1.3%</td>
<td>-3.0%</td>
</tr>
<tr>
<td>7. Deposit rate C</td>
<td>-6.6%</td>
<td>-13.0%</td>
</tr>
<tr>
<td>8. Convenience Yield S</td>
<td>+3.6%</td>
<td>+8.0%</td>
</tr>
<tr>
<td>9. Convenience Yield C</td>
<td>+8.4%</td>
<td>+16.4%</td>
</tr>
<tr>
<td>10. Default Rate S</td>
<td>+6.7%</td>
<td>+15.1%</td>
</tr>
<tr>
<td>11. Default Rate C</td>
<td>-85.6%</td>
<td>-98.7%</td>
</tr>
<tr>
<td>12. GDP</td>
<td>+0.0%</td>
<td>+0.1%</td>
</tr>
<tr>
<td>13. Liquidity Services</td>
<td>-3.8%</td>
<td>-7.3%</td>
</tr>
<tr>
<td>14. Consumption</td>
<td>+0.17%</td>
<td>+0.20%</td>
</tr>
<tr>
<td>15. Welfare</td>
<td>+0.127%</td>
<td>+0.116%</td>
</tr>
</tbody>
</table>

This table presents the results from increases in the capital requirement using different liquidity preference calibrations. With each table entry we show for each variable of interest the percentage change in the mean relative to the economy with an capital requirement of 10%. The benchmark calibration reprints the results in Table 2 in columns 1 and 2. The columns headed with “$\epsilon = 0$” show the results for the capital requirement increase experiment for an economy with a low elasticity of substitution between C-bank and S-bank debt. The columns headed with “$\gamma_H = 0$” show the results for the capital requirement increase experiment for an economy with a low elasticity of substitution between C-bank and S-bank debt. The columns headed with “gammaH = 0” show the results for the capital requirement increase experiment for an economy without decreasing returns to scale in total liquidity.

Table 7 shows how the results for the capital requirement experiment depend on the liquidity parameters $\epsilon$ and $\gamma_H$. We present the results for our main capital requirement experiment using our benchmark calibration, and for the two variations described in the main text ($\epsilon = 0$ and $\gamma_H = 0$). For each variable in the table, we report the percentage change of its mean relative to an increase in the capital requirement from 10% to 15% and from 10% to 20%.

First of all, the table shows that the welfare results are not affected by changes in these parameters. The optimal capital requirement remains around 15% as the increase in consumption and the decrease in liquidity services are roughly identical across parametrizations. However, for the
question of financial fragility looking at the results using different liquidity function parametrizations is quite instructive.

When the capital requirement is increased, commercial banks reduce their leverage and therewith their supply of liquidity services. All else equal, aggregate liquidity falls in the economy. This increases the convenience yield of C-banks across all parametrizations and lowers C-banks’ deposit rate. What is the effect on shadow banks? We know from Section 2 that S-bank leverage is a function of the marginal benefit of leverage. With $\gamma_H = 0$, we turned off the feature that the marginal value of aggregate liquidity services is decreasing in the amount of liquidity services. Thus a reduction in aggregate liquidity services does not increase the marginal benefit of S-bank liquidity services. Without this feature, the convenience yield on S-banks’ falls, causing the deposit rate of S-banks to rise. Faced with higher debt-financing costs, S-banks reduce their leverage, which makes them safer. In this scenario, the effect of capital requirements on financial fragility are unambiguous: the economy is less risky even though more financial intermediation is performed by now less risky shadow banks. However, the overall amount of financial intermediation decreases, as can be seen from the reduction in the aggregate capital stock. This could be interpreted as a reduction in lending, which also translates into a small reduction in GDP. Together with a larger crunch in liquidity production, higher capital requirements lead therefore to lower welfare gains.

When we set $\epsilon$ to zero, households do not like to substitute between the two types of liquidity services. However, because $\gamma_H > 0$, a reduction in aggregate liquidity raises the marginal value and thus the convenience yield of any type of liquidity service. Relative to the benchmark, lower substitutability means that the increase (decrease) in S-banks’ convenience yield (deposit rate) is slightly smaller. Lower debt financing costs provide S-banks with an incentive to ramp up leverage and risk. The effect on overall financial fragility is thus ambiguous. In this calibration (as in the benchmark calibration) it turns out that aggregate deadweight losses decrease with tighter regulation. Another effect of $\gamma_H > 0$ is that the capital stock increases. This is due to the collateral value of capital in liquidity production. When the marginal value of liquidity increases, the collateral value of capital increases as well. This is also the reason for the small positive response in GDP to higher capital requirements.