

**Quantum Information and Accounting Information:  
A Revolutionary Trend and the Role of Topology**

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For presentation at the accounting mini conference on August 26, 2006

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**Section 1. Introduction**

This present paper is a sequel to the paper prepared last summer by the same five authors, entitled "Quantum information and accounting information: Their salient features and conceptual applications." This first paper has been published in the July-August issue of *the Journal of Accounting and Public Policy* and is available online at sciencedirect.com by Elsevier. The purpose of this earlier paper was to explore the latest developments in quantum information and to see whether some conceptual applications can be made that will improve our understanding of accounting information.

This present paper will focus on a revolutionary trend in quantum information and accounting information. The word, "trend," is used to emphasize that not just one but many revolutionary changes are occurring. On quantum information, we will focus on one particularly salient recent development, namely the use of topology in quantum computation. On accounting information, we discuss the changes brought on by the Sarbanes-Oxley Act of 2002 which made accounting information more legalistic and blurred the boundary of financial and managerial accounting. We wish to explore topology's potential applications to accounting as we believe

that this development will most likely aid an important segment of current accounting information, especially internal controls that are mandated by Sarbanes-Oxley.

To preserve continuity, we shall first present a synopsis of the last year's paper before we discuss the present paper. We started the paper with Part I entitled "Understanding quantum information." Here, we examined such concepts as quantum superposition and quantum parallel processing, by which a huge multiplicity of states can be transformed almost instantly. In addition to speed, we learned that quantum information must deal with irreducibly random phenomena and inexhaustible uncertainty, meaning that there are no complete events that are under complete certainty. Furthermore, there have been many quantum phenomena that just seem to be impossible to explain, such as quantum entanglement and quantum transportation where particles can travel long distance "instantly." Finally, we discussed quantum cryptography and its use of the Heisenberg uncertainty principle to create unbreakable keys.

The way physicists are dealing with quantum experiments is also interesting. They can now by means of Bose-Einstein condensate, see and manipulate particles which they could only imagine in theory. Quantum algorithms have also been developed, allowing us to cope with errors inherent in the processing of qubits. Physicists are exploring these applications while still thinking about the fundamental laws in physics. The best progress seems to occur when stretching the classical description without damaging it, thus uncovering something profound about Nature.

In Part II, we dealt with quantum information and double-entry bookkeeping. Arthur Cayley (1821-95), the founder of modern British school of pure mathematics, developed matrix algebra, later used by Werner Heisenberg in 1925, and it became indispensable in quantum mechanics. Interestingly, Cayley wrote a short book on double-entry bookkeeping. Furthermore,

he stated in its preface that double-entry bookkeeping is "Like Euclid's theory of ratios, an absolutely perfect one." We considered the basis for his glorious praise and concluded that Cayley saw the isomorphism between 'ratio matrix' and 'double-entry matrix,' thus transferring his praise for theory of ratios to the framework of double-entry bookkeeping.

It was useful to consider why there are differences in quantum information and accounting information and to trace the reasons. Thus in Part III, we analyzed seven perspectives from accounting information, namely 1) recognition and aggregation issues, 2) measurement vs. information content, 3) physical vs. social sciences perspective, 4) uncertainty and probability assessment, 5) endogenous expectation, 6) error-correction mechanism, and 7) environmental issues.

## **Section 2. Topological quantum computation**

We now take up the topic of topological quantum computation to start the present paper. In particular, we begin with a brief history of obstacles to quantum computation. First of all, the fundamental unit of classical computation is a "bit," while the fundamental unit of quantum computation is a "qubit". Both bit and qubit take on the value of 0 or 1 but qubit can take on the numerous values between 0 and 1. While this richness of the values it can take is an advantage over classical computation, the negative side of qubits is that they are extremely fragile, subject to heat and other environmental disturbances. The data must be repeatedly updated every fraction of a second to avoid "quantum decoherence" that leads to the information being corrupted. The issue of data corruption led some early enthusiasts of quantum computing to give up on its future prospects. But this disappointment was greatly saved by Peter Shor.

In 1994, Shor came up with an error correction algorithm improving the chances of useful quantum computers considerably. People began to accept errors in computation because as long as the system can have multiple copies of the information, the correct answer can be derived with a margin of error practically as small as we wish. But soon another disappointment occurred. This is because the error correction algorithm does not function unless the error before correction is less than one in 10,000 steps. For this and other reasons, quantum computers have been limited to prototypes with a maximum of 7 or so qubits. Theorists say about 32 qubits are needed to run and beat the conventional super computers.

Topological quantum computation was then introduced by Alexei Kitaev of the Landau Institute near Moscow in 1997 (Kitaev 1997). This is a revolutionary proposal to use the robustness of topology<sup>1</sup> for quantum computation instead of worrying about decoherence in quantum computing. In particular, using knot theory in topology, researchers are now considering a pair of particles called “anyons,<sup>2</sup>” which can be envisioned as chasing each other in circular motion on a plane. When the time dimension is added to the motion, the anyons can be depicted as circling on the surface of the horizontally laid cylinder with time passing from left to right. A “braid” is created by the movement of anyons, which is a historical record of the circular motion as shown in Figure 1. The diagram is a combination of space-time dimensions, called “world lines” which the anyons traverse. Computations are contained in the braid formed

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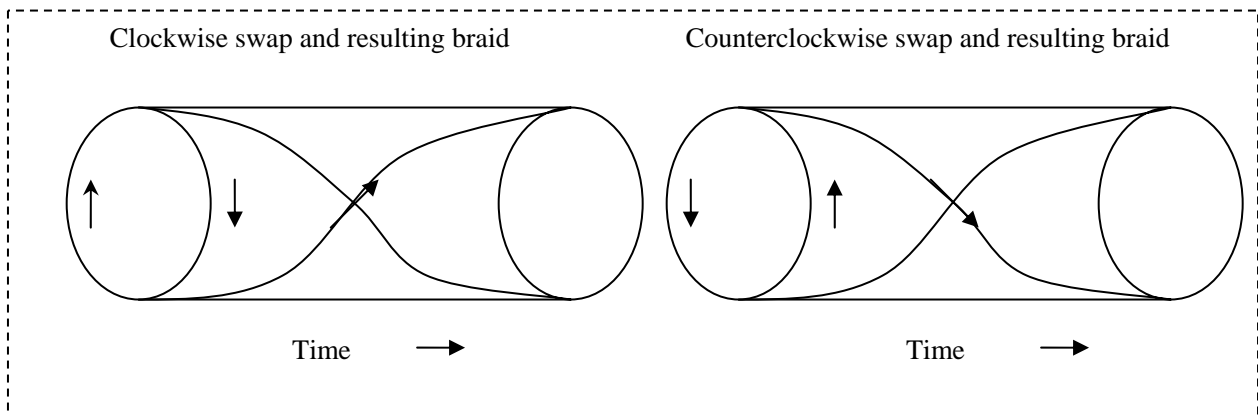
<sup>1</sup> Topology is defined as “the branch of mathematics concerned with those properties of geometric configurations that are unaltered by elastic deformation” (Das Sarma, Freedman, and Nayak, 2006). A donut and coffee cup are topologically indistinguishable as one can be identical to the other by means of gradual deformation without cutting or joining.

<sup>2</sup> In quantum theory all particles are classified either as bosons or fermions, particles that either allow or disallow multiple particles in the same energy state, depending on their spin or rather the distinguishability of the particles. “Anyons” are exceptions proposed 30 years ago. (See Lindley 2005 and Wilczek 1991). They are fractional particles and can take on “Any” phases, thus ‘anyons,’ while bosons and fermions always take on given phases. Anyons can be “abelian” or “nonabelian” depending upon whether the order in which particles are swapped is not important (abelian) or important (nonabelian). It is nonabelian anyons that play the important role in topological quantum computing.

by threads traversed by anyons and the final answer is, too. Hence the computation is topologically protected from decoherence or other disturbances.

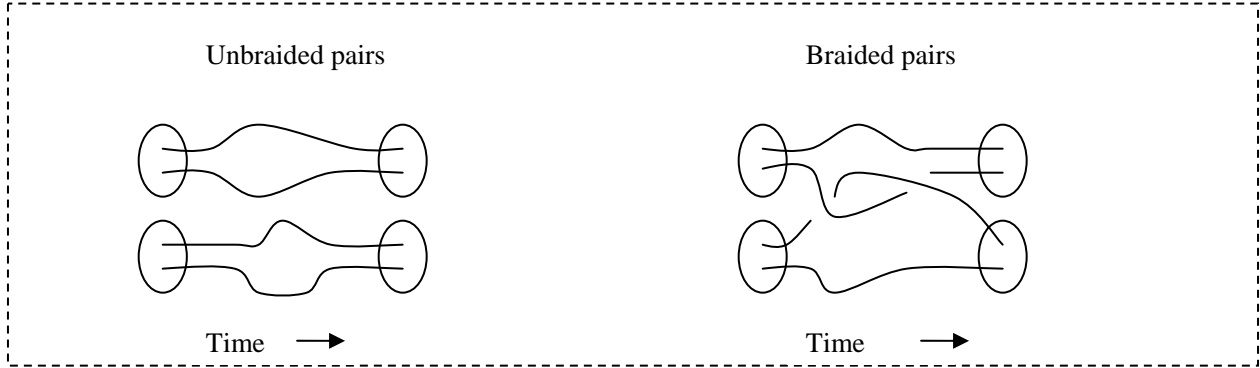
There are two kinds of circular motion--clockwise and counterclockwise, which offers a base for distinguishing 0 and 1 in classical computation. Clockwise swap creates one type of braid and counterclockwise swap creates another type. They are clearly distinguishable. Not only that, it is generally not possible to deform the braid to change its type from one to the other without cutting and/or joining which is prohibited in standard processing in topology. The two braids are topologically distinct since the paths cannot be continuously deformed in one way only without making the particles collide.

**Figure 1: Two different braids from clockwise and counterclockwise swaps**



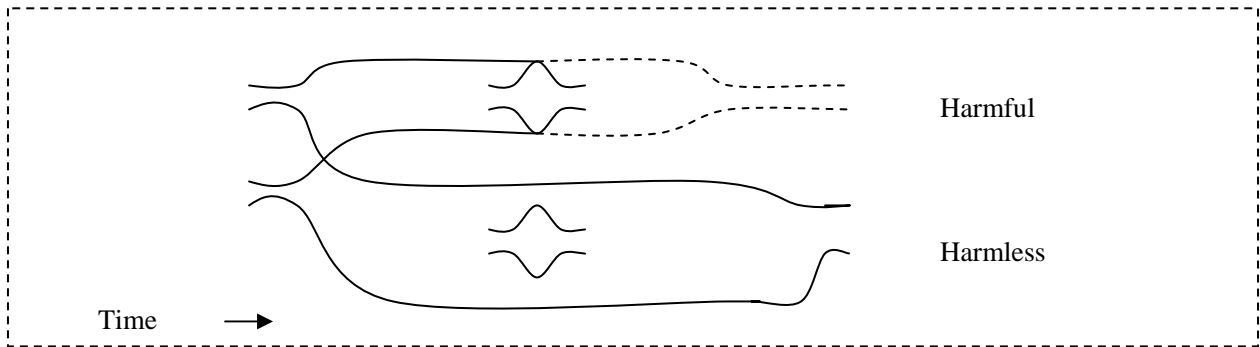
Another example is an unbraided pair of anyons (left) and a braided pair of anyons (right) in Figure 2. They cannot be deformed into each other without cutting and/or joining of parts. Also the two pairs are clearly distinguished; likewise, a topological qubit's state cannot be changed without the violent actions mentioned earlier.

**Figure 2: A simple example of two pairs topologically distinguishable**



Computation is done by pairs of anyons lined up at the start that are moved around by swapping adjacent anyons in a specific sequence. Errors can still creep in by strays as shown in Figure 3 (a pair of upper short anyons) but they can be controlled by widening the space between anyons (a pair of lower short anyons). Topological properties are said to be unchanged by small perturbations.<sup>3</sup>

**Figure 3: Strays causing error (upper), avoided by widening the space (lower anyons)**

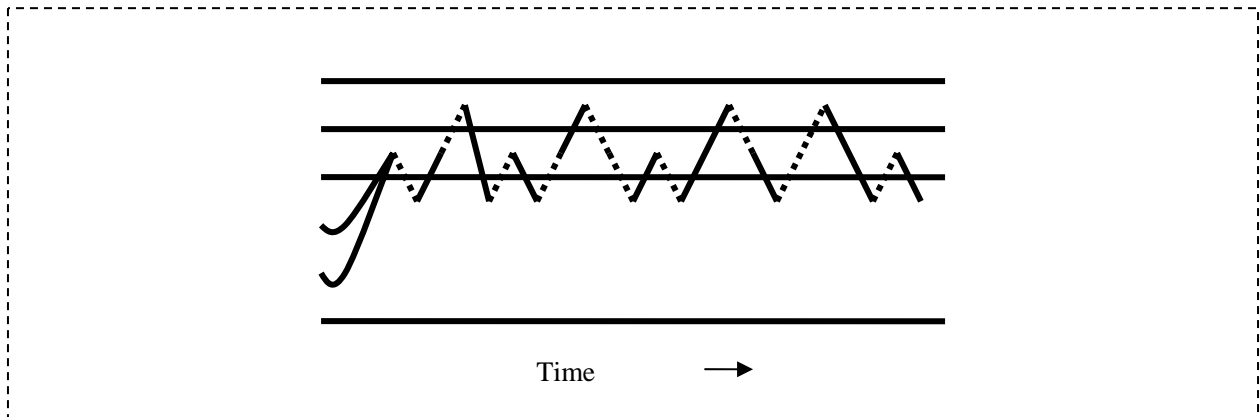


<sup>3</sup> “.If a physical system has topological degrees of freedom that are insensitive to local perturbations, then information contained in those degrees of freedom would be automatically protected against errors caused by local interactions with the environment. If the system happens to be a quantum system, one should, in principle, be able to perform fault-tolerant topological quantum computation without worrying about decoherence--the topological robustness provides quantum immunity...Most physical systems do not have topological degrees of freedom...This pessimistic conclusion is wrong, however, because of one of the most amazing discoveries of the past 30 years...Remarkably, condensed phases of matter do exist that are insensitive to local perturbations. They are topologically invariant at low temperature and energies and at long distance. (Das Sarma, Freedman, and Nayak, 2006).

Such local disturbances are extremely sensitive to distance (and energy) and the probability of contamination goes down exponentially with the widening distance. This exponential decay is the essential contribution of topology as no analogue in the traditional approaches to quantum computing has been found. Thus, for example, the error rate for the NOT gate is said to be  $10^{-30}$  or less (Das Sarma, Freedman, and Nayak, 2005).

Figure 4 deals with logic gates that are created by 6 anyons criss-crossing for operations. A starting portion of a logic gate, CNOT (controlled-not) is shown in Figure 4 where a pair of anyons move by going up or down or going over or under the other anyons' lines. After the input is run through the gate, the output of the computation is determined, which serves as an input to the next step of computation.

**Figure 4: Starting portion of the logic gate CNOT in braids using six anyons**



This process using topology not only avoids decoherence but also is a tremendous shortcut compared with conventional quantum computation. The idea was discovered by Michael Freedman. While visiting Harvard in 1988, he lectured on the possibility of using quantum topology for computation. Collins (2006) states “These ideas, published in a research paper in 1998, built on the discovery that certain knot invariants were associated with the quantum

physics of a two dimensional surface evolving in time. If an instance of the physical system could be created and an appropriate measurement carried out, the knot invariant would be approximately computed automatically instead of via an inconveniently long calculation on a conventional computer. Equally difficult problems of more real-world importance would have similar shortcuts.”

Day (2005) gives further insight into Freedman’s ideas. “Freedman connected the two proofs and had an epiphany: Rather than trying to solve the Jones polynomial (and, by extension, all the other hard problems), why not simply measure it by manipulating whatever system Witten’s quantum field theory applied to?”<sup>4</sup> <sup>5</sup> Collins (2006) also highlights an important analogue. “In a conventional computer, the state of the computer is represented by the combined state of all its bits...Similarly, a quantum computer is represented by the combined state of all its qubits. In a topological quantum computer, the qubits may be represented by groups of anyons...In a quantum computer, the process of going from the initial state of all the qubits to the final state is described by a matrix that multiplies the joint wave function of all the qubits. The similarity to what happens in a topological quantum computer is obvious: in that case, the matrix is the one associated with the particular braid corresponding to the sequence of anyon

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<sup>4</sup> Day (2005) follows the quote with an episode. “A friend brought him back down to earth. ‘What Witten thinks of as physics has nothing to do with what you learned in high school,’ said the physicist. ‘The stuff probably doesn’t exist in the real world.’ Deflated, Freedman shelved his idea. Fortunately, the stuff does exist--in the bizarre, low-temperature physics of the fractional quantum Hall (FQH) effect.” Freedman recovered and published Freedman and Meyer (1998). The two proofs are 1) the traveling salesman problem is equivalently difficult to solve as calculating the Jones polynomial, an invariant of knots and links in three dimensions and 2) Edward Witten’s proof that a certain conformal field theory in two spatial dimensions plus time mathematically resembles the Jones polynomial. Freedman’s idea of bypassing middle computations seems to have a wide applicability--If computations are hard and approximate solutions are available, why not use them and work backwards from them? Day (2005) also states “Fractional statistics and other FQH properties arise from the unique topology of two-dimensional space. In 2D, particles can’t pass above or below each other; they must go around. As they do so, their world lines form braids in the three dimensions of the 2D plane plus time.”

<sup>5</sup> Collins (2006) comments on experiments: “Topological quantum computing would make use of theoretically postulated excitation called anyons, bizarre particle-like structure that are possible in a two-dimensional world. Experiments have recently indicated that anyons exist in special planar semiconductor structure cooled to near absolute zero (on the order of millikelvins) and immersed in strong magnetic fields.

manipulations. Thus we have verified that the operations carried out on the anyons result in a quantum computation.”

If topological quantum computers are so good, can they outperform conventional quantum computers? Freedman, Kitaev, and Wang answered the question negatively, just as Alonzo Church and Alan Turing proposed in 1930s for classical computers. Topological quantum computers can also be simulated by conventional quantum computers to an arbitrary accuracy. A conjecture is: any computation system that makes use of quantum resources has the same computational abilities.

Das Sarma, Freedman, and Nayak (2006) starts with the headline: “The search for a large-scale, error-free quantum computer is reaching an intellectual junction at which semiconductor physics, knot theory, string theory, anyons, and quantum Hall effects are all coming together to produce quantum immunity.” An amazing number of important discoveries have occurred and will continue to occur. Switching from conventional quantum computing to topological quantum computing was a paradigm shift in thinking about quantum computation. We should examine such examples of paradigm shifts to see whether they offer useful conceptual applications in our own field. This is what we wish to explore next in Section 3.

Needless to say, topological quantum computing is now becoming a vast field involving many disciplines. This section is intended to provide very cursory highlights without any formulas. Nevertheless, it is hoped that the section provides a taste for the astonishing discoveries.

### **Section 3. Accounting information and its revolutionary trend**

We shall now change our focus from quantum to accounting information. We should note that fundamentally, quantum and accounting information are both under the broad definition of “information science.” Thus the two have a considerable overlap in objectives. If quantum information seeks speed and accuracy, accounting information seeks the same goals. If topological quantum information improves in speed and accuracy over conventional quantum information, the benefit of improvements in speed and accuracy will be shared by accounting information as well.

Among several candidates for paradigm shifts, we wish to take “forecasts” in financial accounting and “internal controls” in management accounting as being the most dramatic changes that have occurred in the recent decades and compare them with the paradigm shifts in quantum information.

Up until the mid 1960s, the Securities and Exchange Commission took the attitude that forecasted financial information should not be included in any filings with the SEC. In the next ten years, however, the SEC’s attitude toward forecasted data changed 180 degrees. Instead of prohibiting disclosure of forecasts, it considered mandatory forecast disclosure of sales and net income for the coming year. After strong opposition from various industries, the SEC backed down on the mandatory forecast disclosure, but continued to allow and encourage forecast disclosure. Financial accounting standards also moved considerably toward a forecast orientation.

In fact, the latest hot issue is the inclusion in the body of the balance sheet the estimated present value of net pension obligations or assets. The obligations of the firm to pay pensions to retired employees require forecasts that are several decades into the future. Undoubtedly, accuracy of the pension forecasts into such a long time into the future is as soft and fragile as

qubits in conventional quantum computing! This analogy does not seem to be fair for quantum information as qubits are real and forecasts are merely someone's opinion. Nevertheless, the quest for speed and accuracy are the same whether information is quantum or accounting.

The contribution of topological quantum computing was in improving the accuracy of quantum information enormously by taking advantage of the robustness of topology. Can the same kind of a paradigm shift occur in financial accounting?

Note that the nature of information is totally different. Topological quantum computing is worthwhile because quantum decoherence can be controlled within a specified accuracy by using topology. Forecast information cannot be controlled as forecasts are so subjective. In any event, we are not likely to discontinue disclosure of financial information that is based on forecasts as there are so many in number and in amount. The best we can do is to give warning to users as to the softness and fragility and strategic vulnerability of accounting information based on forecasts.<sup>6</sup> Hence, the chances of forecast-based information being helped by use of topology will be remote at least in the near future.

So let us now turn to management accounting focusing in internal controls. Sarbanes-Oxley (SOX) Act became law in July 2002. In particular, what brought a paradigm shift was in the Act's section 404 mandating transparency in the firm's internal control systems. Until 2002, the firm's responsibility for financial data was limited to the published financial statements. Internal control and related management accounting issues are considered to be internal matters

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<sup>6</sup> See Ijiri (2005) on a transition from history-based accounting to forecast-based accounting. See also Glover, Ijiri, Levine, and Liang (2005) for an attempt to separate facts from forecasts in financial statements as well as in journal entries and other accounting records, thereby providing users with suitable warning on the reliability of financial data.

to the firm and no disclosures had been required.<sup>7</sup> Section 404 changed all that in a short period of time.

Section 404 is a short section containing only 169 words. The whole content is reproduced below.

**Sec. 404. MANAGEMENT ASSESSMENT OF INTERNAL CONTROLS.**

(a) Rules Required.--The Commission shall prescribe rules requiring each annual report required by section 13(a) or 15(d) of the Securities Exchange Act of 1934 (15 U.S.C. 78m or 78o(d)) to contain an internal control report, which shall--

(1) state the responsibility of management for establishing and maintaining an adequate internal control structure and procedures for financial reporting; and

(2) contain an assessment, as of the end of the most recent fiscal year of the issuer, of the effectiveness of the internal control structure and procedures of the issuer for financial reporting.

(b) Internal Control Evaluation and Reporting.--With respect to the internal control assessment required by subsection (a), each registered public accounting firm that prepares or issues the audit report for the issuer shall attest to, and report on, the assessment made by the management of the issuer. An attestation made under this subsection shall be made in accordance with standards for attestation engagements issued or adopted by the Board. Any such attestation shall not be the subject of a separate engagement.

We are now interested in exploring internal control systems of a firm and what it means to their attestation. Note that for medium to large corporations, internal control and “Information Technology (IT)” are used almost interchangeably. This is because internal control without

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<sup>7</sup> The only exception may be the Foreign Corrupt Practices Act in mid-1970s that requires recordkeeping obligation of bribes of foreign officials.

computers is unthinkable these days. There are small businesses which may not use IT because of high initial cost. They have been lobbying the SEC to give them an exemption or at least postponement of the deadline for implementing Sec. 404 requirements. The SEC has finally decided to allow postponement for small businesses (Reilly 2006).<sup>8</sup>

The old concept of internal control is mostly oriented toward controlling accounting and financial transactions and the related flow of documents. Now most IT control means control of the entire operations of the firm as a whole as far as the information flow is concerned. The key is the focus on the firm's entire operations. Before SOX, the whole system was operated internally (i.e., management accounting as against financial accounting) to the firm without outside intervention. Now SOX requires attestation by a public accounting firm about the effectiveness of the internal control structure and procedures of the issuer.

This "big picture" orientation is very much in line with topology. When you throw a ball upwards and try to trace its trajectory, this is easy. But if you have to throw hundreds of balls upwards at once, they become intractable. Here, you do not focus on a single ball, you focus on the cloud of balls and the changing patterns of the cloud. French mathematician, Henri Poincaré developed topology with such qualitative characterization in mind.

Topology has long been considered for mathematicians only. However, practical applications of topology are indeed appearing. One example is entitled *Topology for computing*, (Zomorodian 2005) and is written specifically for computer scientists. Its last chapter on "Applications" covers such topics as topological feature detection, structure determination,

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<sup>8</sup> See, however, a new trend in support of Section 404 in Badal and Dvorak (2006) which says, "Expensive, onerous and unnecessary--those are a few of the complaints U.S. publicly traded companies have leveled at the Sarbanes-Oxley Act, designed to keep companies honest. Yet auditors say an increasing number of closely held companies are complying with parts of the corporate-reform law--even though they don't have to. Some of these private-sector companies have investors who hope the company will one day go public, or will be acquired. Others think adhering to the law's requirements will make their businesses more efficient. Still others face pressure from lenders or customers, including government agencies, who value the strong internal controls required by Sarbanes-Oxley."

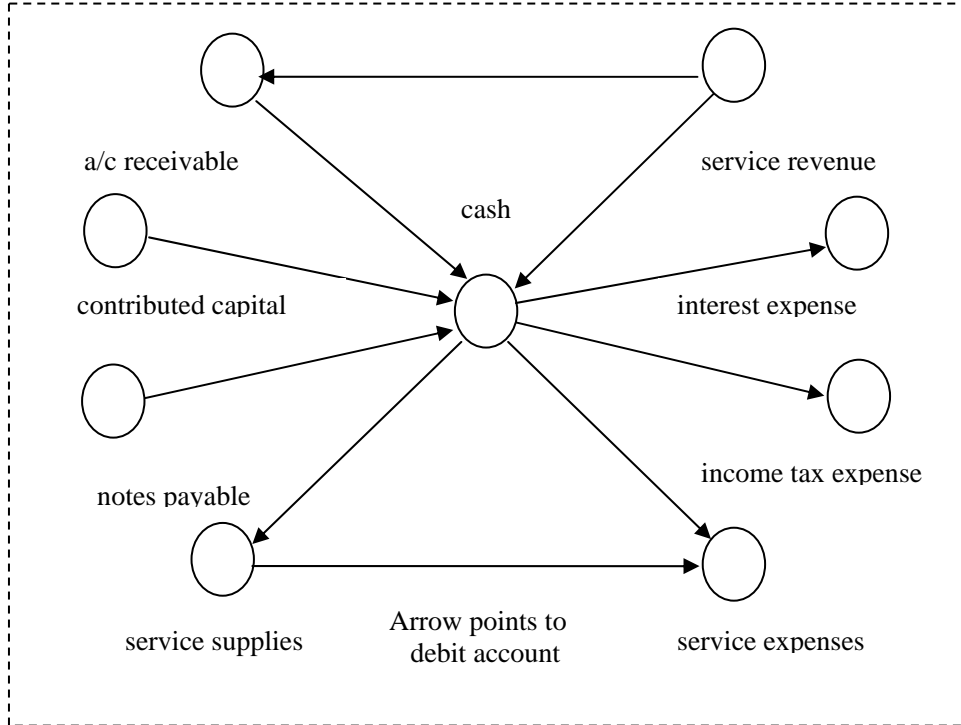
terrain simplification, iso-surface denoising, time-varying data, surface reconstruction, shape description, and I/O efficient algorithms. One of the goals here is to approximate, for example, a biological object such as protein using a combination of prototypes in the library, so that the object can be identified more easily.

Another interesting book is entitled *Topology and Its Applications*, by W. Basener (2006, forthcoming in September) from Wiley. In its abstract, the author stated: “It covers recent applications of topology in computer graphics, economics, dynamical systems, condensed matter physics, biology, robotics, chemistry, cosmology, material science, computational topology, population modeling and other areas of science and engineering.” As we can see, applicability of topology in such wide fields is impressive.

Zomorodian (2005) states that we think of a topology as the knowledge of the connectivity of a space: Each point in the space knows which points are near it, that is, in its neighborhood; we know how the space is connected. Hence, a mere collection of dots is a space but is not a topological space until dots are connected. This makes topology much closer to accounting information by double-entry bookkeeping. Single-entry bookkeeping is merely unconnected dots. Double-entry bookkeeping is a topological space. Figure 5 below is to highlight this point.

As mentioned in the introductory section, in the previous paper, we explored the relationship between ratio matrix and double-entry matrix. We conjecture the same kind of connection that might exist between double-entry network and topological operations, applied to the network in order to reduce the network’s yet undetected errors.

**Figure 5: Double-entry network is a topological space**



We would also like to explore double-entry network, which as in Figure 5 is normally presented as a 2D graph. Here, fortunately, we have, a head start in these matters, provided by Butterworth's [1972] emphasis on the accounting network and Arya et al's [2000] emphasis on exploiting that network to infer transactions and, subsequently, to compensate for possible errors in the underlying recording. Considering topology's ability to deal with higher dimensions might give us new perspectives of double-entry network. In particular, double-entry network can have faces such as the top triangle or the bottom triangle in Figure 5. Currently no attempts have been made to consider their use. But the "face" of the top triangle, for example, can take a name such as "sale-collection cycle." If the graph can be extended to 3D graph, many of such new variations might occur.

In closing, we wish to quote from Ueno et. al, (2003). "From the standpoint of the twentieth century mathematics, geometry can be roughly divided into two parts; (differential) geometry---where lengths and angles are considered important, and topology---where slight differences in lengths and angles are considered insignificant. However, these two fields of modern mathematics are not independent of one another. Rather, they represent two different ways of looking at figures. When our readers study a figure, the use of both methods allows them to see the properties of the figure more clearly. These two points of view enhance each other and often bring out hidden properties of figures that might not be visible when using one of these mathematical methods alone." The same relationship should hold between "conventional accounting" and "topological accounting." Furthermore, the search for complementarity need not stop here.

Deeper still is the "cut" between the endogenous and the exogenous. It seems clear we must treat the firm's underlying transactions as endogenous, just as we must treat the auditor's behavior as endogenous. And it is this imbedding in a presumably goal seeking social network, replete with human foibles, that distinguishes accounting from quantum information.

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