

Ralph's Consistent Framing

Ralph is the owner of a manufacturing firm that is technologically able to manufacture two products. The manufacturing process requires that product 2 be passed through Machine 1 and Machine 2. Product 1, however, requires no work on Machine 2. The following production standards are provided.

<u>Costs:</u>			
Direct materials		\$ 5	per pound
Direct labor and variable overhead		13	per hour
Fixed cost of Capacity		\$ 50	
<u>Usage:</u>			
	Product 1	Product 2	
Direct materials (in pound)	1	2	
Direct labor (in hours)	2	1	
Machine 1	1	1	
Machine 2	0	1	
	Mach. 1	Mach. 2	
Machine hours available	150	50	

The selling prices of Products 1 and 2 are \$38 and \$33, respectively.

1. Formulate an LP to solve for the optimal amount of inputs and outputs to maximize profits. Include the inputs direct materials (DM) and direct labor (DL) as well as the outputs q_1 and q_2 as decision variables. Be sure to include constraints that ensure the amount of input required by the production plan do not exceed the amount of input supplied. Solve for the decision variables and the shadow prices on all four of the constraints. Notice you are not explicitly allocating any costs.
2. Now use the shadow prices from the direct material and direct labor constraints to allocate the costs to the products and repeat 1. That is, here include only the machine capacity constraints and decision variables q_1 and q_2 .
3. Ralph suddenly remembers that he also has two service departments, power and repair. His engineers tell him the following are the usage of these services by the other outputs.

		Users			
		Product 1	Product 2	Power	Repair
Service	Power	1	2	0.1	0.2
Department	Repair	2	1	0.3	0.0

Interpreting the table, 1 unit of power is used to make 1 unit of product 1, 2 units of power are used to make 1 unit of product 2, 0.1 units of power are used to make 1 unit of power, and 0.2 units of power are used to make 1 unit of repair. The following are the direct costs of the services.

$$\text{Power cost} = 50 + 0.90 (\text{kilowatt-hrs}) \qquad \text{Repair cost} = 50 + 2.60 (\text{repair-hrs})$$

- a. Assume Ralph ignores reciprocation and allocates the direct material, direct labor, power and repair costs using 5, 13, 0.90 and 2.60 as the cost per unit of input. What would he choose as the "optimal" amounts of the outputs? Using these quantities, what level of the inputs must be provided (be careful)? Provide an itemized income statement, indicating the amount of revenue, direct material cost, etc. (Hint: total usage of power = 297.62.)
- b. Now do as you did in part 1, including decision variables for all of the inputs and also including constraints to ensure the amount of input required by the production plan do not exceed the amount of input supplied. Be sure to include such constraints for power and repair as well as for direct materials and direct labor. Why are the shadow prices greater than 2.60 and 0.90 for power and repair, respectively?
- c. To ensure a consistent framing, how should you have allocated the costs in part 3.a?