Accounting Structure
Chapter 1 - Accounting as an Information Science

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accounting as an information science

The point of view adopted in this book is that accounting is appropriately treated as an information science. Prior to beginning the journey it probably makes sense to define how the term is used, or at least offer some properties of "information science." Here are four:

1. The discipline is already a scientific discipline prior to the topic of information coming into view.
2. As the discipline evolves, information occupies a more central place.
3. The discipline possesses some ideas and results which can be useful to other information sciences.
4. The discipline can profitably use ideas from other information sciences.

The first item in the list distinguishes this approach from some others. For example, David Luenberger's excellent book entitled "Information Science" commences immediately with information and information related results. While many of the same topics are encountered herein, the line of attack differs: here we start with accounting questions and encounter the information ideas along the way.

To illustrate the above four properties of accounting as an information science the book is, roughly speaking, organized around three example inquiries, along with variations and applications.

1. Double entry accounting as a communication channel.

2. Information - and accounting - stocks and flows.
3. Production, accounting, and information induced synergy.

1.1 double entry accounting as a communication channel

The first step in the construction of financial statements is the collection of a set of numbers reflecting the transactions and prospects of the entity under consideration. The numbers are collected in journal entries. We will denote the set of journal numbers as the vector y. (A vector is an ordered set of numbers; linear algebra and other technical terms will be defined more carefully in the body of the book.) The process of transforming the journal entry numbers into financial statements is a linear process, and can be represented by a transformation matrix. The double entry system gives the matrix a particularly simple and elegant form; most notably, it is what is called an incidence matrix as each column of the matrix contains a "plus one," a "minus one," and the remainder are zeros. We denote the double entry matrix as A.

The journal entry numbers y are transformed linearly by the double entry matrix A to yield a new set of numbers, the financial statement numbers denoted as a vector x. The accounting process can then be represented by the matrix equation

$$Ay = x$$

The first question of interest is: How much of the information in the journal entry set of numbers y finds its way to the financial statement vector x? In other words, how much gets through the double entry accounting channel?

The question can be answered quite simply using the ideas of linear algebra. In the language of linear algebra the matrix A has a well defined "row space" consisting of all the vectors which can be constructed by combining the rows of A. The answer is "only the component of y in the row space gets through the channel."

The idea that "the row component gets through" is a convenient solution to the accounting channel question. But it also has implications for other problems and contexts. In fact, it is a central idea in three other topics covered in the book.

1. Accounting valuation of derivative securities (chapters 4 and 5). Arbitrage free pricing offers a consistent valuation method for derivatives, and the key is to specify the value in the row space of the primitive securities upon which the derivative is "derived."
2. Error detecting and error correcting codes (chapter 9). Linear codes specify a row space. Whenever the received message is not in the

row space, an error has occurred, and, hence, is detected. With some ingenuity the error can not only be detected but also corrected by changing the message so that the corrected message does reside in the row space.

3. Quantum encryption (secret codes in chapters 10 and 11). Quantum units are the irreducible units in nature, so now it is not possible to think about the fraction in the row space. They can't be divided up. A quantum unit is either entirely in the row space, or it is not. So now we think of the *probability* the quantum unit is in the row space. And it is precisely this uncertainty about where the quantum unit is that makes quantum encryption so powerful.

1.2 information - and accounting - stocks and flows

T-accounts are a simple and elegant way to keep track of accounting stocks and flows. At the beginning of the time period under consideration, a T-account contains a beginning balance (stock). As time goes by, the balance is adjusted for various transactions (flows), resulting in an updated ending balance (revised stock). Information is embodied in another stock and flow structure, and it is the notion of probability that holds the structure together. A decision maker begins the time period with probabilities over states of the world - what might happen. These are called prior probabilities and are stocks. During the time period evidence is accumulated and probabilities are revised The evidence is a flow, and the revised probabilities are the updated stock. A consistent way to revise probabilities is the use of Bayes' theorem, and the updating is called Bayesian revision.

It is tempting, and suggestive, to contemplate the possibility of representing the stocks and flows of information updating using the stocks and flow mechanics of the accounting T-account. As it turns out, for illuminating special cases it is possible to do the entire Bayesian updating in a T-account. All that is required isto specify an appropriate depreciation rate. Then just do the cash and depreciation entries the regular accounting T-account way.., and the updated probabilities pop out naturally.

The particular problem set-up is actually quite complicate (at first glance, anyway). The T-account procedure accomplishes Bayesian revision with a minimun of fuss. That's the sort of contribution an information science is capable of. Accounting and information stocks and flows are the topics of chapters 6 and 7.

While the T-account accomplishes probability updating, there is another issue to deal with: where do the prior probabilities come from? The question highlights a duality in information analysis that is comparable to the duality in accounting. For accounting problems, either income or asset value can be computed intitially. But once one is determined, so is the other. In other words, income determination and asset valuation are dual processes.

In information analysis the dual processes are Bayesian revision and probability assignment. The T-account process takes prior probabilities as given, and then updates in a Bayesian fashion to obtain the revised probabilities. The dual process would be to assign the (revised) probabilities directly.

One way to assign probabilities directly to uncertain states of the world is the method of maximum entropy. "Entropy" is a measure of uncertainty proposed by Claude Shannon. It has turned out to be extremely useful and productive for a variety of information problems. In fact, if accounting is to be treated as an information science it seems almost necessary that the concept of entropy should show up. It is useful for us in probability assignment exercises, since it consistent with Bayes theorem - property demonstrated by Shannon on one of his (many) theorems. Entropy is a topic covered in chapter 8.

Also covered in chapter 8 are eigenvalues. The fundamental descriptors of dynamic systems, as embodied in stocks and flows, are eigenvalues and eigenvectors. Their appearance in the stock and flow example is natural. And they prove to be useful in a number of accounting related problems.

A particular, and important, topic which utilitzes both probability assignment techniques and eigenvalues is the recover theorem of Steve Ross. Surprisingly it turns out to be possible to compute implied market state probabilities directly from the observation of arbitrage free security prices. This seems especially surprising, since arbitrage pricing itself is free of state probabilities.

The content of chapter 13 describes Ross' method for state probability assignment. The technique is a particularly elegant application of eigenvalues.

1.3 production, information, and synergy

The final example has to do with the subtle,and powerful, effects of information in the production process. The example differs a bit from the first two. Instead of using accounting problems and structure to illuminate related accounting and information problems, the direction is different. In this example the intent is to use tools from another information science to illuminate and accounting related issue.

The other information science is quantum physics. We will encounter the topic in chapter 11 as a solution to an encryption problem. The tools fundamental to quantum process are linear algebra, vector spaces, and eigenvalues, all of which we will acquire doing the accounting.

In chapter 12 some basic quantum results are covered. How quantum units interact and transfer information is very mysterious. Einstein, for example, thought things too "spooky" to be entirely true. However, the exchange of information in a production environment seems not quite so mysterious. Quantum physics supplies a formalism to bring to bear on our thinking about information and production. A question of interest is how production of multiple products can be more efficient than one at a time. Is there synergy? If so, is the synergy induced by information effects? The quantum formalism allows addressing the questions. And allows thinking about how production accounting should look to be helpful in an information rich environment. Or, indeed, how the accounting can be corrosive.

1.4 a little linear algebra

A major idea of the manuscript is that serious consideration of accounting issues is a good way to learn lots of stuff. All the mathematical theorems will be illustrated by, and often derived from, an accounting problem. In that sense, there is not much in the way of math prerequisites, except, of course, a tolerance for abstraction, and a certain eagerness to learn. Nonetheless, it would not hurt to introduce some linear algebra concepts, as that comprises the first few results.

1.4.1 vectors

A vector is an ordered set of things; for us the things will (almost) always be numbers. And we will often use a letter to denote the whole set. Such as

$$a = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$b = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$$

$$x = \begin{bmatrix} 7 \\ 6 \\ 1 \\ 3 \\ 9 \end{bmatrix}$$

Vectors are typically thought of as columns where the numbers are stacked one on top of the other, as illustrated above. They can be tipped over using an operation called transpose, and denoted with a superscript T.

$$a^T = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix}$$

Two vectors with the same number of elements can be multiplied times each other; that is called vector multiplication. The conventional way to accomplish this is to multiply a row vector times a column vector, so we have to be a little careful which one gets transposed. The answer to the multiplication is one number obtained by multiplying the two vectors term for term, and then adding up the products.

$$a^T b = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} = 1\,(5) + 2\,(1) + 4\,(3) = 19$$

Much of the time it won't matter much to us which is the row and which the column, but we will try to be careful so we can communicate with people who have gotten used to the conventions. The first time we encounter vector multiplication will be the accounting task of posting journal entries to T-accounts.

1.4.2 matrices

A matrix is a set of vectors, and all the matrices we will deal with are rectangular, that is, every row (column) has the same number of elements. Typically matrices are denoted with a capital letter, to contrast with lower case vectors.

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 2 \\ 7 & 1 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

A matrix can be multiplied times a vector using the regular vector multiplication. Since a matrix can have more than one row, there will be more than one answer, and it will be a vector.

$$Ca = \begin{bmatrix} 1 & 1 & 2 \\ 7 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1\,(1) + 1\,(2) + 2\,(4) \\ 7\,(1) + 1\,(2) + 3\,(4) \end{bmatrix} = \begin{bmatrix} 11 \\ 21 \end{bmatrix}$$

Matrices can be multiplied together using the same vector multiplication operation. Each row in the first matrix is vector multiplied by each column in the second matrix, and the vector product is placed in the appropriate place in a result matrix.

$$BD = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1\,(1) + 1\,(0) & 1\,(0) + 1\,(4) \\ 2\,(3) + 1\,(0) & 2\,(0) + 1\,(4) \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 6 & 4 \end{bmatrix}$$

$$\begin{aligned} BC &= \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 7 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1\,(1) + 1\,(7) & 1\,(1) + 1\,(1) & 1\,(2) + 1\,(3) \\ 2\,(1) + 1\,(7) & 2\,(1) + 1\,(1) & 2\,(2) + 1\,(3) \end{bmatrix} \\ &= \begin{bmatrix} 8 & 2 & 5 \\ 9 & 3 & 7 \end{bmatrix} \end{aligned}$$

To be able to multiply matrices together the number of elements in the rows and columns must match up, so we will have to be careful about that.

There's one more linear algebra concept that will come up after a while. For numbers, the number 1 serves as a multiplicative identity. That is, any number multiplied times 1 leaves the number unchanged. Also, any number, say x, has an inverse $1/x$, such that x times its inverse, $1/x$, returns the multiplicative identity 1. correspondingly, square matrices have a multiplicative identity; for the 2 by 2 case the identity matrix is

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

And a square matrix, A, might have an inverse, denoted with a -1 superscript, such that multiplying A times its inverse returns the identity.

$$AA^{-1} = I$$

The concept of an inverse will be quite useful, but we won't actually compute inverses, except for simple cases.

That is plenty to get us started in terms of linear algebra. As far as other prerequisites go, we won't require much, except, of course, a curiousity about things scholarly. Surprisingly, not much in the way of accounting experience is required. The manuscript will contain most (I hope nearly all) of the accounting concepts required to proceed. Since the treatment is atypical, accounting experience ranging from novice to quite experienced will be appropriate for the development herein.

As we will be dealing with a number of mathematical theorems, a word about proofs is in order. The manuscript will not contain rigorous proofs of most of the results. However, the accounting illustrations will often supply demonstrations of the logic in a proof. The accounting special cases will sometimes be surprisingly general. In cases where demonstrations of the logic of the proof is not supplied, accounting illustrations of its use should get across how the theorem works. In any case, rigourous proofs for all the results can be easily found elsewhere.

1.5 summary

The main idea is that we will treat the study of accounting as an academic discipline. It is hoped our understanding and appreciation of what is meant by "academic discipline" will be enhanced by the journey. Along the way we will encounter old ideas and new: products of the history of the University. A part of scholarship - a very important part - is the joy of discovery, so, at some level, this should be a fun trip.

1.6 exercises

Exercise 1.1 *Multiply some vectors.*

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}^T \quad \textit{times} \quad \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}^T \quad \textit{times} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 0 \\ 7 \\ 8 \\ 7 \\ 9 \end{bmatrix}^T \quad \textit{times} \quad \begin{bmatrix} 2 \\ 7 \\ 1 \\ 8 \\ 2 \\ 8 \end{bmatrix}$$

Exercise 1.2 *Multiply a matrix times a vector.*

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Exercise 1.3 *Multiply a matrix times a matrix.*

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$