

## **INVENTORY CONSIDERATIONS IN NETWORK DESIGN**

by

**Keely L. Croxton**  
*The Ohio State University*

and

**Walter Zinn**  
*The Ohio State University*

## **INVENTORY CONSIDERATIONS IN NETWORK DESIGN**

Optimization is accepted by logisticians as the appropriate approach to network design. While network design is used to make decisions regarding many aspects of logistics networks, one set of important decisions relates to the number, location, and size of warehouses in the network. Managers must also determine the customers to be supplied from each warehouse. Because of the costs involved in operating a network, managers often achieve significant cost reductions through network design (Jimenez, Brown, and Jordan 1998).

The logistics network decision is usually modeled as a trade-off between transportation and fixed warehousing costs. When there are few warehouses in the network, fixed warehousing costs are low, but transportation costs are high. As you add warehouses to the network, the fixed costs increase, but the transportation costs are reduced due to two factors. First, adding warehouses generally decreases the number of miles traveled because the total distance a unit is shipped, from the supplier to the warehouse and then to the customer, gets closer to a straight-line. In addition, the most expensive portion of that journey is often from the warehouse to the customer as those are usually less-than-truckload shipments. The average distance of this leg of the journey is reduced when there are more warehouses in the network.

What is missing in this standard trade-off is the inclusion of inventory cost. It is well known from the square root law and portfolio effect theory that inventory levels increase as the number of warehouses in the system increases (Maister 1976; Zinn, Levy, and Bowersox 1989). Accordingly, the main purpose of this research is to show that including inventory often significantly affects the optimal design of the network and that consequently managers should account for inventory when designing logistics networks.

This research also includes a model which can be used to optimally solve network design problems with inventory cost included. While logistics researchers acknowledge the importance of including inventory cost simultaneously with transportation and fixed warehousing cost in network design (Ballou 2001), there are currently no published studies that solve such problems to optimality. Ballou (2001) suggests that this absence is explained by the use of mixed-integer linear programming, which requires that cost relationships be linear, when in reality the relationship between number of warehouses and inventory is not linear. An approach using a piecewise linear approximation has been considered to circumvent this problem. However, instead we treat the relationship between the number of warehouses and inventory as a discrete function. This approach has a smaller impact on the size of the problem than modeling a piecewise linear function because it requires fewer constraints and variables. In fact, the size of the problem is not that much larger than the standard network design model (see Appendix A), and we are able to easily solve it to optimality.

More specifically, the goals of this paper are to: (1) propose a model that firms may use to include inventory in network design; (2) illustrate the application of the model with data collected from a national retailer; and (3) demonstrate the potential impact of the proposed model on network design. We show that benefits can be large or small under different conditions. The paper begins with a literature review. Next is the introduction of the proposed model that firms may use to include inventory cost in network design. This is followed by a scenario analysis that uses data from a national retailer based in the Midwest to illustrate the use of the proposed model and to examine how changes in key parameters influence its output. The final section contains the conclusions and implications.

## LITERATURE REVIEW

The network design literature is reviewed first with particular emphasis on the treatment of inventory cost. This is followed by a review of the inventory theories considered in this research: the square root law and the portfolio effect.

### Network Design

Practitioners and researchers have been using optimization to design logistics systems since the seventies (Geoffrion and Powers 1995). Most initial models focused on the distribution network, that is, locating warehouses and determining which customers should be supplied from each warehouse. As computing power increased, firms used similar models to design increasingly larger portions of the network (Arntzen et al. 1995; Pooley 1994).

In the standard network design problem, the objective is to minimize the sum of transportation and fixed warehousing costs, subject to capacity and demand constraints. This is usually modeled as a mixed-integer linear problem and solved using a variety of optimization techniques, most frequently branch and bound algorithms. It is assumed in standard models that inventory carrying costs are independent of the network design. That is, inventory levels remain constant regardless of

the number and location of warehouses. The inventory deployment decision is made once the network configuration has been decided (Graves, Willems, and Zipkin 2000; Miranda and Garrido 2004).

It is well recognized that ignoring inventory costs in network design models is a mistake. This was acknowledged as early as 1966 (Heskett). Ballou (2001) recognizes it as one of the primary limitations of today's network design models. Practitioners consider the lack of inventory treatment as one of the "worst features" of the models they currently use (Ballou and Masters 1999). Software companies have also recognized the limitation. Insight, Inc., specializing in network design software, and Optiant, specializing in inventory deployment software, have recently formed an alliance to link their systems so that the two problems can be solved in an iterative, but still serial, fashion (Insight, Inc. 2004).

In recognition of this limitation, there have been a few attempts to include inventory costs directly into a network design model. Both Baumol and Wolfe (1958) and Ballou (1984) include non-linear inventory costs. Each develops a heuristic to solve their resulting models. More recently Jayaraman (1998) adds inventory costs to a network design model, but only considers in-transit inventory and linear cycle stock, not safety stock. Miranda and Garrido (2004) include both cycle and safety stock but the resulting model has a non-linear objective function. It therefore cannot easily be solved to optimality, so they develop a Lagrangian-based heuristic. Their computational results are consistent with ours in terms of the magnitude of savings that results from including inventory cost, but the advantage of the model proposed in this research is that it can be solved to optimality using standard mixed-integer linear methodologies.

In summary, the issue of ignoring inventory cost when designing a logistics network has been recognized. However, researchers have not yet developed ways to solve problems that include inventory cost to optimality.

### **The Square Root Law and the Portfolio Effect**

The square root law (SRL) estimates safety stock savings from inventory centralization. Savings are proportional to the square root of the ratio of the new number of stocking locations over the original number of stocking locations. If, for instance, inventory is decentralized from one to two stocking locations, safety stock is increased by a factor of  $\sqrt{2}$ . Early proponents of the SRL (Brown 1967; Heskett, Glaskowsky, and Ivie 1974) focused on the SRL as an estimate of safety stock savings. Maister (1976) extended its application to cycle stocks in the special case where the cycle stock is determined with the economic order quantity formula (EOQ). In this case, the formulation remains the same, but the input values in the SRL equation reflect the total inventory rather than only the safety stock.

The portfolio effect offers a more precise estimation of safety stock savings from inventory centralization by eliminating two of the assumptions embedded in the SRL. The first assumption is that the demand variance for an item is the same at all inventory locations and the second is that sales for the same item in all inventory locations are independent (Zinn, Levy, and Bowersox 1989).

The portfolio effect was extended to include transportation and procurement costs (Mahmoud 1992), multiple consolidation points (Evers and Beier 1993), variable lead times (Tallon 1993), and the effect of transshipments (Evers 1996, 1997).

It is important to note that the two above mentioned assumptions of the square root law are problematic to estimate safety stock savings from centralization for a specific item and often lead to substantial error (Zinn, Levy, and Bowersox 1989). However, the SRL is adequate for this research because network design uses aggregate sales data as input. The sales data are aggregated for multiple items over extended periods of time (usually yearly). As a result, the assumptions of equal demand variances and independence of demand have a limited impact on network design because the demand variance and correlations across individual items disappear at the aggregate level. Therefore, we will use the SRL and model network-wide inventory as a function of the number of warehouses in the network.

### THE PROPOSED MODEL

As noted earlier, the proposed model simultaneously incorporates inventory cost into the standard network design model, which minimizes transportation and fixed warehousing costs subject to capacity, demand, and flow balance constraints (see Appendix A). In this research, we include supplier capacities but not warehouse capacities.

To this standard model we add inventory cost, which we model with the SRL. In this first application of the proposed model, we take a conservative approach and assume that cycle stock is independent of the number of warehouses. We therefore focus on the impact of safety stock on network design. Nevertheless, as demonstrated by Maister (1976), the SRL also applies to cycle stock if an EOQ ordering policy is used. Therefore, if managers order according to the EOQ, the SRL can be applied to the total inventory (cycle plus safety stock) and the proposed model still applies. Adding cycle stock would amplify the benefit of using the proposed model.

We have chosen to model inventory cost using the square root law, not only for its simplicity, but because the assumptions that underlie it are appropriate. As previously noted, the restrictive assumptions of the square root law, namely that it assumes equal demand variance and independent demand among items, are reasonable assumptions at the level of aggregation required for network design.

Note that the SRL also implicitly assumes that the lead time to all warehouses is the same regardless of the number of warehouses. This assumption, which may be removed in future research, has only a limited effect on this research. There are three main reasons for this. First, lead time is not the same as transit time. When reducing the number of warehouses, the average transit time increases, but this results in a less than proportional increase in lead time, because the time to receive, process and fill orders remains the same. Second, when the change in the number of warehouses is relatively small, say, from 4 to 3 or 2 warehouses as it is in this research, the change in transit time is relatively minor. Finally, it is the square root of the lead time that is used to

compute safety stock, which helps to further reduce the impact of lead time changes on the level of inventory maintained.

The proposed model offers two fundamental contributions. First is the simultaneous inclusion of inventory in network design. The second contribution is that the model allows different network configurations for different product classifications. That is, a firm might want to keep C items in fewer warehouses than A or B items, due to the differences in demand variability and customer service levels for different product classifications. While this issue is often handled in inventory deployment models, to the best of our knowledge it has never been included as part of a network design model. At best, network design models might include multiple product families with varying cost structures, but since they don't consider demand variability and customer service levels, they are not basing the location decisions on the varying inventory requirements of each classification.

**The Formulation**

We model inventory with the square root law (Maister 1976). It states that:

$$SS_n = SS_1 \sqrt{n} \tag{1}$$

where:

$SS_n$  = the total network-wide safety stock required if inventory is decentralized in  $n$  warehouses

$SS_1$  = the total safety stock required if inventory is centralized in a single warehouse

The equation for  $SS_1$  is below (Fetter and Dalleck 1961):

$$SS_1 = k \sqrt{t \sigma_d^2 + d^2 \sigma_t^2} \tag{2}$$

where:

$k$  = number of standard deviations of demand required to achieve a given level of service

$t$  = average lead time, in days

$\sigma_d$  = standard deviation of daily demand

$d$  = average daily demand

$\sigma_t$  = standard deviation of lead time, in days

We represent the SRL as a discrete function of the number of warehouses by adding binary variables and appropriate forcing constraints (for the complete formulation, see Appendix A). The technique is a modification of one frequently used to model piecewise linear functions (Croxtton, Gedron, and Magnanti 2003). The advantage to modeling the inventory cost as a discrete function, rather than a piecewise linear one, is that it involves fewer variables and constraints, keeping the formulation to a more manageable size. This means that the proposed model requires less computing power to solve.

For simplicity, in this application of the proposed model we assume no lead time variability. Note that this is a conservative assumption because including lead time variability increases the inventory level and therefore the impact of the proposed model. If we relax this assumption, the model remains valid because  $SS_1$  can be recalculated with lead time included.

In addition to including inventory cost directly into the formulation, we consider the impact of product classifications on network design decisions. To accomplish this, we allow the model to choose whether to include A, B, and/or C items at each warehouse by replicating the variables and constraints for each product classification. The service level, average demand, and variability of demand vary for each classification. In essence, the model is solving the network design problem for each product classification, but the fixed warehouse costs are shared across all classifications.

### SCENARIO ANALYSIS

Our goal in this section is to illustrate the effect that including inventory cost can have on network design. We use data obtained from a national retailer based in the Midwest to establish a base case scenario. We then created a set of 13 diverse but realistic scenarios. For each scenario we compare the output of the standard model and the proposed model. The results of this comparison highlight the potential impact of including inventory cost in network design. To build the scenarios, we consider the effect of changing the values of seven input parameters:

- Coefficients of variation of demand (CVD) for A, B, and C items
- Service levels (SL) for A, B, and C items
- Inventory carrying cost (CC)
- Transportation cost factor
- Demand factor
- Fixed warehouse cost factor
- Product value factor

The values for the first three input parameters used in the illustration are based on similar applications in the literature. More specifically, these were Zinn and Marmorstein (1990) and Closs and Law (1984) for the coefficient of variation of demand; Zinn, Mentzer, and Croxton (2002) for the service levels; and Zinn and Marmorstein (1990) for the inventory carrying cost. To simplify displaying the data, we refer to coefficients of variation of demand and service levels as low or high. Table 1 provides the values used. The last four input parameters are represented as relative factors over the base case, which uses actual demand and cost data from the national retailer. For example, a demand factor of 1.5 increases the base-case demand by 50%.

**TABLE 1**  
**CVD AND SL VALUES**

		A	B	C
CVD	Low	.25	.35	.45
	High	.38	.53	.68
	C-High	.25	.35	1.35
SL	Low	97%	94%	91%
	High	99%	97%	95%

To derive scenarios, we first consider the effect of changing one input parameter at a time with respect to the base case scenario (Scenario 1). This is shown in Scenarios 2-8 in Table 2. The bolded cells indicate which parameter was changed. The goal in Scenarios 9-11 is to illustrate further the impact that adding inventory cost to network design can have. To that end, we progressively increase the impact of inventory cost by increasing the values of inventory related parameters. Thus, in Scenario 9, we increase the values of CVD and CC. In Scenario 10 we increase the product value without returning to the base case values of CVD and CC. In Scenario 11 we add further to the importance of inventory by increasing SL. In Scenarios 12-14 we illustrate cases where the demand variability of C items is quite high. The goal is to illustrate situations where using the proposed model results in a different network for C items than for A and B items.

For each scenario, we compare the solution provided by the standard network design model to the solution provided by the proposed model. The four solution values that provide the basis for this comparison are:

- Total cost
- Number of warehouses selected
- Locations of selected warehouses
- Distribution of selected warehouses among product classifications

**TABLE 2**  
**PARAMETER VALUES FOR SCENARIOS**

ID	CVD	SL	CC	Multiplicative Factors			Value
				Trans	Dem	Fixed	
1	Low	Low	20%	1	1	1	1
2	<b>High</b>	Low	20%	1	1	1	1
3	Low	<b>High</b>	20%	1	1	1	1
4	Low	Low	<b>30%</b>	1	1	1	1
5	Low	Low	20%	<b>1.5</b>	1	1	1
6	Low	Low	20%	1	<b>1.5</b>	1	1
7	Low	Low	20%	1	1	<b>0.5</b>	1
8	Low	Low	20%	1	1	1	<b>1.5</b>
9	<b>High</b>	Low	<b>30%</b>	1	1	1	1
10	<b>High</b>	Low	<b>30%</b>	1	1	1	<b>1.5</b>
11	<b>High</b>	<b>High</b>	<b>30%</b>	1	1	1	<b>1.5</b>
12	<b>C-High</b>	Low	20%	1	1	1	1
13	<b>C-High</b>	<b>High</b>	20%	1	1	1	1
14	<b>C-High</b>	Low	<b>30%</b>	1	1	1	1

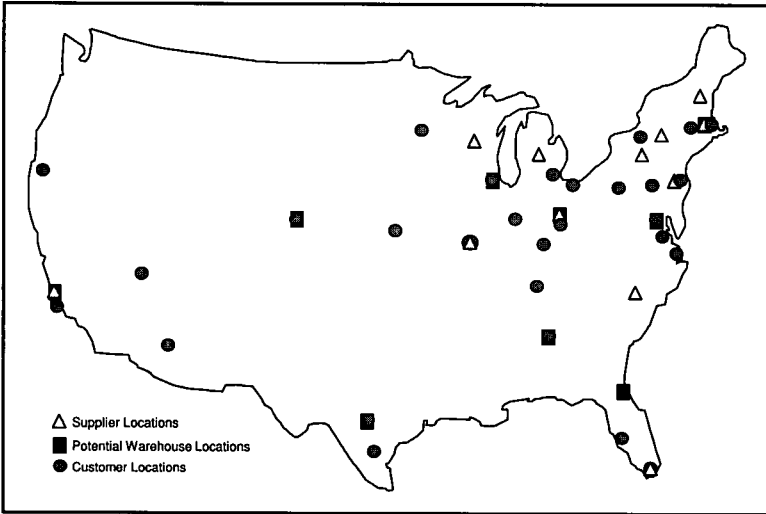
### The Data

The data used in the scenario analysis were collected from a national retailer with approximately 150 stores in about 30 states. Specifically, we were provided supplier locations and annual volumes, store locations and annual volumes, transportation rates for existing lanes, and fixed costs for the current warehouse structure. We then determined the set of potential warehouse locations, the transportation rates to apply to new lanes, the fixed warehousing costs, the supplier capacities, and the demand for each product classification. Each of these data elements is described below. Large suppliers were aggregated by state into 12 locations, representing 90% of total inbound volume. Three of those locations are ports used by suppliers from overseas. Stores were aggregated by location into 29 sites, which represented 130 of their stores. Nine potential warehouse locations were identified by considering supplier and customer locations and based on a published list of common warehouse sites (King and Keating 2003). This provided the network structure shown in Figure 1.

Estimates of transportation cost are based on the truckload rates provided by the retailer. We plotted transportation cost per mile and approximated the concave function with a piecewise linear one. In other words, the cost per mile decreased as the total distance traveled increased. We used the same rates for moves between suppliers and warehouses and those between warehouses and stores.

We calculated distance between locations based on longitude and latitude and estimated lead time based on distance.

**FIGURE 1**  
**NETWORK STRUCTURE**



The retailer gave us information on their fixed costs for a typical warehouse. We then adjusted the fixed costs for the other locations based on lease-cost information in other cities (CB Richard Ellis 2004). To include supplier capacities, we set capacities to be 150% of the volume shipped in 2003. Since the retailer did not have total volume broken down by product classification we divided the capacity up between the three product classifications. To obtain demand data, we used 2003 demand and assigned that to be 80% A, 15% B, and 5% C products.

The data for each problem instance were saved in a data file, which was used to create an MPS file. CPLEX 9.0 was used to read and solve the MPS files. We used all of the default settings in CPLEX and solved the problem using standard branch and bound. All instances were solved within three seconds on a Pentium 4, 1.8 GHz Dell desktop.

**Results**

Table 3 provides the results for the 14 scenarios described. The first eight columns of Table 2 have been duplicated to ease interpretation. The seven new columns show the results. Each scenario from Table 2 is shown on two lines. The first line shows the results for the standard

network design formulation, without inventory costs included. To the cost of this solution, we add the appropriate inventory cost so that we can compare this solution with the one resulting from the proposed model. The inventory cost is determined with the same discrete function as in the proposed model, which uses the square root law and is based on the number of warehouses selected. The second line for each scenario (denoted by A) shows the solution to the proposed model, which already includes inventory cost. In the final column we show the cost savings expressed as a percentage; that is, the incremental savings derived from including inventory cost when designing the network.

The key results from the scenario analysis may be grouped into three broad themes. The first is the general effect of the proposed model on network design. The second is the impact of individual parameters. Finally, the third theme examines the possibility of warehousing C items in fewer locations than A and B items. Each theme is discussed below.

#### *General effect of including inventory cost in network design*

The general effect of including inventory cost in network design is that the number of warehouses in the solution either stays the same or is reduced when compared to the solution obtained with the standard model. The same is true for the total cost of the network. The base case presented in Scenario 1 is representative of this general effect. The number of warehouses is reduced from four to three and the total cost of the network decreased by 1.4%.

TABLE 3

RESULTS

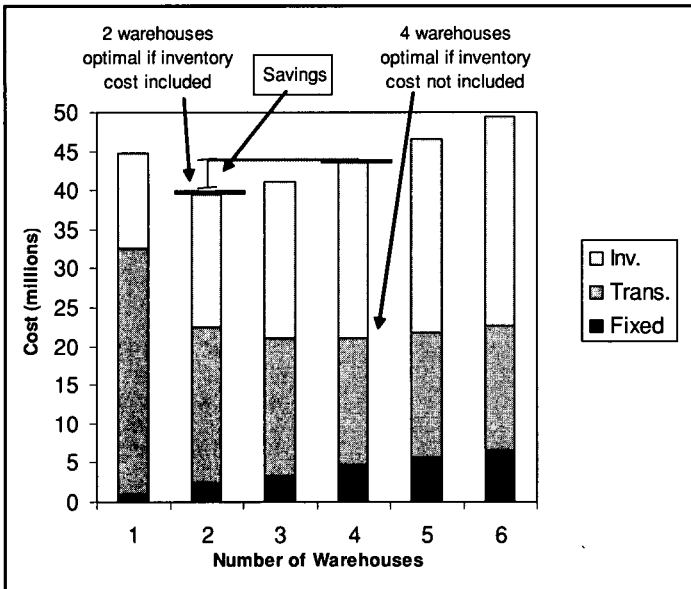
ID	CVD	SL	CC	Mult. Factors				# WHS			Cost in millions		% Diff.	
				Trans	Dem	Fixed	Value	A	B	C	Model	Inv.		Total
1	Low	Low	20%	1	1	1	1	4	4	4	19.820	5.722	25.542	
1A	Low	Low	20%	1	1	1	1	3	3	3	25.173	–	25.173	1.4
2	High	Low	20%	1	1	1	1	4	4	4	19.820	8.689	28.509	
2A	High	Low	20%	1	1	1	1	3	3	3	27.743	–	27.743	2.7
3	Low	High	20%	1	1	1	1	4	4	4	19.820	7.080	26.900	
3A	Low	High	20%	1	1	1	1	3	3	3	26.349	–	26.349	2.0
4	Low	Low	30%	1	1	1	1	4	4	4	19.820	8.584	28.404	
4A	Low	Low	30%	1	1	1	1	3	3	3	27.651	–	27.651	2.6
5	Low	Low	20%	1.5	1	1	1	4	4	4	27.363	5.722	33.085	
5A	Low	Low	20%	1.5	1	1	1	4	4	4	33.085	–	33.085	0.0
6	Low	Low	20%	1	1.5	1	1	4	4	4	27.359	8.584	35.943	
6A	Low	Low	20%	1	1.5	1	1	4	4	4	35.943	–	35.943	0.0
7	Low	Low	20%	1	1	0.5	1	4	4	4	17.449	5.722	23.172	
7A	Low	Low	20%	1	1	0.5	1	4	4	4	23.172	–	23.172	0.0
8	Low	Low	20%	1	1	1	1.5	4	4	4	19.820	8.584	28.404	
8A	Low	Low	20%	1	1	1	1.5	3	3	3	27.651	–	27.651	2.6
9	High	Low	30%	1	1	1	1	4	4	4	19.820	13.034	32.854	
9A	High	Low	30%	1	1	1	1	3	3	3	31.506	–	31.506	4.1
10	High	Low	30%	1	1	1	1.5	4	4	4	19.820	19.551	39.371	
10A	High	Low	30%	1	1	1	1.5	2	2	2	36.285	–	36.285	7.8
11	High	High	30%	1	1	1	1.5	4	4	4	19.820	24.189	44.009	
11A	High	High	30%	1	1	1	1.5	2	2	2	39.565	–	39.565	10.1
12	C-High	Low	20%	1	1	1	1	4	4	4	19.820	6.429	26.249	
12A	C-High	Low	20%	1	1	1	1	3	3	3	25.786	–	25.786	1.8
13	C-High	High	20%	1	1	1	1	4	4	4	19.820	7.950	27.770	
13A	C-High	High	20%	1	1	1	1	3	3	2	27.085	–	27.085	2.5
14	C-High	Low	30%	1	1	1	1	4	4	4	19.820	11.926	31.746	
14A	C-High	Low	30%	1	1	1	1	3	3	2	30.424	–	30.424	4.2

The general effect is explained by the fact that the model is simultaneously optimizing all relevant costs: inventory, fixed warehousing, and transportation. First, inventory cost is reduced because the number of stocking locations is reduced. The relationship between inventory cost and number of stocking locations is explained by the square root law and the portfolio effect (Zinn, Levy, and Bowersox 1989). Second, because the optimum number of warehouses is reduced, the fixed cost

of warehousing is reduced. Finally, the transportation cost increases because it is inversely related to the number of warehouses. The general effect and the impact on each relevant cost are illustrated in Figure 2.

FIGURE 2

EFFECT OF THE PROPOSED MODEL ON NETWORK DESIGN



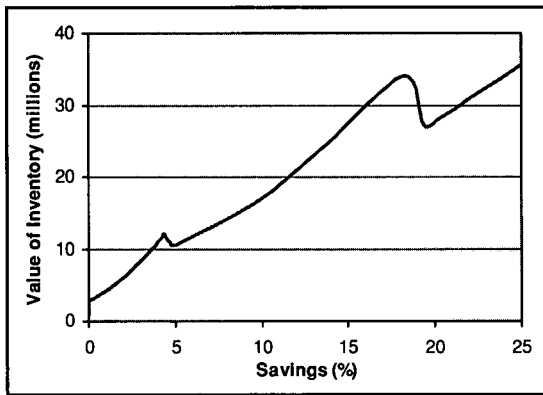
In addition to the number of warehouses and the cost of the network, we examine the effect of the proposed model on the location of the warehouses in the network. There were no location changes in any of the 14 scenarios considered in the research. In other words, in scenarios where the number of warehouses was reduced with the proposed model, the locations selected were always a subset of the locations obtained with the standard model. This effect, however, cannot be generalized beyond the scenarios included in this research.

The overall impact of inventory cost on network design is illustrated in Figure 3. It shows the savings a firm might obtain from including inventory cost in network design as a function of the value of its inventory. As expected, the higher the value of the inventory, the higher the savings. Note, however, that there are “steps” in the function where savings continue to climb as the value of the inventory decreases. These steps occur at points where the number of warehouses is reduced.

At each of these points, the value of the inventory goes down due to the portfolio effect. It is also observable that the size of each step increases as the value of the inventory increases. This is because the monetary value of the savings is proportional to the size of the inventory and also because the portfolio effect is more pronounced as the inventory becomes more centralized.

FIGURE 3

VALUE OF INVENTORY AND SAVINGS FROM THE PROPOSED MODEL



*Impact of individual parameters*

The results presented in Scenarios 2 through 8 in Table 3 illustrate how different parameters alter the impact of inventory cost on network design. Note that an increase in the values of transportation, demand, and fixed warehousing cost (Scenarios 5, 6 and 7) produce no savings at all. This may be explained by the fact that these parameters are also included in a standard network design model. Conversely, coefficient of variation of demand, service level, carrying cost, and product value produce savings to different degrees (Scenarios 2, 3, 4 and 8). The last four parameters are only included in the proposed model.

Note also, in Scenarios 9 to 11, that savings increase as we simultaneously increase the values for more than one of the four parameters mentioned above. Thus, Scenario 9 has increased values for coefficient of variation of demand and carrying cost, and Scenario 10 has increased values for the two previous parameters plus product value. At the extreme, Scenario 11 shows the highest level of savings because it includes increased values for all four parameters.

### *Network design and C items*

C items have unique characteristics of low demand and high coefficient of variation of demand that can warrant a different network from A and B items. The inventory cost for C items is relatively high, particularly in cases where customers require almost equal availability for all product classifications (Zinn, Mentzer, and Croxton 2002). As a result, there are cases where C items should be inventoried in fewer locations than other product classifications. This is illustrated in Scenarios 12 to 14. The output of the network design model in Scenario 12 indicates that C items should be inventoried in the same locations as other product classifications. In Scenarios 13 and 14, however, when service level and carrying costs are high, results show that the firm may reap additional savings if C items are inventoried in two locations instead of three.

## CONCLUSIONS AND IMPLICATIONS

Recall that the goals of this paper are to: (1) propose a model that firms may use to include inventory in network design; (2) illustrate the application of the model with data collected from a national retailer; and (3) demonstrate the potential impact of the proposed model on network design. The illustration shows that the proposed model can be successfully applied in a realistic setting. In addition, the following conclusions and implications are relevant to managers considering whether or not to include inventory cost in network design.

One general effect of including inventory in network design is that the number of warehouses in the solution is reduced. An important implication of this result is that including inventory cost in network design is more beneficial, or even practical, when the number of warehouses in the network is relatively large. Clearly, if the effect of considering inventory cost is to reduce the number of warehouses, than a network with few original warehouses is less likely to benefit. At the extreme, a network with only one warehouse will not benefit at all from adding inventory cost to network design unless the location of the warehouse is changed.

Another important general effect of including inventory cost in network design is that the total cost of the network is lower. Note also, in Table 3, that cost savings from including inventory in network design only occurs when the inclusion results in fewer warehouses. Savings can be substantial. In the scenarios exhibited in Table 3 the reduction in total network cost varied from zero to ten percent and up to 4.5 million dollars.

We also observe that the proposed model did not select different warehouse locations than the ones selected by the standard model. This is true for all 14 scenarios considered in this research.

A further implication of this research is that network design may yield different results for different product classifications. As noted earlier in the results section, C items typically have lower demand and higher coefficient of variation of demand. This fact is overlooked whenever inventory is not included in network design. Table 3 illustrates two scenarios where C items are inventoried in fewer warehouses than A or B items.

The results above also imply that the flexibility often sought by logistics managers is warranted. They suggest that networks might need to be adjusted when new products are introduced, demand characteristics change, or new sources of supply are identified. The model proposed in this research is helpful in adjusting the network to these changes.

Finally, the proposed model is particularly relevant for firms seeking to increase flexibility in their networks. Such firms typically use the services of third parties to manage down the fixed warehouse cost because that allows for faster location changes in response to market uncertainties. As a result, the relative importance of fixed warehousing cost in the design of their network declines. If inventory cost is included in network design, the network will increasingly be determined by the trade-off between transportation and inventory cost. Consequently, managers who reduce the fixed warehousing cost and fail to include inventory cost in network design will adopt a solution that has too many warehouses and is unnecessarily costly.

In summary, we propose a model to simultaneously include inventory cost in network design. We demonstrate the application of the proposed model with an illustration based on data obtained from a national retailer headquartered in the Midwest. The key results and implications obtained in the research are summarized below.

- Inventory costs, calculated according to the square root law, can be added to a standard network design model with limited impact on the size of the formulation. Therefore, this approach should be applicable to large-scale problems. In fact, the model is not restricted to using the square root law. The same approach can be used with other inventory functions, as long as inventory is described as a function of the number of warehouses.
- Inventory cost matters in network design by generally reducing the number of warehouses in the solution. By applying the proposed model, we were able to reduce the number of warehouses in the network.
- The total cost of the network often declines when inventory is included in its design. In the application of the proposed model provided in this research, the network cost declined by up to ten percent or 4.5 million dollars.
- The solution generated with the proposed model did not change the location of selected warehouses included in the illustration.
- If the number of warehouses is small without considering inventory in network design, the expected benefit of the proposed model is small.
- Application of the proposed model might yield a different network configuration for C items than for A and B items.
- The expected benefit from using the proposed model is higher when the fixed cost of warehousing is small.

## NOTES

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## APPENDIX A

### THE MODELS

We will use the following notation for both the standard model and the proposed model. Assume a network with  $m$  suppliers,  $n$  potential warehouses,  $p$  customers, and  $r$  product classifications.

#### Data

$F_j$  = fixed cost associated with operating warehouse  $j$

$T_{ij}$  = per unit transportation cost for shipments between location  $i$  and  $j$

$C_i^h$  = capacity of supplier  $i$  for product classification  $h$

$D_k^h$  = demand of product classification  $h$  at customer location  $k$

$I_s^h$  = inventory cost associated with storing product classification  $h$  in  $s$  warehouses (this is equivalent to  $SS_n$  and is calculated using equation (1) on page 8)

#### Variables

$V_{ij}^h$  = units of product classification  $h$  shipped from supplier  $i$  to warehouse  $j$

$X_{jk}^h$  = units of product classification  $h$  shipped from warehouse  $j$  to customer  $k$

$Y_j$  = binary variable indicates if warehouse  $j$  is selected

$Y_j^h$  = binary variable indicates if warehouse  $j$  is selected to store product classification  $h$

$W_s^h$  = binary variable indicates if  $s$  warehouses are used for product classification  $h$

**The Standard Formulation**

$$\text{Min: } \sum_{j=1}^n F_j Y_j + \sum_{i=1}^m \sum_{j=1}^n \sum_{h=1}^r T_{ij} V_{ij}^h + \sum_{j=1}^n \sum_{k=1}^p \sum_{h=1}^r T_{jk} X_{jk}^h \tag{1}$$

$$\text{s.t.: } \sum_{i=1}^m V_{ij}^h = \sum_{k=1}^p X_{jk}^h \quad \forall j = \{1, \dots, n\}, h = \{1, \dots, r\} \tag{2}$$

$$\sum_{i=1}^m V_{ij}^h \leq C_i^h \quad \forall i = \{1, \dots, m\}, h = \{1, \dots, r\} \tag{3}$$

$$\sum_{i=1}^m X_{jk}^h \geq D_k^h \quad \forall k = \{1, \dots, p\}, h = \{1, \dots, r\} \tag{4}$$

$$V_{ij}^h \leq C_i^h Y_j \quad \forall i = \{1, \dots, m\}, j = \{1, \dots, n\}, h = \{1, \dots, r\} \tag{5}$$

$$X_{jk}^h \leq D_k^h Y_j \quad \forall j = \{1, \dots, n\}, k = \{1, \dots, p\}, h = \{1, \dots, r\} \tag{6}$$

$$Y_j^h \leq Y_j \quad \forall j = \{1, \dots, n\}, h = \{1, \dots, r\} \tag{7}$$

The objective function (1) minimizes the sum of the fixed-warehouse cost, and the transportation cost both from supplier to warehouse and from warehouse to customer. Constraints (2)–(4) are the standard network design constraints that maintain flow balance while meeting customer demand and not exceeding supplier capacity. Note that warehouse capacity constraints could easily be added to the formulation if needed. Constraints (5) and (6) are forcing constraints that assure that product can flow in and out, respectively, of a warehouse only if that warehouse is open. Note that only one set of these constraints is needed, and the other is redundant in the mixed-integer program. However, when  $C_i > D_k$ , Constraint (6) can tighten the linear relaxation. Therefore, both sets of constraints remain in the formulation. Constraint (7) is a forcing constraint that defines the  $Y_j$  variables. If a warehouse is used for any of the product classifications, then the fixed cost is incurred. Note that in this formulation, which does not incorporate inventory costs, the solution will use the same network for all products. Therefore, it is only necessary to define one classification, which could simplify the notation. However, the formulation is defined for each classification to facilitate a comparison to the proposed model.

**The Proposed Formulation**

$$\text{Min: } \sum_{j=1}^n F_j Y_j + \sum_{i=1}^m \sum_{j=1}^n \sum_{h=1}^r T_{ij} V_{ij}^h + \sum_{j=1}^n \sum_{k=1}^p \sum_{h=1}^r T_{jk} X_{jk}^h + \sum_{s=1}^r \sum_{h=1}^r I_s^h W_s^h \tag{8}$$

$$\text{s.t.: } \sum_{i=1}^m V_{ij}^h = \sum_{k=1}^p X_{jk}^h \quad \forall j = \{1, \dots, n\}, h = \{1, \dots, r\} \tag{9}$$

$$\sum_{i=1}^m V_{ij}^h \leq C_i^h \quad \forall i = \{1, \dots, m\}, h = \{1, \dots, r\} \tag{10}$$

$$\sum_{i=1}^m X_{jk}^h \geq D_k^h \quad \forall k = \{1, \dots, p\}, h = \{1, \dots, r\} \tag{11}$$

$$V_{ij}^h \leq C_i^h Y_j^h \quad \forall i = \{1, \dots, m\}, j = \{1, \dots, n\}, h = \{1, \dots, r\} \quad (12)$$

$$X_{jk}^h \leq D_k^h Y_j^h \quad \forall j = \{1, \dots, n\}, k = \{1, \dots, p\}, h = \{1, \dots, r\} \quad (13)$$

$$Y_j^h \leq Y_j \quad \forall j = \{1, \dots, n\}, h = \{1, \dots, r\} \quad (14)$$

$$\sum_{j=1}^n Y_j^h = \sum_{s=1}^n s W_s^h \quad \forall h = \{1, \dots, r\} \quad (15)$$

$$\sum_{s=1}^n W_s^h = 1 \quad \forall h = \{1, \dots, r\} \quad (16)$$

The objective function (9) minimizes the sum of the fixed-warehouse cost, the transportation cost both from supplier to warehouse and from warehouse to customer, and the inventory cost.  $W_s^h$  is a binary variable indicating whether  $s$  warehouses are chosen, so the last expression in the objective function takes on the value of the discrete inventory function. Constraints (9) – (14) are the same as Constraints (2) – (7) of the standard model and are described above. Constraints (15) and (16) are unique to this formulation and they define the  $W_s^h$  variables so that inventory costs are appropriately applied. Since by (16), only one of the  $W_s^h$  variables can equal one, the right hand side of (15) will equal the number of warehouses that product classification  $h$  uses, which must be equal to the sum of the  $Y_j^h$  variables.

The difference in the size of problem between the standard and proposed formulations is relatively small. The proposed formulation adds  $rn$  binary variables and  $2r$  constraints. The test problems in this research had very small LP gaps and were solved within three seconds, so this formulation should be applicable to other commercial-sized problems. We also did not exploit any special structure to solve the test instances, so if the size of the formulation did become an issue, future research could develop algorithms to solve the models more efficiently.

### ABOUT THE AUTHORS

**Keely L. Croxton** is an Assistant Professor of Logistics in the Department of Marketing and Logistics at The Ohio State University. Her research interests are in logistics optimization and supply chain management. Her research has been published in the *Journal of Business Logistics*, *Transportation Science*, *Management Science*, and *The International Journal of Logistics Management*. She has worked in the automotive, paper and packaging, and third-party logistics industries. She holds a BS in Industrial Engineering from Northwestern University and a Ph.D. in Operations Research from MIT.

**Walter Zinn** (Ph.D. Michigan State University) is Associate Professor of Logistics and Marketing at The Ohio State University. He is a former Systems Section Editor of the *Journal of Business Logistics*. Dr. Zinn’s research has been published in academic journals such as the *Journal of Business Logistics*, *European Journal of Operational Research*, *Journal of the Operational Research Society*, and *The International Journal of Logistics Management*.

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