Search and Collusion in Electronic Markets

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We examine the impact of reduced search costs on prices of commodity products in electronic marketplaces. Conventionally, reduced consumer search costs may be expected to engender stronger price competition between firms, resulting in lower prices and improved consumer welfare. This notion was formalized in Stahl (1989, “Oligopolistic pricing with sequential consumer search,” American Economic Review, Vol. 79, No. 4, pp. 700–712) in a model of static firm competition. In this paper, we show that these standard welfare conclusions may be neutralized or reversed in a dynamic environment. We focus on self-enforcing collusion by firms and characterize the conditions under which collusive equilibria exist. We show that less costly consumer search can facilitate firms’ abilities to collude, resulting in higher prices and reduced consumer welfare, even with imperfect or no monitoring by sellers of each other’s prices. If the same technology that eases consumer search also allows firms to monitor each other’s prices more easily, then firms can more easily detect cheating on a collusive price arrangement, allowing an even greater scope for collusion. This raises antitrust concerns with respect to the electronic marketplace and suggests that at least some of the anticipated competitive gains from electronic market systems may be difficult to realize.

Key words: electronic markets; search costs; collusion; repeated games; monitoring; pricing; Internet

History: Accepted by G. Anandalingam and S. Raghavan, special issue editors; received May 2002. This paper was with the authors 3 months for 2 revisions.

1. Introduction

Electronic marketplaces such as the Internet or business-to-business (B2B) e-hubs bring sellers and buyers together to facilitate trade. On the Internet, for example, sellers hold themselves out to the world, electronically presenting their products and prices to global buyers. Buyers can quickly and cheaply reach many sellers, without the constraints of geography, and easily discover and compare publicly posted prices. Technology can therefore dramatically lower the buyer’s cost of search. Indeed, the proliferation of Web-based price comparison agents,1 often called “shopbots” (automated software agents that scan multiple online merchant websites to gather information about prices and other attributes), portends a future in which the search cost for buyers who use them can become negligibly small.

The implication of these changes for price competition has been widely discussed in both the academic literature (e.g., Malone et al. 1987; Bakos 1991, 1997, 1998; Alba et al. 1997; Bailey 1998; Smith et al. 2000) and the business press (e.g., Fortune 1996, The Economist 1997). A frequent prediction is that a reduction in buyers’ search costs subjects vendors to increased price competition, resulting in reduced seller profits and increased consumer welfare. For example, Bakos (1997) examines the impact of reduced buyer search costs on equilibrium prices in the context of an electronic marketplace. In Bakos’ model, each buyer incurs the same fixed search cost to learn the price and characteristics of a product offered by a seller in a horizontally differentiated market. He finds that prices decline smoothly as buyers’ search costs are reduced. He further finds that, even for horizontally differentiated products, making prices freely available to buyers can eliminate all seller profits in a Bertrand-type equilibrium.

The logic via which reduced consumer search costs may result in stronger price competition is clear. Search costs are a friction that bestow to a firm market power over consumers who know that firm’s prices, but who must undertake costly search of its

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1 These software agents, such as PriceScan, mySimon, PriceGrabber, and DealTime, cover a wide range of products, including books, CDs, and movies.
competitors’ prices. Technology such as the Internet, that reduces the friction, compels firms to price more aggressively to avoid losing customers. The reduced buyer search cost argument thus suggests that electronic marketplaces would result in lower prices for consumers, especially for commodity products. However, empirical studies present mixed results (see Smith et al. 2000 for an excellent review). For example, while some studies (e.g., Brynjolfsson and Smith 2000) suggest that prices for commodity-like products such as books and CDs are lower on the Internet, others (e.g., Clay et al. 2002) find no significant differences in price levels across sales channels. Studies also find a high degree of price dispersion in electronic marketplaces (Brynjolfsson and Smith 2000, Clemons et al. 2002), even when the good is relatively simple and has an unambiguous product description.

In a static Bertrand pricing game with homogeneous products, zero buyer search costs induce the traditional Bertrand equilibrium. Diamond (1971) shows that if buyer search costs are always strictly positive, then the equilibrium price is the monopoly price. If a general proportion $\mu \in [0, 1]$ of consumers have zero search costs, Stahl (1989) demonstrates that the Nash equilibrium distribution of prices changes continuously from the degenerate distribution at monopoly price (Diamond) to the degenerate distribution at marginal cost (Bertrand) as $\mu$ goes from 0 to 1. Stahl’s model will provide a benchmark for our analysis, and we will elaborate on its details in a later section.

The aforementioned papers posit a static environment and focus on examining the impact of reduced buyer’s search friction. Our paper departs by considering dynamic competition by infinitely lived firms. As is well known, long-term interactions can give rise to equilibrium behavior that looks quite different from static equilibrium predictions. In the context of seller competition, the possibility that firms can engage in self-enforcing collusion is often of interest. The questions we explore in this paper are the following: What is the effect of reduced buyer search costs on firms’ ability to collude? What is the effect of reduced buyer search costs on prices when firms are already colluding? In particular, do reduced buyer search costs unambiguously increase competition and consumer surplus in a dynamic environment as they do in a static environment?

We address these questions for two specifications of a dynamic extension of the Stahl model for homogeneous product competition. In one specification, we assume that a firm cannot observe its competitors’ pricing behavior directly, but can make partial inferences based on observed demand for its goods. This noisy observability is similar to that of Green and Porter (1984), in which a firm can only probabilistically infer what output other firms have produced from a common price with a random component. Because of imperfect observability, firms cannot collude perfectly, but must sometimes revert to static equilibrium behavior for certain periods. In the second specification, we allow that firms may be able to monitor competitors’ pricing behavior at some cost, and costlessly at least some of the time. This specification is particularly relevant if the price search technology in question is, like electronic markets, publicly available, so that firms may be presumed to search industry prices as readily as do consumers. Firms may achieve perfect collusion under this specification if the technology allows costless monitoring sufficiently frequently.

Our results show that in a dynamic environment, firm collusion may overturn the standard comparative static that reduced-price search costs result in lower prices. Search costs in our model are represented by two parameters, the proportion $\mu$ of consumers who can search prices for free, and the cost of search, $s$, for consumers who cannot search for free. For the specification in which a firm’s pricing is unobservable to other firms, there are regions of parameters such that firms can sustain collusion only if $\mu$ is not too large or too small. Thus, despite the fact that an increase in $\mu$ always reduces prices in the static equilibrium, such an increase can in some cases result in higher average discounted prices by permitting firms to collude. A reduction in search cost $s$ may similarly enable firm collusion, to the detriment of consumers; furthermore, if the firms are already colluding, where an increase in $\mu$ unambiguously helps consumers, a reduction in $s$ has no impact on average discounted prices and consumer welfare.

When firms can partially monitor each other’s prices, any technology change that increases the frequency of costless monitoring must weakly enable collusion. Furthermore, even when the technology only improves consumer search, it may widen the scope for collusion with monitoring. This is always true in the case of reductions in $s$, and, much like for the first specification, can be true of increases in $\mu$, as collusion is feasible only when $\mu$ takes on intermediate values. Thus, we are able to identify numerous cases across the two specifications in which reduced-price search costs for consumers result in higher dynamic prices by making self-enforcing collusion by firms viable. We present our model in the second section. We conclude in §3 with a discussion of the implications of our results.

2. The Model
Two identical firms, A and B, compete over an infinite discrete-time horizon by choosing prices for an indivisible homogeneous good with constant unit cost,
normalized to zero. In each time period there are two one-period-lived unit demand consumers. Each consumer has a two-dimensional type. One dimension is the consumer’s cost of searching for prices. With probability $\mu \in [0, 1]$ a consumer has zero search cost, and with probability $1 - \mu$ a positive search cost $s > 0$. The second dimension is the consumer’s reservation value for the consumption of one good. This reservation value equals a number $r$ (high demand) with probability $\pi$, and equals 0 (low demand) with probability $1 - \pi$.

Consumers are paired with firms in the following fashion: In a given period, each firm has measure one of associated consumers who are able to observe that firm’s price at no cost. The consumer associated with Firm A (B) can observe the price charged by Firm B (A) only by incurring her search cost; furthermore, it is not possible to purchase a firm’s good without first observing its price. Individual consumers’ search costs are independently determined, but within a given time period, the pair of consumers who live in that period have perfectly correlated reservation values. Periods in which the consumers have reservation value $r$ we call high-demand periods; periods in which consumers have reservation value 0 are low-demand periods. The assumption about correlated reservation values emphasizes aggregate demand shocks that affect all firms in an industry similarly, rather than idiosyncratic shocks to consumers associated with a particular firm because of search factors. Consumers’ reservation values are, however, independent across time periods, and reservation values are independent of search costs.

The timing of the game in a given period is as follows. Each firm announces a price for a single unit of its good. A given firm’s announcement is immediately observable to its associated consumer, but is not observable to its competitor’s associated consumer without search and, until monitoring is introduced in §2.3, is never observable to its competitor. Each consumer then learns her two-dimensional type. At no time can firms observe consumers’ types directly, though they may be inferred via behavior. Each consumer then decides whether to search for the price of the firm with which she is not associated. Search is with perfect recall, so a consumer who chooses to search may buy from the firm with the lower price. After the search phase, consumers decide whether to

2 We assume two firms for simplicity. We discuss later how the number of firms may affect the magnitude and scope of the results. The total measure of consumers is irrelevant to the analysis because costs are linear; assuming one consumer per firm may be thought of as a normalization.

3 One may wish to classify the goods as being horizontally differentiated because of this asymmetry, in the tradition of the Hotelling (1929) “travel cost” model of product differentiation.

buy a good and from which firm, firms fill all demand at their announced prices, consumers consume (with a given consumer with reservation value $r$ who buys a good at price $p$ realizing utility $r - p$, gross of any search costs), and time proceeds to the next period.

2.1. Static Equilibrium

We first examine equilibrium in the one-period game. This game is a special case of the one analyzed by Stahl (1989), who considers the general setting of $M$ firms, and whose analysis we replicate here for $M = 2$ to facilitate comparisons with the dynamic models.

A play of this game is characterized by pricing strategies for the firms, search and purchase strategies for the consumers, and consumer beliefs about firm pricing. Consumer beliefs take the form of a probability distribution on the real numbers, representing a consumer’s assessment of the price charged by the firm with which she is not associated, and hence whose price she cannot observe without searching. A consumer’s search strategy is optimal if (a) when her search cost is 0, she always searches when she attaches a positive probability to the event that the firm she is not associated with charges a strictly lower price than the firm she is associated with; and (b) when her search cost is $s$, she always searches when the price charged by the firm she is associated with exceeds the expected price charged by the firm she is not associated with by strictly more than $s$, and never searches when this difference is strictly less than $s$. A consumer’s purchase strategy is optimal if she never purchases a more expensive good when she observes both prices, and if she never purchases a good whose price is strictly higher than her reservation value, and always purchases when the lowest price she observes is strictly less than her reservation value. A firm’s pricing strategy is a best reply if it maximizes expected revenue given the other firm’s pricing strategy, and given the consumers’ search and purchase strategies. Consumers’ beliefs are consistent if each consumer’s assessed probability distribution over prices of the firm with which she is not associated is identical to the (potentially mixed) pricing strategy of that firm. An equilibrium of the game is a play in which firms use best-reply pricing strategies, consumers use optimal search and purchase strategies, and consumer beliefs are consistent.

Stahl (1989) shows that there is a unique equilibrium of this game. The firms’ equilibrium pricing strategies are mixed, except when $\mu = 0$ or $\mu = 1$. If $\mu = 0$ (i.e., all consumers have positive search costs), then both firms charge a price of $r$ and no search takes place. If $\mu = 1$, all consumers have zero search costs, and the firms are essentially playing a Bertrand pricing game; the
equilibrium entails both firms charging a price equal to their unit cost of 0.4

For \( \mu \in (0, 1) \), the firms use symmetric, strictly mixed pricing strategies. The support of the firms’ mixing is an interval of prices \([p_L, p_H]\) \( \subset [0, r] \). Consumers with zero search cost always search for the lower price in this equilibrium, while consumers with positive search cost never search. We note immediately that in all equilibria we study in this paper, including those in the dynamic games treated later, consumers with positive search cost do not search. Given this consumer behavior, \( p_L \) and the distribution of the mixed strategy can be determined as functions of \( p_H \) via the condition that each firm must earn the same expected profit for all prices in the support. If a firm charges \( p_H \), it will only sell to its associated consumer, and only when that consumer has a positive search cost. If it charges \( p_L \), it will always sell to its associated consumer, and will also sell to the other consumer when that consumer has zero search cost. \( p_L \) must therefore satisfy the condition \((1 - \mu)p_H = (1 + \mu)p_L \), or

\[
p_L = [(1 - \mu)/(1 + \mu)]p_H. \tag{1}
\]

If a firm charges a price between \( p_L \) and \( p_H \) and that price is higher than its competitor’s, it will only sell to its associated consumer, and only when that consumer has a positive search cost. If a firm charges a price between \( p_L \) and \( p_H \) and that price is lower than its competitor’s, it will always sell to its associated consumer, and will sell to the other consumer when that consumer has zero search cost. Thus, letting \( F(p) \) be the equilibrium price distribution, the isoprofit condition can be expressed as

\[
(1 - \mu)pF(p) + (1 + \mu)p(1 - F(p)) = (1 - \mu)p_H \quad \text{or} \quad F(p) = [(1 + \mu)p - (1 - \mu)p_H]/(2\mu p). \tag{2}
\]

The associated density function is

\[
f(p) = (1 - \mu)p_H/(2\mu p^2). \tag{3}
\]

The expected selling price is

\[
P(p_H) \equiv \int pf(p) dp = [(1 - \mu)p_H/2\mu]\ln[(1 + \mu)/(1 - \mu)]. \tag{4}
\]

The final step is to determine the equilibrium \( p_H \). If for \( p_H = r \) consumers with positive search cost do not wish to search, then all the requirements of the equilibrium are satisfied, and \( p_H = r \) supports an equilibrium. Thus, if \( r \leq P(r) + s \), then \( p_H = r \). If \( r > P(r) + s \), then \( p_H = r \) cannot support an equilibrium, because consumers with positive search cost would optimally search, and by conjecture they do not search in equilibrium. In this case, \( p_H^* \) will be the price such that if a consumer with positive search cost observes this price at her associated firm, she is indifferent between searching and not searching. That is, if \( r > P(r) + s \), then \( p_H^* \) solves \( p_H^* = P(p_H^*) + s \), or

\[
p_H^* = s/[1 - [(1 - \mu)/2\mu]\ln[(1 + \mu)/(1 - \mu)]].
\]

Combining the two cases yields

\[
p_H^* = \min \left\{ r, \frac{s}{1 - [(1 - \mu)/2\mu]\ln[(1 + \mu)/(1 - \mu)]} \right\}. \tag{5}
\]

As stated above, if all consumers have zero search cost, then the unique equilibrium is the Bertrand equilibrium; and if all consumers have positive search cost, then the unique equilibrium entails monopoly pricing. It is readily verified that the expression \([(1 - \mu)/2\mu]\ln[(1 + \mu)/(1 - \mu)] \) is continuous and strictly decreasing in \( \mu \), approaching 1 as \( \mu \) approaches 0, and approaching 0 as \( \mu \) approaches 1, so that the extreme cases properly represent limits of the general specification of \( \mu \). This yields two important comparative statics. Conditions (4) and (5) suggest that any technology that reduces the search costs of a large proportion of consumers to zero also reduces the expected selling price. Condition (5) suggests that any technology that reduces the search cost of consumers who have positive search cost, even if those search costs continue to be above zero and those consumers are not induced to search, can also result in lower prices and never in higher prices. These results support the standard search cost hypotheses that predict lower prices in electronic marketplaces.

### 2.2. Dynamic Equilibrium Without Monitoring

We now consider the dynamic game. Repeated interaction will allow the firms to coordinate on prices via punishment strategies. Our first specification is one in which a firm cannot directly observe, or monitor, its competitor’s price decisions or sales. The dynamic equilibrium without monitoring is therefore in the spirit of the noncooperative collusive equilibrium under imperfect information proposed by Green and Porter (1984). In their model, colluding firms observe a common signal (e.g., industry price) that permits them to coordinate on the timing of collusive and noncooperative behavior. Although our model contains no such public signal, there is sufficient structure to ensure that firms’ beliefs over this

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4When \( \mu = 1 \), there is a second equilibrium in which all firms charge \( r \) and no consumer searches. However, because consumers who search costlessly can never be harmed by searching, this equilibrium fails to satisfy an appropriate modification of trembling-hand perfection. Thus, Stahl (1989) rejects this equilibrium; we do the same.
timing are coordinated and consistent in equilibrium, even without common knowledge of past outcomes.

As is true of virtually all repeated games, this game has a very large set of subgame-perfect Nash equilibria, including, for instance, one in which the firms simply use their static equilibrium pricing rules in every period. Because ours is a paper about firms’ ability to engage in self-enforcing and mutually beneficial price coordination, we naturally focus on dynamic equilibria in which prices may exceed their static equilibrium levels, resulting in greater firm profits. The exercise we perform may therefore be thought of as an exploration of the limits of feasible self-enforcing firm collusion, which is in the spirit of much of the economics of industrial organization literature on collusion (e.g., Green and Porter 1984, Bernheim and Whinston 1990). To this end, we focus on the subgame-perfect equilibrium that maximizes firms’ (symmetric) profits. While we do not invoke a formal equilibrium refinement to distinguish this equilibrium from others, its obvious desirability to the firms may give it a focal quality that makes it a natural candidate for study, in addition to its status as a bound on what firms may achieve by colluding.

The equilibrium is characterized by two “states,” a coordination state and a punishment phase. The coordination state will be one in which both firms charge a price of \( r \). Each firm uses the following strategy: I believe that we are in the coordination state if (a) it is Period 0; or (b) if we were in a coordination state last period, I charged a price of \( r \), and I sold exactly one good; or (c) if we have just finished \( N \) periods of punishment. If I believe we are in a coordination state, I charge a price of \( r \) this period. I believe that we are in the first period of a punishment phase if we were in a coordination state last period and I charged a price less than or equal to \( r \) and sold zero or two goods. If I believe that we are in the first period of a punishment phase, I use the static equilibrium play for exactly \( N \) consecutive periods, at which point I revert to believing that we are in a coordination state and charge \( r \) accordingly. If I believed that we were in a coordination state last period and I charged a price in excess of \( r \), my beliefs over the current and all future states evolve according to Bayes’ rule, and I optimize accordingly. This last event does not occur in equilibrium, nor will the specification of the actions ever contradict the competitor’s equilibrium beliefs, so we do not solve for the off-the-equilibrium-path behavior in this event.

Under these strategies, firms will begin the game by both charging a price of \( r \). They will continue to do so as long as demand is high, in which case each sells one good (in equilibrium all consumers will buy at a price of \( r \) from their associated firms in coordination periods). The first time they fail to make a sale, the firms will play the static equilibrium punishment for \( N \) periods. Then they will return to charging \( r \) until after the next no-sale period. Note that although it is never common knowledge between the firms that they are in a particular state because no behavior or outcomes are publicly observable, under equilibrium behavior a firm will sell one good if and only if the other firm sells one good. Thus, if a firm sells zero or two goods in a coordination state, it believes that the other firm has also sold zero or two goods, and hence will share the belief that punishment will begin the following period. This equilibrium in the absence of public signals distinguishes our model from a simple application of Green and Porter in a subtle but nontrivial way.

We note that if \( p^*_{hi} < r \), it is important that consumers in a given period be able to observe firms’ prices and sales in previous periods, an assumption we make implicitly. If not, a firm that is supposed to play its punishment strategy would find it worthwhile to charge \( r \) because a consumer with positive search cost would assume that the firms were in a collusive state and would not search. As an alternative to unbounded hindsight by finitely lived consumers, we could assume a continuum of infinitely lived consumers, and obtain the same results without the issue of observability of history. If \( p^*_{hi} = r \), then the observability of history, or lack thereof, by consumers does not affect our equilibrium.

Let \( V(\alpha) \) be a firm’s expected discounted profit of the entire game starting in the first period of a punishment phase, and let \( V(\beta) \) be a firm’s expected discounted profit for the entire game starting in the coordination state. Recall that in each of the \( N \) periods of a punishment phase, firms use their static equilibrium pricing rules, under which a firm’s expected profit equals \( \pi(1-\mu)p^*_{hi} \). Assuming a common discount factor \( \delta \in [0,1] \), \( V(\alpha) \) and \( V(\beta) \) can be solved for via the defining equations

\[
V(\alpha) = \left[\frac{(1-\delta^N)}{(1-\delta)}\right]\pi(1-\mu)p^*_{hi} + \delta^N V(\beta) \quad (6)
\]

and

\[
V(\beta) = \pi r + \delta[\pi V(\beta) + (1-\pi)V(\alpha)]. \quad (7)
\]

Formally, there are \( N+1 \) states, a coordination state and one punishment state for each of \( N \) rounds of punishment. We refer to the punishment states collectively as a phase for simplicity.

If \( p^*_{hi} = r \), then off the equilibrium path any firm with uncertainty about the current state will charge a price of \( r \), as it is in the best-reply set in all states.

\[ \]
Solving (6) and (7) yields:

\[ V(\alpha) = \frac{\pi (1-\delta)^N + \pi p_H^*(1-\delta)^N(1-\delta \pi)(1-\mu)}{(1-\delta)(1-\delta \pi - \delta^{N+1}(1-\pi))} \]  

(8)

\[ V(\beta) = \frac{\pi r(1-\delta)^N + \pi r p_H^*(1-\delta)^N(1-\delta \pi)(1-\mu)}{(1-\delta)(1-\delta \pi - \delta^{N+1}(1-\pi))}. \]  

(9)

Because the firms play the static equilibrium in the punishment phase, the only condition that must be checked is that a firm optimizes by charging \( r \) in the coordination state. The optimal one-period deviation when your competitor charges \( r \) is to charge marginally less than \( r \), as this will induce your competitor’s associated consumer to purchase your good whenever she has high demand and zero search cost (recall that consumers with positive search costs do not search in equilibrium; thus, there is no possibility of deviating by charging a much lower price and inducing the competitor’s associated consumer with a positive search cost to search your price, as the consumer will not anticipate your deviation). The supremum of the expected gain from such a deviation is therefore \( \pi \mu r \). The cost of such a deviation is that if demand is high, you will have precipitated a punishment phase next period when you would otherwise have remained in a coordination state. The expected cost of such a deviation is therefore \( \delta \pi [V(\beta) - V(\alpha)] \), where \( V(\beta) - V(\alpha) \) implicitly depends on the length \( N \) of the punishment phase; this difference is readily verified to be strictly increasing in \( N \). Because punishment phases occur with positive probability, firms benefit if these phases are as short as possible, while still generating the proper incentives not to deviate from the coordination strategy. As we are interested in the best equilibrium for the firms, we focus on \( N^* \), defined as the minimum \( N \) satisfying

\[ \pi \mu r \leq \delta \pi [V(\beta) - V(\alpha)]. \]  

(10)

Substituting (8) and (9), \( N^* \) is the minimum \( N \) satisfying

\[ \mu r \leq \frac{\delta \pi (1-\delta^N)(r - (1-\mu)p_H^*)}{1-\delta \pi - \delta^{N+1}(1-\pi)}. \]  

(11)

Because we model time as discrete, \( N \) can only take on integer values. Thus, \( N^* \) will generically not satisfy (11) with equality. However, as the time between periods grows shorter, which may be represented by a discount factor \( \delta \) that approaches 1, the right-hand side of (11) evaluated at \( N = N^* \) will converge to the left-hand side. Thus, for the purpose of comparative statics, we use an approximation of \( N^* \) equal to the (noninteger) value of \( N \) that solves (11) with equality. Combination of (8) and the equality version of (11) yields \( V(\alpha) \) and \( V(\beta) \) solely in terms of exogenous fundamentals:

\[ V(\alpha) = r[\delta \pi - \mu(1-\delta \pi)]/[\delta(1-\delta)]; \]

(12)

\[ V(\beta) = r[\pi - \mu(1-\pi)]/[1-(1-\delta)]. \]  

(13)

Recalling that the cost of punishment is increasing in the punishment duration \( N \), this equilibrium (and indeed any collusive equilibrium) exists as long as there is some \( N \) large enough for which Condition (11) holds (i.e., it exists if the firms would not wish to deviate if the punishment phase were infinite); a necessary and sufficient condition for existence is therefore that

\[ \mu r \leq [\delta \pi (r - (1-\mu)p_H^*)]/(1-\delta \pi), \]  

(14)

where the right-hand side of (14) is the right-hand side of (11) for infinite \( N \).

Noting that \( p_H^* \leq r \), so that \( r - (1-\mu)p_H^* \geq \mu r \), (14) must hold for all \( \mu \) and \( s \) if \( \delta \) and \( \pi \) satisfy \( \delta \pi \geq 1/2 \). A large value of \( \delta \) implies that firms put relatively high weight on future payoffs, making future punishment more costly. A large value of \( \pi \) implies that demand is high and the firms make sales frequently, so that the expected difference between being in the collusive state and being in the first stage of a punishment phase is high, which also makes punishment costly.

If \( \mu \geq \delta \pi/(1-\delta \pi) \), then because \( p_H^* > 0 \), (14) cannot hold, and the collusive equilibrium does not exist. A high value of \( \mu \) means that a firm that cheats in the collusive state by undercutting its rival’s price is very likely to gain by doing so, as the probability that the rival’s consumer will search is high; if this probability is too high, the punishment cannot be made sufficiently costly to deter such cheating. If \( \mu < \delta \pi/(1-\delta \pi) \), then holding \( \mu \) fixed, the collusive equilibrium exists only if \( p_H^* \) sufficiently small, and hence for \( s \) sufficiently small, as a smaller search cost translates into a smaller static equilibrium profit, and hence greater punishment. If \( s \) is large, then \( p_H^* \) is large as well, and (14) is harder to satisfy; indeed, there is a critical \( s' \) such that for \( s > s' \), the condition \( \delta \pi \geq 1/2 \) is not only sufficient for the collusive equilibrium to exist for all \( \mu \), but also necessary. However, for any fixed \( s \leq s' \) and for \( \delta \pi < 1/2 \), the equilibrium does not exist if \( \mu \) is too small, in which case \( p_H^* \) and the static equilibrium profits are too high and the punishment is not sufficiently unattractive to deter cheating, or if \( \mu \) is too large, so that \( \mu \geq \delta \pi/(1-\delta \pi) \) and the expected gain from deviation is too large relative to the discounted cost of punishment.

We have asserted that our equilibrium maximizes the firms’ joint profits among all subgame-perfect equilibria. The only candidates for this status are
equilibria in which the firms charge some common price $p^* \leq r$ in coordination states. The effect on the firms’ payoffs would be substitution of $p^*$ for $r$ in Expressions (12) and (13); because these expressions are increasing in $p^*$, they are clearly maximized for $p^* = r$. That is, the subgame-perfect equilibrium that maximizes the firms’ joint profits entails that the firms maximize joint profits in the coordination state.\(^8\)

We now turn to comparative statics of this equilibrium, first on the search cost $s$. Interestingly, for $s$ such that the collusive equilibrium exists, when $N^*$ can be chosen to make (11) hold with very close to equality, the search cost $s$ has no effect on firm profits, as it did in the static case. This is most easily seen in Expressions (12) and (13) for equilibrium discounted firm profits $V(\alpha)$ and $V(\beta)$, which do not depend on $s$ or $p_{1t}^*$. Although it remains true that $p_{1t}^*$ may fall with $s$, resulting in lower prices during punishment phases, firms can maintain incentives with shorter punishments (see Figure 1, where $N^*$ is determined by substitution of (12) and (13) into (6)). These effects offset exactly. Because sales take place under the same conditions irrespective of $s$ (i.e., if and only if demand in a given period is high), total welfare does not change for changes in $s$. Thus, because firm profits are unchanged, conditional on colluding, consumer welfare must also be the same for varying $s$ under optimal collusion (recall that consumers with positive search costs never search in equilibrium, so there is no equilibrium effect on incurred search costs when $s$ changes). Furthermore, as shown above, if $\mu < \delta \pi / (1 - \delta \pi) < 1$, the collusive equilibrium exists only for $s$ sufficiently small, in which case having to play the static equilibrium in the punishment phase is relatively costly. Thus, when this condition holds, a fall in the search cost $s$ that allows firms to collude rather than to play the static equilibrium harms consumers.

Comparative statics on $\mu$ also may change from the static to the dynamic environment. If $\delta \pi \geq 1/2$, then the collusive equilibrium exists, and expected prices are necessarily declining in $\mu$, just as in the static case; this can be seen in the fact that the firm’s equilibrium profits, expressed in $V(\alpha)$ and $V(\beta)$, are declining in $\mu$. However, if $\delta \pi < 1/2$, then for fixed $s$, the collusive equilibrium is possible only if $\mu$ is not too large or too small. In particular, for a sufficiently small value of $s$, it is possible to find two values of $\mu$ such that firms can collude when $\mu$ takes on the larger value, but cannot when $\mu$ takes on the smaller. Prices in the static equilibrium are higher for the smaller $\mu$, but this acts as a deterrent to collusion, because high prices in the static equilibrium means that the punishment of playing the static equilibrium is not sufficient to enforce monopoly pricing during collusive phases. If the two values of $\mu$ are close, consumers must suffer from a discrete jump in the frequency of collusive play, from zero to a positive amount, which offsets any gain from lower prices in the static equilibrium. Figure 2 demonstrates a typical region in $(\mu, s)$ space for which a collusive equilibrium exists when $\delta \pi < 1/2$. Note that a smaller $s$ always expands the range of $\mu$ for which collusion is possible, and that for any $s$, the needed values of $\mu$ are intermediate.

These comparative statics of search costs on consumer welfare are radically different from those for the static case. In the static equilibrium, consumers benefit from reduced search costs via lower prices. In a collusive dynamic equilibrium, the static equilibrium prices used as punishment are also lower as a result of a smaller search costs, but these lower

\[^8\] This is not a general feature of environments to which the Green and Porter (1984) analysis applies; for example, firms playing a Cournot game maximize their discounted expected payoffs by producing more than the joint monopoly output in the coordination state.
prices are charged less often. Furthermore, firms may be able to collude successfully only if search cost is sufficiently small, so that the punishment from deviating is large enough; if search cost falls below the critical level that permits collusion, consumers suffer as a result. In addition, an increase in the fraction of consumers able to search for free, from a low level to an intermediate level, may encourage collusion under some circumstances, to consumers’ detriment. Thus, the standard conclusion about reduced search costs in the static environment may be neutralized or reversed in a dynamic environment.

2.3. Dynamic Equilibrium with Monitoring

As discussed in the introduction, a technology that reduces consumers’ costs of discovering a firm’s prices is also likely to reduce a competitor’s costs of monitoring this same information. In the previous subsection, we demonstrated that reduced search costs for consumers may induce higher prices in a dynamic environment with noncooperative collusion, even if the technology that facilitates consumer search has no effect on firms’ abilities to monitor and verify each other’s collusive behavior. In this subsection we show that when technology also improves firm monitoring, the consequences for consumers may be worse yet. With no observability of prices, firms cannot maintain a fully collusive outcome, and in equilibrium must alternate between periods of high and low prices. It is also well known that with perfect observability, a large enough punishment may ensure a maximally collusive equilibrium. Here we study an environment in which firms can achieve this maximum with imperfect observability.

To incorporate firm monitoring, we assume that each firm uses a technology that permits its competitor to monitor its prices. In particular, we assume that the technology generates a probability \( \lambda \in [0, 1] \) that a firm’s competitor can costlessly observe the firm’s pricing decisions in a given period, while with probability \( 1 - \lambda \), monitoring is costly.

Even with imperfect monitoring (\( \lambda < 1 \)), the firms may be able to sustain an equilibrium in which both firms charge the monopoly price of \( r \) forever. The strategies that support this are as follows. Each firm monitors its competitor’s price only in periods in which its own cost of monitoring is zero. If a given firm has charged at least \( r \) in every period, and its competitor has charged \( r \) in every period in which that firm has monitored its competitor’s price, then that firm charges \( r \) in the next period; otherwise, the firm plays the static equilibrium strategy forever.

The condition needed for these strategies to comprise an equilibrium is that a firm does not wish to charge a price below \( r \) and gamble that its rival will not monitor in that period. Because being detected results in an infinite punishment, the relevant constraint is

\[
\pi \mu r \leq \lambda \delta \left[ \pi r - \pi (1 - \mu) p^*_m \right] / (1 - \delta).
\]  

(15)

The left-hand side of (15) is the supremum of the one-period expected gain from undercutting the rival’s price of \( r \). The right-hand side is the difference between the expected profit from both firms charging \( r \) in every period forever, and the expected profit from the firms playing the static equilibrium in every period forever, starting in the next period. \( \lambda \) appears in the right-hand side, because a firm will provoke punishment by deviating and charging less than \( r \) only when its rival can costlessly observe the deviation, which occurs with probability \( \lambda \).

We examine comparative statics in this environment from two different benchmarks. Note that if the firms play this equilibrium, the search cost measures \( s \) and \( \mu \) have no bearing on the firms’ profits provided the constraint holds: Both firms charge \( r \) and sell in every period in which demand is high. Furthermore, any increase in the parameter \( \lambda \) relaxes the constraint, ceteris paribus. Thus, if a technology that reduces consumer search costs also eases price monitoring by sellers, consumers may not benefit at all from reduced search costs. In particular, consumers can be worse off than if their search costs and firm-monitoring costs had both remained high, as an increase in \( \lambda \) may result in the satisfaction of Constraint (15), and hence the possibility of full collusion, where it was impossible without the technological change.

A second approach is to ask whether consumers are better off if technology reduces \( s \) and increases \( \mu \) but keeps \( \lambda \) constant. That is, even if a new technology reduces search costs for consumers without improving firms’ abilities to monitor each other’s prices (if some monitoring technology already exists), are consumers necessarily better off? For \( s \), the answer is the same as in the dynamic equilibrium without monitoring: A lower \( s \) can in fact be worse for consumers. To see this, suppose that under current parameters, Condition (15) is violated. As a result, the firms do not monitor, and use strategies that at least sometimes entail prices below \( r \). However, if \( s \) is reduced, which in turn reduces \( p^*_m \), Condition (15) may be satisfied, and prices can rise to \( r \) in every period. Figure 3 shows firm profits as a function of \( s \) for the static and two dynamic regimes, under the assumption that firms collude when it is an equilibrium to do so. The effect of a smaller \( s \) is that punishment is worse, which makes firms more reluctant to cheat on cooperative pricing in the face of monitoring. By reducing static equilibrium profits, a reduced search cost in essence acts as an enforcement mechanism for firms to avoid price wars.
also 1. However, for $(1-\mu)r_s>0$, Figure 4 Values of $s$.

**Figure 3** Equilibrium Profits as Functions of $s$ $(r = 10, \mu = 0.5, \pi = 0.5, \lambda = 0.15, \delta = 0.8)$

Comparative statics on $\mu$ are also similar to those in the case without monitoring. Condition (15) can be rewritten

$$(1-\delta)/(\lambda \delta) \leq [r - (1-\mu)p^*_1]/(\mu r).$$

The condition is thus satisfied if the ratio $[r - (1-\mu)p^*_1]/(\mu r)$ is sufficiently large. When $\mu$ is small, $p^*_1 = r$, and this ratio is 1. If $\mu = 1$, then the ratio is also 1. However, for $\mu < 1$ and $p^*_1 < r$, the ratio is strictly greater than 1. Therefore, if $(1-\delta)/(\lambda \delta)$ is less than 1, firms can collude for all values of $\mu$, as $[r - (1-\mu)p^*_1]/(\mu r)$ is always greater than $(1-\delta)/(\lambda \delta)$. This occurs for high values of $\lambda$ and $\delta$, in which case the probability and cost of future punishment are relatively high. However, if $(1-\delta)/(\lambda \delta)$ is greater than 1, firms cannot collude perfectly for values of $\mu$ that are too high or too low; they can collude perfectly only for intermediate values of $\mu$. These conclusions are very close to those from the dynamic model with no monitoring; note the similarity between Conditions (15) and (14). Figure 4, analogous to Figure 2 in the previous subsection, shows a representative space of $(\mu, s)$ pairs such that a fully collusive equilibrium exists.

As before, if $\mu$ is too low, firms charge prices very close to $r$ even in the static equilibrium, and the static equilibrium punishment is not strong enough to prevent deviation. Similarly, when $\mu$ is very high, the one-period gain from deviation is very high. Therefore, firms can collude perfectly only for intermediate values of $\mu$. This again stands in contrast to the static equilibrium, where firm profits are strictly decreasing in $\mu$. Here, the extra incentive to cheat created by a higher $\mu$ balances against the fact that punishment is worse the higher $\mu$ is. Unlike in the previous subsection, when some punishment was inevitable, in the monitoring equilibrium punishment never occurs, making a worse punishment strictly beneficial via the relaxed incentive constraint. The profit functions are such that possibly only for intermediate $\mu$ is the proper balance struck.

Consider now the comparative statics when firms are assumed to be playing the equilibrium that maximizes profits given the parameter array. If $(1-\delta)/(\lambda \delta)$ is less than 1, firms play the monitoring equilibrium for all values of $\mu$. If $(1-\delta)/(\lambda \delta)$ is greater than 1 and $\delta \pi \geq 1/2$, firms play the best nonmonitoring equilibrium whenever the constraint that permits the monitoring equilibrium is violated, and play the monitoring equilibrium otherwise. If $(1-\delta)/(\lambda \delta)$ is greater than 1 and $\delta \pi < 1/2$, then for some values of $\mu$, neither form of collusion will be viable, and the firms must play the static equilibrium in each period. Therefore, the firms’ expected profit functions may actually be discontinuous in $\mu$, decreasing at first as the firms play the static equilibrium, then increasing discontinuously as Constraint (14) or (15) becomes satisfied, and decreasing discontinuously as Constraint (14) or (15) is violated.
Figure 5 demonstrates what may happen for certain parameter arrays: For very low or very high values of $\mu$, collusion is impossible, and the firms can earn only the static equilibrium profits; for medium-low or medium-high values of $\mu$, perfect collusion with monitoring is infeasible, but collusion with no monitoring is viable, and firms earn profits greater than the static profits; and for medium values of $\mu$, perfect collusion with monitoring is viable and the firms are able to charge $r$ in every period, earning their maximum possible profit. Of particular note is that firms may benefit at consumers’ expense from an increased $\mu$ if it allows firms to collude more effectively, thus reversing the comparative static on $\mu$ in the static case, just as in the case without firm monitoring. Table 1 summarizes our key results.

Before concluding, we comment on how allowing for a more general number of firms may affect the scope of our results. The mechanics of the optimal collusive equilibrium, with or without monitoring, are the same for any number of firms $M$: Each firm charges $r$ when it is colluding, and uses its static equilibrium pricing rule when it is punishing. What changes is the set of parameters for which such play is an equilibrium. When there are many firms colluding at a common price, the benefits from undercutting and capturing full market share are high. In particular, for fixed values of the other parameters, it is necessarily true that a collusive equilibrium cannot exist for $M$ too large, as the size of the per-period punishment is bounded below by $r$. Thus, if, for instance, it were thought that free entry would drive the number of firms in the market to infinity, then the collusive equilibria would not exist, and our conclusions about the effect of changes in search costs would not apply. However, for any fixed number of firms $M$, our qualitative results do hold, albeit for a smaller range of parameters the larger $M$ is. If there are fixed costs of entry, for example, then the number of firms that will enter must be bounded, because static equilibrium profits converge to 0 as the number of firms increases. If it is the case that the cost of entry exceeds the stream of discounted static equilibrium per-firm profits when the number of firms is the largest for which collusion is viable, then collusion is necessarily viable for the equilibrium number of entrants.

### 3. Conclusion

Reduced buyer search costs are invoked as a reason that electronic marketplaces might result in lower prices, particularly for commodity products. In this paper we examine how the magnitude and distribution of search costs in the buyer population, and seller monitoring costs, affect equilibrium prices. Two metrics—search cost $s$, and the proportion of consumers $\mu$ who search for free—are used to model heterogeneity in buyer search costs (a la Stahl 1989). If the impact of electronic marketplace is to reduce $s$ and increase $\mu$, prices do in fact fall in a static environment.

In a dynamic environment, the conclusions are different. When firms cannot monitor each other’s prices, reducing $s$ may have no impact on average prices when firms collude, as firms can compensate for lower punishment prices with shorter spells of such prices. If $\mu$ increases, then average prices do fall in a collusive equilibrium. However, under some circumstances, equilibrium collusion may be enabled by a reduction in search frictions, in which case consumers suffer higher average prices because periods of (lower) static equilibrium prices are now mingled with periods of collusive prices. In the dynamic model with monitoring, improved access to each other’s prices make it easier for rival sellers to achieve anticompetitive coordination, even under conditions of imperfect observability. Furthermore, a reduction in buyers’ search costs, $s$, necessarily gives firms a greater scope for collusion. An increase in $\mu$ causes firms to balance between the extra incentive to cheat and a worse punishment; possibly only for middling values of $\mu$ does collusion dominate the incentive to cheat and allow monopoly pricing, so that consumers may suffer from an increase in $\mu$, just as in the case with no monitoring. Therefore, in a dynamic environment, reduced search costs that are characteristic of electronic marketplaces can lead to higher prices.

The notion that reduced search costs may harm consumers has also been advanced in Lee (1994). In his model, a single strategic seller interacts with a buyer who has the ability to search for an alternative price drawn from a fixed probability distribution. If the buyer’s search cost is sufficiently high, then the seller offers the buyer the maximum price that
does not induce the buyer to search. If the buyer’s search cost is sufficiently low, then the seller offers the buyer a high price that induces search, in the hope that the randomly drawn price is higher than the one the seller offers. As the buyer’s search cost increases, the shift in the seller’s pricing regime at a critical value of the search cost causes the buyer’s equilibrium expected surplus to increase discontinuously at that critical value; thus, two values of search costs may be found such that the buyer’s payoff is greater for the higher value. Although the conclusion that reduced search costs may harm a consumer in equilibrium is the same, the forces driving the results in Lee (1994) and in our model are quite different. In our model, all prices are chosen endogenously by optimizing firms, and as the static model to which we refer demonstrates, reduced search costs in such an environment always induces more competition between firms and lower prices. Thus, the phenomenon of reduced search costs harming consumers in our dynamic models is wholly a product of the effect of reduced search costs on firms’ ability to collude.

We have assumed a simplified market structure in order to gain insight, recognizing that real markets are more complicated. Our analysis, nevertheless, shows that an electronic marketplace can facilitate implicit collusion, with serious antitrust implications. Indeed, prior experience with two electronic market systems, the NASDAQ exchange (Christie and Schultz 1994, 1995; Barbosa 1997) and computerized airline reservation systems (Nomani 1989; Davidson 1992, 1994) suggests that this is more than a theoretical possibility. There are, however, important differences between the electronic marketplace setting we model in this paper and the NASDAQ and the airline price-fixing cases. In both cases, it was relatively easy for the antitrust enforcers and the courts to infer an agreement among rivals to fix prices and, more importantly, to frame a satisfactory remedy. In the setting described here (e.g., online merchants on the Internet or a B2B e-marketplace), it would be more difficult for the courts to infer an agreement because the availability of pricing information to rivals is an unavoidable byproduct of normal business conduct involving legitimate communication with customers in the open electronic marketplace. Further, even if a tacit agreement via price signalling is inferred, it is not clear what the courts could order the parties to do in order to correct it. Therefore, in the absence of an appropriate legal and regulatory remedy, at least some of the anticipated gains in market efficiency from electronic markets may be difficult to realize.

Acknowledgments
The authors thank Anil Arya, John Butler, Barrie Nault, Doug Schroeder, and Richard Young, participants at the 1999 Workshop on Information Systems and Economics, and the anonymous referees and editors for helpful comments on earlier versions of this manuscript.

References