EDITOR'S NOTE

In the Spring 1989 issue (Volume 10, Number 1) of the *Journal of Business Logistics*, an article by Zinn, Levy, and Bowersox reexamined the so-called "square root law" used to approximate safety stock requirements when making changes in the number of stocking locations. They demonstrated that the square root law is a special case of the "Portfolio Effect." The authors derived two rules that can be easily applied by managers when considering whether to add or delete stocking locations. The authors purport to show how it is possible to improve customer service by adding stocking locations but without the usual additions in aggregate safety stock.

Professor David Ronen contends in his article that the Zinn, Levy, and Bowersox analysis of the square root law and its impact on safety stock is not practical. He bases his argument upon how they choose to measure the customer service level, *viz.*, the probability of not running out of stock during lead time. Ronen argues that their definition of inventory availability is not precise enough and therefore their analysis is useless.

Zinn, Levy, and Bowersox counter that Ronen incorrectly presumes that they assume that they used the EOQ model in their analysis, when they actually assumed that the order quantity was equal to demand during lead time. They also contend that Ronen's analysis of the EOQ is flawed. What do you think?

INVENTORY CENTRALIZATION/DECENTRALIZATION – THE "SQUARE ROOT LAW" REVISITED AGAIN

by

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In a recent paper, Zinn, Levy, and Bowersox[1] analyzed the effects of inventory centralization/decentralization on aggregate safety stock. The basic guideline of their analysis and comparisons was to maintain a given customer service level, as measured by inventory availability. Unfortunately they used the probability to run out of stock, *during lead time* as the measure of inventory availability. This measure is convenient and allows for a neat but impractical analysis. Probability to run out of stock during lead time neither reflects how often the specific item is in lead time, nor the length of the lead time; thus, this measure is irrelevant both to customers and to inventory managers.[2] Moreover, centralization/decentralization of inventory changes the number of lead times per year if inventory is replenished according to the Economic Order Quantity (which seems to be a common practice in industry and is implied in their paper). In this paper, a more relevant measure of inventory availability is used, namely, fraction of demand supplied from stock, and the issue of inventory centralization is analyzed with it.

ANALYSIS

We deal with an inventory system under an Order Quantity/Reorder Point (Q,R) inventory control policy. Demand during stock replenishment lead time has some statistical distribution due to variability of demand, lead time, or both. The order quantity is determined by the Economic Order Quantity (EOQ) and the reorder point, R, consists of average demand during lead time plus a safety stock, which is a certain number (k) of standard deviations of demand during lead time. (Thus k is a policy decision and determines the probability to run out of stock during lead time.)

The following notation is used:

D - Annual demand for the item (units)

S - Ordering cost ($/order)

H - Holding cost ($/unit per year)

Q - Economic order quantity (units)

N - Number of order cycles (lead times) per year

σ - Standard deviation of demand during lead time

μ - Average demand during lead time

k - Number of standard deviations of lead time demand in safety stock

E(k) - Expected number of units short during lead time (in standard deviations).

u - Fraction of units supplied from stock during lead time (or share of lead time when stock is available)

U - Overall fraction of units supplied from stock (or share of time stock is available)

p - Probability to not run out of stock during lead time.

Thus, the reorder point is:

$$R = \mu + k\sigma \tag{1}$$

the order quantity is the EOQ[3]:

$$Q = \sqrt{\frac{2DS}{H}} \tag{2}$$

and the number of order cycles per year is:

$$N = \frac{D}{Q} = \sqrt{\frac{DH}{2S}} \tag{3}$$

The specification of k determines the probability not to run out of stock during lead time, and if the statistical distribution of demand during lead time is known, E(k) can be calculated by:[4]

$$E(k) = \sum_{x>k} (x-k)p(x) \qquad \text{for a discrete distribution,}$$

or

$$E(k) = \int_{x>k} (x-k)f(x)\,dx \qquad \text{for a continuous distribution}$$

First, let us see that the number of order cycles per year changes with centralization/decentralization. Consider two stock locations, 1 and 2. From (3):

$$N_1 = \sqrt{\frac{D_1H_1}{2S_1}}, \quad N_2 = \sqrt{\frac{D_2H_2}{2S_2}}$$

assuming that the two locations have the same stock ordering and holding costs ($H_1 = H_2 = H$, $S_1 = S_2 = S$) and letting:

$$b = \sqrt{\frac{H}{2S}} \tag{4}$$

then: $N_1 = b\sqrt{D_1}$, $N_2 = b\sqrt{D_2}$

after centralizing the stock of the two locations, the centralized facility will face their joint demand:

$$D_0 = D_1 + D_2 \tag{5}$$

and its number of order cycles per year (assuming it has the same H and S) will be:

$$N_0 = b\sqrt{D_1 + D_2} \tag{6}$$

Thus, the centralized stock will have more order cycles (i.e., lead times) per year than any of the decentralized facilities. Therefore, using the probability to run out of stock during lead time as a measure of inventory availability is meaningless.

A MORE RELEVANT SERVICE MEASURE

Let us turn now to more relevant measures of inventory availability. The fraction of demand supplied from stock *during lead time* is given by:[5]

$$u = 1 - \frac{E(k)\sigma}{\mu} \tag{7}$$

and the overall fraction of demand supplied from stock is:[6]

$$U = 1 - \frac{\mu}{Q}(1-u) \tag{8}$$

for the case of backorders (BO), and

$$U = \frac{1}{\frac{\mu}{Q}(1-u) + 1} \tag{9}$$

for the case of lost sales (LS).

Substituting (7) into (8) and (9), we get:

$$U = 1 - \frac{E(k)\sigma}{Q} \qquad \text{(BO)} \tag{10}$$

$$U = \frac{1}{1 + \frac{E(k)\sigma}{Q}} \qquad \text{(LS)} \tag{11}$$

Thus the service level defined as fraction of demand supplied from stock cannot be computed independently of the order quantity.

In both cases (BO and LS) in order to equalize the service level (U) between two locations we need:

$$\frac{E(k_1)\sigma_1}{Q_1} = \frac{E(k_2)\sigma_2}{Q_2} \tag{12}$$

or, assuming equal ordering and holding costs and using (2) we get:

$$\frac{E(k_1)\sigma_1}{\sqrt{D_1}} = \frac{E(k_2)\sigma_2}{\sqrt{D_2}} \tag{13}$$

In (13) only k_1 and k_2 are managerial decisions, but unfortunately E(k) is seldom a linear or even closed form function of k; therefore, numerical methods must be used to equalize the service levels. A simple computer program may be written to do this.

If we centralize the stock of the two locations, the centralized facility will face their joint demand:

$$D_0 = D_1 + D_2 \tag{14}$$

and a standard deviation of demand during lead time of:[7]

$$\sigma_0 = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho_{12}} \tag{15}$$

and if we want the same service level we get:

$$\frac{E(k_1)\sigma_1}{\sqrt{D_1}} = \frac{E(k_2)\sigma_2}{\sqrt{D_2}} = E(k_0)\sqrt{\frac{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho_{12}}{D_1 + D_2}} \tag{16}$$

where ρ_{12} is the correlation coefficient between the lead time demands in the 2 locations. Again, k_0 is a policy decision which can be set at any desired level, but its effect on service is determined by reference to the specific joint probability distribution.

Using this more appropriate measure of inventory availability, no general results can be derived concerning the desirability of inventory centralization/decentralization without knowing the order quantities and the probability distributions involved.

EXAMPLE

Let us take a case where demand during lead time is normally distributed, sales are lost if the item is not in stock, and the desired fraction of demand supplied from stock is .98 (U=.98). The data and results of this example are presented in Table 1. It is seen in the table that the value of k and the level of safety stock ($k\sigma$) do change for the various cases in order to retain the same service level (U).

As the coefficient of correlation ρ decreases, so does the safety stock and p. (Remember that all cases in this example provide the same fraction of overall demand supplied from stock.)

TABLE 1

ONE CENTRALIZED VS. TWO STOCKING FACILITIES, ALL WITH THE SAME SERVICE LEVEL

	Location		1	2	Centralized				
	ρ_{12}		—	—	1.0	0.5	0.0	-0.5	-1.0
D	D		3000	2000	5000	5000	5000	5000	5000
a	σ	(15)*	40	30	70	61	50	36	10
t	S		50	50	50	50	50	50	50
a	H		1	1	1	1	1	1	1
R	Q	(2)*	548	447	707	707	707	707	707
e	N	(3)*	5.5	4.5	7.1	7.1	7.1	7.1	7.1
s	E(k)	(11)*	.279	.304	.206	.237	.288	.400	1.44
u	k	**	.28	.20	.48	.36	.24	0	-1.40
l	p	**	.61	.58	.68	.64	.59	.50	.08
t	$k\sigma$		11	6	34	22	12	0	-14
s									

*Number of the equation used for the calculation
**From Standard Normal tables

EFFECT OF CENTRALIZATION ON SAFETY STOCK

As seen in the former example, the value of k (which determines the safety stock) in the centralized location is not easy to relate to the equivalent k value in the decentralized locations.

Let us assume that we have 2 locations where: $D_1 = D_2 = D$ and $\sigma_1 = \sigma_2 = \sigma$, and both have the same distribution of demand during lead time, i.e., if $k_1 = k_2 = k$ then $E(k_1) = E(k_2) = E(k)$. When we centralize the stock into a single location, in order to get the same service level as in the formerly decentralized locations we need, from (16):

$$\frac{E(k)\sigma}{\sqrt{D}} = E(k_0)\sqrt{\frac{\sigma^2 + \sigma^2 + 2\sigma\sigma\rho_{12}}{D + D}} \tag{17}$$

or

$$E(k_0) = \frac{E(k)}{\sqrt{1 + \rho_{12}}} \tag{18}$$

Thus, when $\rho_{12} = 0$ (no correlation) then $E(k_0) = E(k)$, and if the distribution of the joint demand is of the same type as that of the separate demands, then $k_0 = k$. But this does not mean the same safety stock, because k_0 is related to a standard deviation of $\sigma\sqrt{2}$, i.e., now the safety stock will be $k\sigma\sqrt{2}$ instead of $2k\sigma$ in the decentralized case, which confirms the "Square Root Law." When $\rho_{12} = 1$ (perfect positive correlation), then $E(k_0) = E(k)/\sqrt{2}$ and the relation between the safety stocks depends on the specific demand distributions. However, in this case $E(k_0) < E(k)$. Although k_0 and k come from two different distributions (the joint and the original), still $k_0 > k$, and the safety stock is now, from (15), $2k_0\sigma$ instead of $2k\sigma$, i.e., larger by a factor of k_0/k. When $\rho_{12} = -1$ (perfect negative correlation), which under our assumptions eliminates all variability in the joint demand, then $E(k_0)$ goes to infinity and k_0 goes to minus infinity. This means that to achieve any service level less than 100% requires very large negative safety stock, because demand is constant.

SUMMARY

The use of inappropriate measures of inventory availability results in misleading conclusions. It has been noted that such measures must be defined precisely,[8] and often inventory availability is higher than intended due to poor definitions.[9] The impact of stock centralization/decentralization on the size of safety stocks cannot be determined simply, because it depends on the order quantities and the specific demand distributions. We could not find a simple formula to relate the number of stocking locations to the size of safety stocks when relevant measures of inventory availability are used. However, we have shown how to use available results to evaluate the impact of stock centralization/decentralization on the level of safety stocks for a given situation.

ENDNOTES

[1]Walter Zinn, Michael Levy and Donald J. Bowersox, "Measuring the Effect of Inventory Centralization/Decentralization on Aggregate Safety Stock: The 'Square Root Law' Revisited," *Journal of Business Logistics*, Vol. 10, No. 1 (1989), pp. 1-14.

[2]David Ronen, "Measures of Product Availability," *Journal of Business Logistics*, Vol. 3, No. 1 (1982), pp. 45-58, and G. Gerson and R. C. Brown, "Decision Rules for Equal Shortage Policies," *Naval Research Logistics Quarterly*, Vol. 17 (1970), pp. 351-358.

[3]W. Stevenson, *Production/Operations Management* (Homewood, Ill.: Irwin, 1986), p. 480.

[4]Tables of E(k) for the normal distribution may be found in many sources. See, for example, Endnote 3, p. 502.

[5]Same reference as Endnote 2.

[6]Same reference as Endnote 2.

[7]Same reference as Endnote 1.

[8]J. F. Magee, *Industrial Logistics* (New York: McGraw Hill, 1968).

[9]R. E. Sabath, "How Much Service Do Customers Really Want?," *Business Horizons* (April 1978), pp. 26-32.