Efficient Evaluation[1]

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Abstract

Performance evaluation with multiple tasks and multiple measures has been explored in a variety of settings. Yet, the efficient design of a portfolio of performance measures remains opaque, largely because of the multidimensional nature of the exercise. Here we focus on the value of adding additional measures to the portfolio of measures in a multi-task LEN style agency model. We offer a geometric interpretation of the problem, and show that additional measures are valuable (are usefully added to the portfolio) if the linear projection of the optimal second-best production onto the first-best production increases.

1 Introduction

Evaluation typically employs a variety of measures, and the metaphor of a "balanced scorecard" has become a commonplace expression for this phenomenon (Kaplan and Norton [1996]). The underlying issues are which measures to use and what "weight" to place on each measure (e.g., Lambert [2001] and Ittner, Larcker and Meyer [2003]).[2]

In particular, optimal production in a multi-task agency setting reflects trade frictions associated with (1) the ability to motivate a particular "balance" among tasks, (2) the compensating wage differential associated with second-best risk sharing, and (3) their interaction. This is most evident in a LEN (linear contract, exponential utility and normally distributed noise) setting, especially Feltham and Xie [1994], Feltham and Wu [2000], Datar, Kulp and Lambert [2001] and Christensen and Feltham [2005].

Here our emphasis on efficiency leads to direct assessment of the usefulness of additional performance measures (e.g., an expanded though efficiently scored score card), and the in-

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[1] Thanks to participants at the 2006 Carnegie Mellon accounting conference, and especially Rick Young, for helpful comments.

[2] The metaphor is somewhat deceptive in that balance implies equipoise and, in the limit, equal distribution of weight.

sight provided by the null space of the associated performance matrix. Baker [2000, 2002] recasts this theme in geometric terms, and provides a crisp, condensed rendering of the economic forces at hand in the labor for compensation trade. Our contribution is to extend Baker's insight to the realm of (efficient) linear projections.[3]

2 Setup

A principal "owns" a production function in which agent action produces uncertain value. The agent's action is multidimensional, with m components, and is denoted $a^T = [a_1, a_2, ..., a_m]$. The principal is risk neutral, and measures the expected value of the productive outcome via $b^T a$, where $b^T = [b_1, b_2, ..., b_m]$ is a vector of valuation coefficients. The cost of providing action a, which is borne by the agent, is quadratic in nature, $\frac{1}{2}a^T a$.

Thus, absent any contracting friction, the first-best solution is given by

$$\max_a b^T a - \frac{1}{2}a^T a,$$

which implies an optimal, first-best solution of $a^{FB} = b$ along with an optimal expected payoff to the principal of $b^T b - \frac{1}{2}b^T b = \frac{1}{2}b^T b$.

Contracting friction is introduced, in the usual LEN setup, by assuming (i) a vector of n performance measures, denoted $y^T = [y_1, y_2, ..., y_n]$, is available; (ii) compensation is an affine function of these measures, $C(y) = \alpha + \beta^T y$; (iii) the performance measures are linear aggregations of the agent's action coupled with additive noise, $y = Ma + \epsilon$; (iv) the noise is distributed $N([0], \Sigma)$; and (v) the agent exhibits constant absolute risk aversion (with Arrow-Pratt measure denoted $r \geq 0$).

From here, the usual analysis exploits a certainty equivalent representation of the agent's induced preferences:

$$CE(\alpha, \beta, a, M, r) = \alpha + \beta^T Ma - \frac{1}{2}a^T a - \frac{1}{2}r\beta^T \Sigma\beta.$$

And it readily follows that the agent's induced action supply is described by

$$\max_a CE(\alpha, \beta, a, M, r),$$

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[3] Use of the LEN setup is not without cost, a point stressed by Hemmer [2004]. Here, however, it allows for a less opaque window into the economic forces at play.

or $a = M^T\beta$. Moreover, the intercept of the contract is used to satisfy the agent's outside employment or individual rationality requirement, which without loss of generality we normalize to a certainty equivalent of 0 :

$$CE(\alpha^0, \beta, M^T\beta, M, r) = 0,$$

where $\alpha^0 = \frac{1}{2}r\beta^T\Sigma\beta + \frac{1}{2}\beta^T MM^T\beta - \beta^T MM^T\beta$.

Given this, the principal's control variables are the piece rates, the β vector, and the optimal second best solution is identified via

$$\max_{\beta} b^T M^T\beta - \{\frac{1}{2}r\beta^T\Sigma\beta + \frac{1}{2}\beta^T MM^T\beta - \beta^T MM^T\beta\} - \beta^T MM^T\beta. \tag{1}$$

This provides the following first order condition,

$$[MM^T + r\Sigma]\beta = Mb. \tag{2}$$

And we readily find optimal piece rates of

$$\beta^* = [MM^T + r\Sigma]^{-1}Mb, \tag{3}$$

coupled with an optimal action vector of

$$a^{SB} = M^T\beta = M^T[MM^T + r\Sigma]^{-1}Mb. \tag{4}$$

Substituting (4) and (2) into (1), the optimal second-best expected payoff to the principal is $\frac{1}{2}b^T a^{SB}$.

Before proceeding, notice the solution presumably identified in (3) assumes $[MM^T + r\Sigma]$ is nonsingular, has rank n.[4] This assumption, in turn, implies the solution in (3) is unique.

The difference between the principal's first- and second-best payoffs is often termed an *efficiency loss*:

$$\frac{1}{2}b^T b - \frac{1}{2}b^T b = \frac{1}{2}b^T b - \frac{1}{2}b^T M^T[MM^T + r\Sigma]^{-1}Mb.$$

Importantly, the second-best solution reflects the effects of risk that is imposed on the agent, de facto increasing the cost to the principal of action supply, and the ability to fine tune the

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[4] The performance matrix, M, does not have to be full row rank, unless of course the agent is risk neutral.

balance among the various actions.[5]

The risk effect, of course, disappears in the case of a risk neutral agent ($r = 0$). Here the efficiency loss reduces to

$$\begin{aligned} & \frac{1}{2}b^T b - \frac{1}{2}b^T M^T [MM^T]^{-1} Mb \\ = & \frac{1}{2}b^T [I - M^T [MM^T]^{-1} M] b. \end{aligned} \tag{5}$$

And, with a risk neutral agent, a positive efficiency loss reflects the inability to fine tune the balance among the various tasks. The first-best action can be achieved in this case (of risk neutrality) if and only if M is full rank ($n = m$).

Conversely, with a risk-averse agent, even if we have as many measures as actions and the special case of the evaluation matrix being the identity matrix, $M = I$, coupled with independent, *iid* error terms (with variance denoted σ^2), we would have perfectly balanced but uniformly under-supplied action due to the compensating wage differential. In particular,

(3) now provides $\beta^* = \frac{1}{1+r\sigma^2} b$, or $\beta^* = kb$, where k is a positive constant less than unity, implying perfectly balanced but uniformly under-supplied action due to the compensating wage differential.

3 Supplemental Contracting Variables

Now suppose an additional or supplemental vector of variables is also available for contracting: $\widehat{y}^T = [\widehat{y}_1, \widehat{y}_2, ..., \widehat{y}_{\widehat{n}}]$ with $\widehat{y} = \widehat{M} a + \widehat{\epsilon}$. The expanded covariance matrix is denoted,

$$\widetilde{\Sigma} = \begin{bmatrix} \Sigma & \overline{\Sigma} \\ \overline{\Sigma}^T & \widehat{\Sigma} \end{bmatrix},$$

and, of course, the combined vector is distributed $N([0], \widetilde{\Sigma})$.

From here we again focus on the piece rates as control variables, which we display in

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[5] These two effects are often labeled congruity and intensity effects (e.g., Feltham and Xie [1994]) or congruity and sensitivity/precision effects (e.g., Lambert [2001]). We will shortly take a parallel attack, but one based on the effect of appending an additional vector of performance measures.

partition form as

$$\widetilde{\beta} = \begin{bmatrix} \beta \\ \widehat{\beta} \end{bmatrix},$$

where $\widehat{\beta}$ denotes the piece rates on the supplemental variables. In usual fashion, we now have first order conditions

$$[\widetilde{M}\widetilde{M}^T + r\widetilde{\Sigma}] \begin{bmatrix} \beta \\ \widehat{\beta} \end{bmatrix} = \widetilde{M}b = \begin{bmatrix} Mb \\ \widehat{M}b \end{bmatrix}, \tag{6}$$

where, importantly, we presume $[\widetilde{M}\widetilde{M}^T + r\widetilde{\Sigma}]$ is nonsingular, and thus has rank $n + \widehat{n}$.

Expanding provides the notational highlight of our paper:

$$\begin{bmatrix} MM^T + r\Sigma & M\widehat{M}^T + r\overline{\Sigma} \\ \widehat{M}M^T + r\overline{\Sigma}^T & \widehat{M}\widehat{M}^T + r\widehat{\Sigma} \end{bmatrix} \begin{bmatrix} \beta \\ \widehat{\beta} \end{bmatrix} = \begin{bmatrix} Mb \\ \widehat{M}b \end{bmatrix},$$

or

$$\begin{aligned} \left[MM^T + r\Sigma\right]\beta + \left[M\widehat{M}^T + r\overline{\Sigma}\right]\widehat{\beta} &= Mb; \\ \left[\widehat{M}M^T + r\overline{\Sigma}^T\right]\beta + \left[\widehat{M}\widehat{M}^T + r\widehat{\Sigma}\right]\widehat{\beta} &= \widehat{M}b. \end{aligned} \tag{7}$$

Now suppose, for the moment, that the supplemental variables are useless. This implies the solution to the above first order conditions is $\beta = \beta^*$ (See [3]) and $\widehat{\beta} = 0$. In turn, this implies, with slight rearrangement, that

$$\widehat{M}\left[M^T\beta^* - b\right] + r\overline{\Sigma}^T\beta^* = 0. \tag{8}$$

And this leads to the following suggestive terminology.

Definition 1. *$\widehat{M}\left[M^T\beta^* - b\right]$ is termed the distance effect and $r\overline{\Sigma}^T\beta^*$ is termed the risk effect of the supplemental contracting variables.*

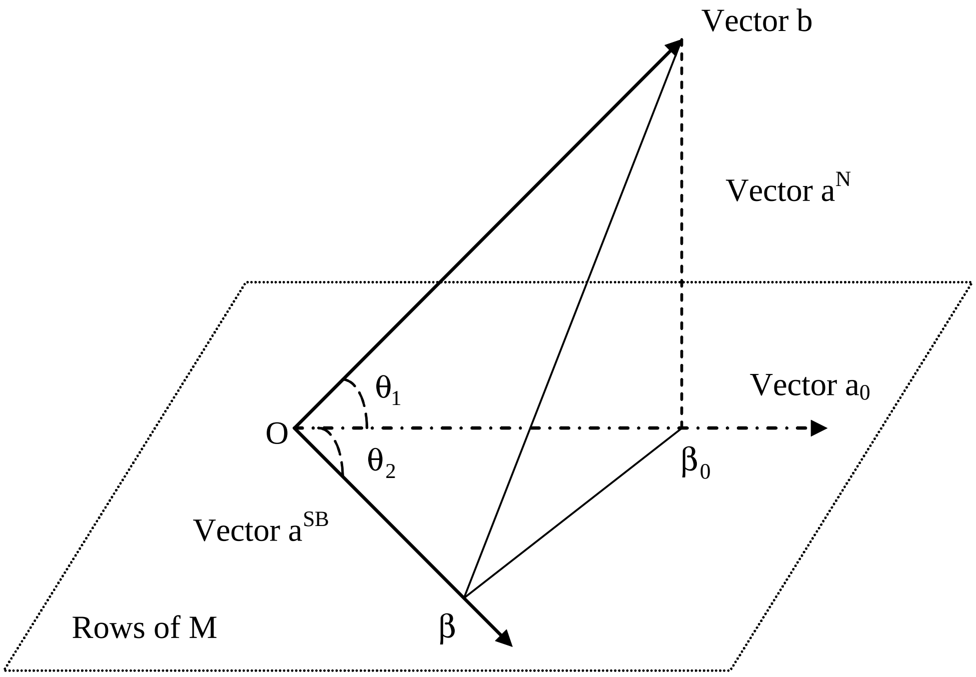
Now recall our earlier concern for nonsingularity of the coefficient matrices in the first order conditions. Nonsingularity, as noted, implies both sets of first order conditions have a solution and that solution is unique. Moreover, $\widetilde{\beta} = \begin{bmatrix} \beta^* \\ 0 \end{bmatrix}$ is feasible in the expanded contracting system. This provides the intuition for:

Proposition 1. *Suppose $rank(MM^T + r\Sigma) = n$ and $rank(\widetilde{M}\widetilde{M}^T + r\widetilde{\Sigma}) = n + \widehat{n}$. The* *supplemental contracting variables are useful if and only if the sum of the distance effect and the risk effect of the supplemental contracting variables non-zero.*

Importantly, the distance effect is zero if the vector $M^T\beta^* - b = a^{SB} - a^{FB}$ is in the null space of supplemental matrix $\widehat{M}$. Likewise, the risk effect is zero if the weighted covariance between the supplemental and original variables is zero, where the weights are given by the original piece rates, β^*. A nil distance effect essentially means there is no way to exploit the additional variables so as to better "balance" the action vector, and a nil risk effect means there is no noise reduction possibility. In this sense, adding the new variables to the portfolio of performance measures hinges on the ability to provide better incentives or less noisy incentives, but not necessarily to trade them off. Also notice the distance and risk effects are similar to the congruity and intensity (Feltham and Xie [1994]) or sensitivity/precision effects (Lambert, [2001]). The difference is we concentrate not on the initial distortions but on their incremental improvement.

4 A Geometric Interpretation

Now we return to the status quo setting, where the supplemental variables are not yet available, and offer a geometric formulation, expanding on Baker [2000, 2002] and Lambert [2001]. Generally speaking, solving for the efficient piece rate is equivalent to projecting (orthogonally or non-orthogonally) vector b into the row spaces of the evaluation matrix, M. To simplify matters, we now assume (here and throughout) that the evaluation matrix (M or $\widetilde{M}$ when it is subsequently reintroduced) is always of full row rank. One of the advantages of the geometric formulation is the ability to visualize how the projection is affected by the balance among actions, the risk sharing concerns and their interaction. Figure 1 is illustrative.



Projection of vector b into rows of M

4.1 Risk Neutral Agent

When the agent is risk neutral ($r = 0$), we have a direct connection between orthogonality and the optimal second-best action supply. Solving for the optimal piece rate is reduced to a least squares problem of $M^T\beta = b$ (Datar, Kulp and Lambert [2001] and Lambert [2001]).[6]

Proposition 2. *With $r = 0$, the optimal second-best action vector is the projection of vector b into the rows of matrix M.*

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[6] If the null space of M is empty ($m = n$), a unique solution exists. When $m > n$, vector b does not lie in the row space of M. A least squares problem looks to minimize the error between two column vectors, $M^T\beta$ and b. The same technique applies when it comes to solving for the optimal piece rate.

Proof. The projection of vector b into the rows of M is β_0 such that

$$M[b - M^T \beta_0] = 0,$$

or

$$\beta_0 = [MM^T]^{-1} Mb,$$

which is precisely (3) when $r = 0$. □

In Figure 1, β_0 is an orthogonal projection of vector b into the rows of evaluation matrix M, which is also the solution to the least square problem of regressing (absent the intercept) the piece rates are the regression of b on M. The second-best action vector is the vector $a_0 = M^T \beta_0 = M^T \beta^*$. Define θ_1 to be the angle between vector b and the second-best action vector. When risk sharing is not a concern, angle θ_1 captures the effect of a less fine-tuned contract due to insufficient measures available in the portfolio. Moreover, the resulting expected payoff to the principal (i.e., $\frac{1}{2} b^T a^{SB}$) is one-half the squared length of vector a_0 (since $a_0 = b \cos \theta_1$)

$$\begin{aligned} \frac{1}{2} b^T a^{SB} &= \frac{1}{2} b^T a_0 = \frac{1}{2} \|b\| \, \|a_0\| \cos \theta_1 \\ &= \frac{1}{2} \|a_0\|^2 = \frac{1}{2} a_0^T a_0. \end{aligned} \tag{9}$$

Now we are ready to show that the efficiency loss is intimately connected to the null space of matrix M.

Proposition 3. *With $r = 0$, the efficiency loss is one-half the squared length of the projection of vector b into the null space of matrix M.*

Proof. Since the first-best action is vector b itself and the second-best action is the projection of b into the rows of M, we can write $b = a_0 + a^N$, where a^N is the null space component of b, the projection of b into the null space of M. We then have an efficiency loss of

$$\begin{aligned} & \frac{1}{2}[a_0 + a^N]^T [a_0 + a^N] - [a_0 + a^N]^T a_0 + \frac{1}{2} a_0^T a_0 \\ = \; & \frac{1}{2}[a_0 + a^N]^T [a_0 + a^N] - \frac{1}{2} a_0^T a_0 \\ = \; & \frac{1}{2} a_0^T a_0 + \frac{1}{2} a^{NT} a^N - \frac{1}{2} a_0^T a_0 \\ = \; & \frac{1}{2} a^{NT} a^N. \end{aligned} \tag{10}$$

The first equality is given by the fact that vectors a_0 and a^N are orthogonal. The second equality is a direct application of the Pythagorean Theorem. $\square$

Back to Figure 1, the efficiency loss can also be expressed as one-half the squared length of vector b multiplied by $\sin^2\theta_1 = 1 - \cos^2\theta_1$.

4.2 Risk Averse Agent

When the agent is risk averse, the risk sharing concern will be factored into the projection. To isolate its aspect, suppose vector b resides in the row spaces of M, i.e., there is an "error free" solution to the least squares problem as $b = M^T\beta_0$. In this case, however, risk sharing prevents the second-best action vector from overlapping with vector b.

In Figure 1, suppose vector b overlaps with vector $a_0 = M^T\beta_0$. The second-best action vector is $a^{SB} = M^T\beta$, which has an angle θ_2 with vector a_0. Immediately, we have the following observation.

Lemma 1. *Suppose vector b resides in the rows of M and $n > 1$. $\theta_2 = 0$ if and only if vector a^{SB} coincides with vector b.*

Proof. By definition,

$$\cos\theta_2 = \frac{b^T a^{SB}}{\|b\|\,\|a^{SB}\|} = 1.$$

The last equality holds if and only if vector a^{SB} coincides with vector b. Thus, in the range of 0 and 2π, $\cos\theta_2 = 1$ gives $\theta_2 = 0$. $\square$

In a single measure setting ($n = 1$), on the other hand, θ_2 is always zero. The efficiency loss is connected only with the length of vector a^{SB}. When there is more than one measure, the angle θ_2 is zero if and only if the vector a^{SB} lies onto vector b with the same length. The efficiency loss is connected with both the angle θ_2 and the length of vector a^{SB}.

By (3), the second-best action vector is written as,

$$a^{SB} = M^T\lambda\beta_0, \text{ where } \lambda = \left[I + [MM^T]^{-1}r\Sigma\right]^{-1}, \tag{11}$$

where λ can be thought of as the risk adjustment matrix. Intuitively, if θ_2 is non-zero, the second-best action vector not only rotates away from vector b, but also changes its length.

Lemma 2. *Suppose vector b resides in the rows of M. With $r > 0$, the efficiency loss is one-half the product of the length of b and the projection of $[b - a^{SB}]$ onto b.*

Proof. Since b resides in the rows of M, we have $b = a_0$. The efficiency loss is $\frac{1}{2}b^T b - \frac{1}{2}b^T a^{SB} = \frac{1}{2}\left[\|b\|^2 - \|b\|\left\|a^{SB}\right\|\cos\theta_2\right]$. But $\left[\|b\| - \left\|a^{SB}\right\|\cos\theta_2\right]$ is the projection of $[b - a^{SB}]$ onto b. $\square$

To see how the risk concern affects the optimal production and the efficiency loss, consider two independent, noisy measures. The evaluation matrix M is 2 by m. A less noisy measure, *ceteris paribus*, would increase the length of a^{SB} and also reduce the angle θ_2, thereby reducing the efficiency loss.

More generally, consider the case in which vector b does not reside in the row space of M. The second-best action vector turns out to be a non-orthogonal projection of vector b into the rows of M, $a^{SB} = M^T\beta$. We can, as a result, decompose the overall efficiency loss into two components: one reflecting the balancing concern and the other the risk-sharing concern.

Proposition 4. *The efficiency loss, in general, includes two parts: (i) one-half the squared length of the projection of b into the null space of M; and (ii) one-half the product of length of a_0 and projection of $[a_0 - a^{SB}]$ onto a_0.*

Proof. As in Figure 1, two steps are taken to arrive at vector a^{SB}. First, project b into the rows of M orthogonally as $a_0 = M^T\beta_0$. Then rotate vector a_0 by angle θ_2 and adjust the length simultaneously to arrive at vector $a^{SB} = M^T\beta$. We show that the expected payoff is one-half the inner product of vector a_0 and vector a^{SB}, since

$$\begin{aligned}
\frac{1}{2}b^T a^{SB} &= \frac{1}{4}\left[\|b\|^2 + \left\|a^{SB}\right\|^2 - \left\|b - a^{SB}\right\|^2\right] \\
&= \frac{1}{4}\left[\|b\|^2 + \left\|a^{SB}\right\|^2 - \left\|a^N\right\|^2 - \left\|a_0 - a^{SB}\right\|^2\right] \\
&= \frac{1}{4}\left[\|a_0\|^2 + \left\|a^{SB}\right\|^2 - \left\|a_0 - a^{SB}\right\|^2\right] \\
&= \frac{1}{2}a_0^T a^{SB}. \qquad (12)
\end{aligned}$$

Now we have an efficiency loss of

$$\begin{aligned}\frac{1}{2}b^Tb - \frac{1}{2}b^Ta^{SB} &= \frac{1}{2}a^{NT}a^N + \frac{1}{2}a_0^Ta_0 - \frac{1}{2}b^Ta^{SB} \\ &= \frac{1}{2}a^{NT}a^N + \frac{1}{2}a_0^Ta_0 - \frac{1}{2}a_0^Ta^{SB}. \end{aligned} \tag{13}$$

The first equality follows from the Pythagorean Theorem and the second equality is implied by (12). □

The noted risk sharing concern is the only source for the non-orthogonality (illustrated in Figure 1). To trace the second best distortion in the action vector, we move initially from vector b to vector a_0, reflecting the component of the efficiency loss (part (i)) that results from an insufficient supply of performance measures being included in the portfolio when the risk concern is absent (as identified in Proposition 3). From there we trace the second component of the efficiency loss, moving from vector a_0 to vector a^{SB}, reflecting the risk sharing concern and its interaction with the balancing concern (as identified in Lemma 2). Once risk sharing is nontrivial, it is impossible to separate the balancing concern and the risk sharing concern.

4.2.1 Value of Supplemental Variables

We now return to our theme in Section 3 of supplementing the existing portfolio of performance measures, essentially asking whether adding the supplemental vector of variables, $\widehat{y} = \widehat{M}a + \widehat{\epsilon}$, to the evaluation portfolio is useful. The efficiency loss, of course, measures the maximal amount the principal would pay for additional information. Reinterpreting Proposition 1 in our geometric formulation now allows us to better disentangle the distance and risk effects of the supplemental variables. This provides:

Proposition 5. *The supplemental variables are useful if and only if any of the following considerations holds,*

(i) $r = 0$ and the row space of $\widehat{M}$ has a non-zero component in the null space of matrix M;

(ii) $\widehat{M} = 0$ and $\widehat{y}$ is correlated with y;

(iii) The projection of vector a^{SB} onto vector b increases.

Proof. For part (i), using the fact that with $r = 0$, the second-best action is the (orthogonal) projection of b into the rows of the information matrix, M or M combined with $\widehat{M}$, we can write $b = a^{SB} + a^N$ in the former case and $b = \widehat{a}^{SB} + \widehat{a}^N$ in the latter case. But the information is useful if and only if $a^{SB} \neq \widehat{a}^{SB}$, or $a^N \neq \widehat{a}^N$. But the respective null components will differ if and only if $\widehat{M}$ has a non-zero component in the null space of matrix M, that is, if and only if $\widehat{M}\left[M^T\beta^* - b\right] \neq 0$.

For part (ii), by Proposition 1, $\widehat{M} = 0$ implies $\widehat{y}$ is valuable if and only if $r\overline{\Sigma}^T\beta^* \neq 0$, where β^* denotes the efficient piece rate when the evaluation matrix is M.

For part (iii), the efficiency loss given M as the evaluation matrix can be written as

$$\begin{aligned} & \frac{1}{2}b^T b - \frac{1}{2}b^T a^{SB} \\ = \; & \frac{1}{2}\|b\|^2 - \frac{1}{2}\|a_0\|\left\|a^{SB}\right\|\cos\theta_2 \\ = \; & \frac{1}{2}\|b\|\left[\|b\| - \left\|a^{SB}\right\|\cos\theta_1\cos\theta_2\right]. \end{aligned} \tag{14}$$

Back to Figure 1, define γ to be the angle between vector b and vector a^{SB}. We have

$$\begin{aligned} \cos\gamma &= \frac{b^T a^{SB}}{\|b\|\,\|a^{SB}\|} = \frac{a_0^T a^{SB}}{\|b\|\,\|a^{SB}\|} \\ &= \frac{\|a_0\|\left\|a^{SB}\right\|\cos\theta_2}{\|b\|\,\|a^{SB}\|} = \cos\theta_1\cos\theta_2. \end{aligned} \tag{15}$$

The projection of vector a^{SB} onto vector b is $\left\|a^{SB}\right\|\cos\gamma$. $\widehat{y}$ is valuable if and only if $\left\|\widetilde{a}^{SB}\right\|\cos\widetilde{\gamma}$ increases. □

Returning to Proposition 1, then, part (i) of Proposition 5 shows the distance effect of supplemental variables, whereas part (ii) shows the risk effect. Part (iii) of Proposition 5 shows the combined effects when distance effect and risk effect are entangled. Also, parts (i) and (ii) are special cases of part (iii) of Proposition 5.

The "dimension" structure of the two information matrices and the "covariance structure" of their noise, then, tell us whether the additional matrix will convey useful information in the action balancing exercise. In this respect it is highly reminiscent of Ashby's Law: "a model system or controller can only model or control something to the extent that it has sufficient internal variety to represent it."

4.3 A Numerical Example

Consider a simple example with the following parameters,

$$\begin{aligned} M &= \begin{bmatrix} 5 & 4 & 3 & 2 \\ 2 & 3 & 7 & 9 \end{bmatrix}; \\ b^T &= \begin{bmatrix} 2 & 3 & 7 & 9 \end{bmatrix}; \\ r &= 0.1; \text{ and} \\ \Sigma &= \begin{bmatrix} 5 & 0 \\ 0 & 25 \end{bmatrix}. \end{aligned}$$

The efficient piece rate is $\beta^T = \begin{bmatrix} 0.04 & 0.97 \end{bmatrix}$, and the optimal second-best act vector is $a^T = \begin{bmatrix} 2.12 & 3.05 & 6.89 & 8.78 \end{bmatrix}$. The efficiency loss is 1.2. And we have $\theta_1 = 0°$, $\theta_2 = 0.93°$, and $\left\|a^{SB}\right\| \cos\theta_2 = 11.756$.

Now consider an additional measure, $\widehat{M} = \begin{bmatrix} 2 & 3 & 4 & 5 \end{bmatrix}$, with $\widehat{\Sigma} = 5$ and $\overline{\Sigma} = 0$. The newly resolved efficient piece rate is $\beta^T = \begin{bmatrix} -0.18 & 0.59 & 0.8 \end{bmatrix}$, and the optimal second-best act vector is $a^T = \begin{bmatrix} 1.86 & 3.43 & 6.76 & 8.92 \end{bmatrix}$. The efficiency loss now is 0.73. And we get $\theta_1 = 0°$, $\theta_2 = 2.43°$, and $\left\|a^{SB}\right\| \cos\theta_2 = 11.836$. The additional measure is useful since the efficiency loss is reduced and since $\left\|a^{SB}\right\| \cos\theta_1 \cos\theta_2$ increases.

5 Conclusion

Accounting has long played an important role in the stewardship arena. This arena is a profoundly complex, multidimensional, multiperiod setting, which begs for an interpretable model of the forces at hand. Projection from what is contractible into the space of managerial behavior provides, we argue, one such approach. This projection, in turn, has a strong parallel in our use of weighted least squares when facing an estimation problem with a non-standard covariance structure. Precisely the same type of weighting surfaces here, and the reweighting when additional measures are added to the portfolio is useful if and only if it alters the projection, which is equivalent to providing a distance or a risk effect. Efficient evaluation, in other words, calls for a weighted assessment of the evaluation portfolio's measures; and the weights in that assessment themselves stem from a weighted least squares type of projection, reflecting, intuitively, their ability to measure that various tasks and their attendant noise patterns.

References

Baker, George. The use of performance measures in incentive contracting. *American Economic Review.* May 2000, 90(2): 415-20.

Baker, George. Distortion and risk in optimal incentive contracts. *Journal of Human Resources.* Fall 2002, 37(4).

Datar, Srikant, Susan Cohen Kulp, and Richard Lambert. Balancing performance measures. *Journal of Accounting Research.* June 2001, 39(1): 75-92.

Feltham, Gerald, and Jim Xie. Performance measure congruity and diversity in multitask principal/agent relations. *Accounting Review.* July 1994, 69(3): 429-53.

Feltham, Gerald, and Martin Wu. Public reports, information acquisition by investors, and management incentives. *Review of Accounting Studies.* June 2000, v5(2): 155-190.

Hemmer, Thomas. Lessons lost in linearity: a critical assessment of the general usefulness of LEN models in compensation research, *Journal of Management Accounting Research, 2004 v16, 149-162.*

Holmstrom, Bengt and Paul Milgrom. Multitask principal-agent analyses: incentive contracts, asset ownership, and job design, *Journal of Law, Economics and Organization*, 1991, v7(3), 24-52.

Ittner, Christopher, David Larcker and Marshall Meyer. Subjectivity and the weighting of performance measures: evidence from a balanced scorecard," *Accounting Review*, 2003, v78(3,Jul), 725-758.

Kaplan, Robert and David Norton, *The balanced scorecard: translating strategy into action.* Harvard Business School Press, 1996.

Lambert, Richard. Contracting theory and accounting, *Journal of Accounting & Economics*, 2001, v32(1-3,Dec), 3-87.

Prendergast, Canice. What trade-off of risk and incentives? *The American Economic Review.* May 2000, v90(2): 421-25.