

Testing the CAPM

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Testing the CAPM: Background

CAPM is a model

- It is useful because it tells us what expected returns should be.
- We want test whether it is a good model.
- Remember, whenever we test a model we are jointly testing market efficiency.

Testable Implication of the CAPM

The market portfolio is the tangency portfolio:

$$E(r_i) = r_f + \beta_{iM}[E(r_M) - r_f], \text{ where } \beta_{iM} = \frac{\text{cov}(r_i, r_M)}{\sigma^2(r_M)}$$

Testing the CAPM: The Approach

Average Return vs CAPM Prediction

- The most common approach is to compare historical average returns to the CAPM's prediction.
- We compute the CAPM's estimated prediction by estimating beta (β), the market premium ($E(r_M) - r_f$), and the risk free rate (r_f).
- We want the estimated prediction error (called $\hat{\alpha}$):

$$\begin{aligned}\hat{\alpha}_i &= \bar{r}_i - \text{CAPM Prediction} \\ &= \bar{r}_i - \bar{r}_f - \hat{\beta}_{im}(\bar{r}_M - \bar{r}_f)\end{aligned}$$

The CAPM and $\hat{\alpha}$

- $\hat{\alpha}$ will not always be zero even if the CAPM is true. Why?
- What can we say about prediction error if the CAPM holds?

A Good Strategy?

Stock tip: Invest in mid-cap stocks.

- It is a good strategy because everyone ignores mid-cap stocks.
- Investors want blue chips, or they want to invest in small start-up companies with growth opportunities.
- Therefore, mid-cap stocks tend to be undervalued and have high returns on average.

Testing

- Is this true? What do you think of this strategy?
- How can we test this empirically?
- How is this related to testing the CAPM?

Monthly Data: 2004-2005

	A	B	C	D	E	F	G
1	date	r_mid	r_M	r_f			
2	Jan-2004	2.79	2.31	0.07			
3	Feb-2004	2.44	1.55	0.06			
4	Mar-2004	-0.06	-1.07	0.09			
5	Apr-2004	-3.44	-2.42	0.08			
6	May-2004	2.1	1.41	0.06			
7	Jun-2004	2.54	2.16	0.08			
8	Jul-2004	-5.28	-3.77	0.1			
9	Aug-2004	-0.61	0.27	0.11			
10	Sep-2004	3.56	2.06	0.11			
11	Oct-2004	2.02	1.78	0.11			
12	Nov-2004	6.7	4.82	0.15			
13	Dec-2004	4.45	3.52	0.16			
14	Jan-2005	-2.19	-2.66	0.16			
15	Feb-2005	2.69	2.27	0.16			
16	Mar-2005	-1.22	-1.69	0.21			
17	Apr-2005	-4.26	-2.52	0.21			
18	May-2005	5.67	3.79	0.24			
19	Jun-2005	2.56	1.15	0.23			
20	Jul-2005	5.64	4.33	0.24			
21	Aug-2005	-1.06	-0.59	0.3			
22	Sep-2005	0.88	1.06	0.29			
23	Oct-2005	-2.74	-2.08	0.27			
24	Nov-2005	4.97	4.04	0.31			
25	Dec-2005	0.86	0.35	0.32			
26							

Monthly Data Description

Variables

- r_{mid} is the return on a portfolio composed of medium sized firms (mid-cap stocks).
- r_M is the return on a proxy for the market portfolio (a value-weight index of all stocks on NYSE, AMEX, and Nasdaq).
- r_f is proxy for the riskfree rate (one-month T-bill rate).

Other data notes

All returns are reported in percent per month.

Excess Returns

Computing excess returns

An excess return usually (but not always) refers to the return on a portfolio in excess of the riskfree rate:

$$r_p - r_f$$

Need excess returns for tests of the CAPM

- Need excess returns (or zero cost portfolios) to test the CAPM.
- Subtract off the riskfree rate (t-bill rate) from both the mid-cap portfolio and the market portfolio.

Monthly Data with Excess Returns: 2004-2005

	A	B	C	D	E	F	G
1	date	r_mid	r_M	r_f	r_mid-r_f	r_M-r_f	
2	Jan-2004	2.79	2.31	0.07	2.72	2.24	
3	Feb-2004	2.44	1.55	0.06	2.38	1.49	
4	Mar-2004	-0.06	-1.07	0.09	-0.15	-1.16	
5	Apr-2004	-3.44	-2.42	0.08	-3.52	-2.5	
6	May-2004	2.1	1.41	0.06	2.04	1.35	
7	Jun-2004	2.54	2.16	0.08	2.46	2.08	
8	Jul-2004	-5.28	-3.77	0.1	-5.38	-3.87	
9	Aug-2004	-0.61	0.27	0.11	-0.72	0.16	
10	Sep-2004	3.56	2.06	0.11	3.45	1.95	
11	Oct-2004	2.02	1.78	0.11	1.91	1.67	
12	Nov-2004	6.7	4.82	0.15	6.55	4.67	
13	Dec-2004	4.45	3.52	0.16	4.29	3.36	
14	Jan-2005	-2.19	-2.66	0.16	-2.35	-2.82	
15	Feb-2005	2.69	2.27	0.16	2.53	2.11	
16	Mar-2005	-1.22	-1.69	0.21	-1.43	-1.9	
17	Apr-2005	-4.26	-2.52	0.21	-4.47	-2.73	
18	May-2005	5.67	3.79	0.24	5.43	3.55	
19	Jun-2005	2.56	1.15	0.23	2.33	0.92	
20	Jul-2005	5.64	4.33	0.24	5.4	4.09	
21	Aug-2005	-1.06	-0.59	0.3	-1.36	-0.89	
22	Sep-2005	0.88	1.06	0.29	0.59	0.77	
23	Oct-2005	-2.74	-2.08	0.27	-3.01	-2.35	
24	Nov-2005	4.97	4.04	0.31	4.66	3.73	
25	Dec-2005	0.86	0.35	0.32	0.54	0.03	

Estimation Issues

How much data should we use?

- Are there ever circumstances where we should only use around 24 months of data?
- When can we use lots of past data?
- Does it ever make sense to use daily or annual data instead of monthly data?

Generally we prefer to work with portfolios

- Why are portfolios usually superior to individual securities for tests?
- In general we like testing with portfolios formed on characteristics like market-cap, P/E, leverage, etc. Why?
- In general using industry or sector based portfolio can be problematic. Why?

Estimation Issues

Other Problems

- Is it a problem that r_M is a value-weight index of all stocks on NYSE, AMEX, and Nasdaq?
- What is the proxy missing?

Exploring the Data

Always start with summary statistics

- Always check your summary statistics.
- Its a way to catch mistakes and get a “feel” for your data.
- In Excel use the average and stdev commands.

Summary Statistics

	r_{mid}	r_M	r_f	$r_{mid} - r_f$	$r_M - r_f$
Mean	1.209	0.836	0.172	1.037	0.665
Std. Dev.	3.284	2.468	0.087	3.280	2.463

What do the summary statistics tell us?

What can we learn from the summary statistics about the mid-cap strategy or about the CAPM?

Estimating Sharpe Ratios

Estimating Sharpe ratios: Method #1

$$\hat{S}R_i = \frac{\bar{r}_i - \bar{r}_f}{\hat{\sigma}(r_i - r_f)}$$

	Mid-Cap	Market
$\hat{S}R$	0.316	0.270

Estimating Sharpe ratios: Method #2

$$\hat{S}R_i = \frac{\bar{r}_i - \bar{r}_f}{\hat{\sigma}(r_i)}$$

	Mid-Cap	Market
$\hat{S}R$	0.316	0.269

Estimating Sharpe Ratios

Which method is better?

- They are both fine.
- I will always use method #1: I prefer working with excess returns in both the numerator and denominator.
- The book uses method #2.
- Often they answers are the same out to 2-3 digits of precision.

Graphing Sharpe ratios

Compute the estimated CAL line:

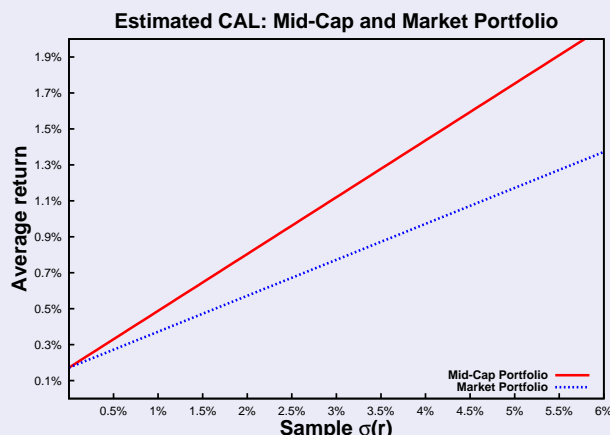
$$\bar{r}_p = \bar{r}_f + \hat{S}R_p \hat{\sigma}(r_p)$$

Back to the Sharpe Ratios

The estimated Sharpe ratios

	Mid-Cap	Market
$\hat{S}R$	0.316	0.270

Graphically



Inferences and Estimated Sharpe Ratios

What do the Sharpe ratios tell us?

- Can we infer that investing in mid-cap stocks is a good strategy based on the Sharpe ratios?
- Can we reject the CAPM based on the Sharpe ratios?
- Suppose, you don't believe the CAPM is a good model, should you still estimate and examine Sharpe ratios?

Weaknesses

Are there any weaknesses to the Sharpe ratio approach?

Regression

Regression

- A regression, least squares, or ordinary least squares fits a line through points.
- For example, we could regress the excess returns of an asset i on the excess returns of the market portfolio (M):

$$r_{it} - r_{ft} = \alpha_i + \beta_{iM}(r_{Mt} - r_{ft}) + \epsilon_{it}$$

- In the regression α refers to the intercept and β refers to the slope of the line
- We can't perfectly fit the data with a line. We are working with random variables.
- ϵ_{it} refers to the error term or how much an observation deviates from the line.

Regression

What a regression does

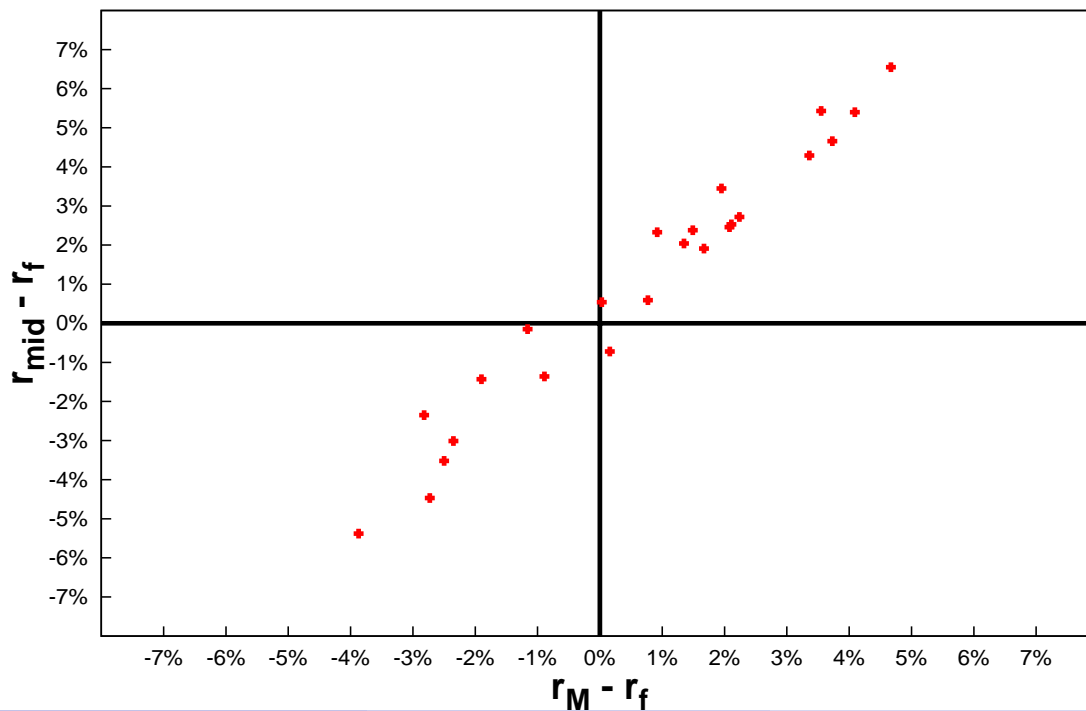
- Regression minimizes the squared error:

$$\min_{\alpha, \beta} \sum \epsilon_i^2$$

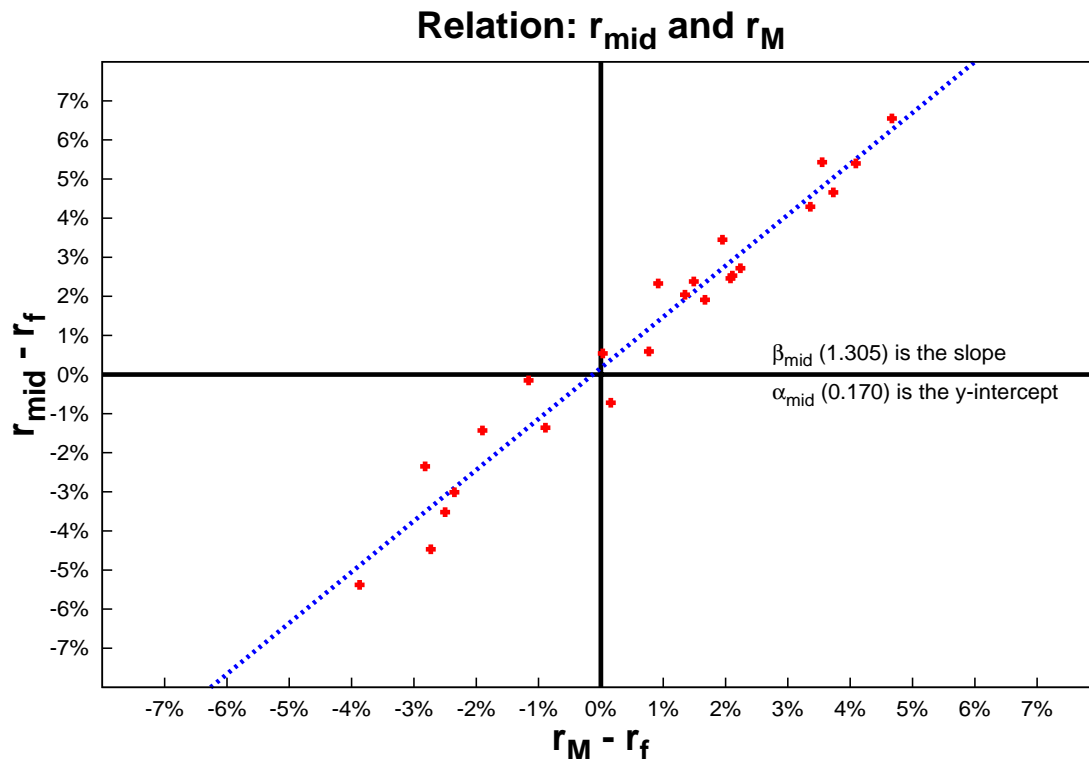
- A regression picks α and β so that the squared error are as small as possible.
- This ensures that we get the best possible estimates of α and β .

Regression: Mid-Cap Stocks

Relation: r_{mid} and r_M



Regression: Mid-Cap Stock



Regression

β and α

- Suppose we run the following regression:

$$r_{it} - r_{ft} = \alpha_i + \beta_{iM}(r_{Mt} - r_{ft}) + \epsilon_{it}$$

- The regression estimate of β is the following:

$$\hat{\beta}_{iM} = \frac{\text{cov}(r_i - r_f, r_M - r_f)}{\hat{\sigma}^2(r_M - r_f)}$$

- Thus we get the an appropriate estimate of beta from the regression.
- The regression estimate of α is the following for the regression:

$$\begin{aligned}\hat{\alpha}_i &= \bar{r}_i - \bar{r}_f + \hat{\beta}_{iM}(\bar{r}_M - \bar{r}_f) \\ &= \bar{r}_i - \text{CAPM Prediction}\end{aligned}$$

- Thus the regression also estimates the prediction error (often called model error)

What is the Regression Testing?

The regression

$$r_{it} - r_{ft} = \alpha_i + \beta_{iM}(r_{Mt} - r_{ft}) + \epsilon_{it}$$

Different ways of saying what the regression tests

- It tests whether the thing on the right hand side is the tangency portfolio.
- It tests whether the thing on the right-hand side has the highest possible Sharpe ratio.

Specifically

The intercept in this regression shows whether the right-hand side portfolio has the highest possible Sharpe ratio out of all possible portfolios consisting of the right-hand side variable, the left-hand side variable, and the risk free asset.

Alpha and Beta

Synonyms for α and β

α	β
Intercept	Coefficient
Constant	Slope
Alpha	Beta
a	b

Alpha

- The beta is interesting because it tells us about risk.
- Alpha is interesting because it tests if the market portfolio is the tangency portfolio.
- If the market portfolio is the tangency portfolio, then the estimated alpha should be zero (well, statistically indistinguishable from zero).

Regression Estimation

Back to mid-cap stocks

We want to run the following regression:

$$r_{midt} - r_{ft} = \alpha_{mid} + \beta_{midM}(r_{Mt} - r_{ft}) + \epsilon_{it}$$

Critical data issue

- Your left hand and your right hand side variables must be excess returns (or zero cost portfolios).
- in this case: $r_{mid} - r_f$ and $r_M - r_f$.
- If you don't use excess returns (or a zero cost portfolio return) then your estimated α will be **wrong**.

Excel: The Regression Menu

Excel: Data Analysis and Regression

- You can run a regression in Excel by going to the "Tools" menu and the "Data Analysis" sub-menu. Once you are in the "Data Analysis" sub-menu pick "Regression".
- If the "Data Analysis" Sub-menu is not available go to "Add-Ins" in the "Tools" menu and add it.
- If you can't add the "Data Analysis" Sub-menu use a lab computer for your homework.

Excel needs the y-variable and x-variable

- In this case: y-variable = $r_{mid} - r_f$ data.
- In this case: x-variable = $r_M - r_f$ data.

Regression Results:

$$r_{midt} - r_{ft} = \alpha_{mid} + \beta_{midM}(r_{Mt} - r_{ft}) + \epsilon_{it}$$

Results From Excel

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	Regression Statistics						
4	R Square	0.960					
5	Observations	24					
6							
7		Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
8	Intercept	0.170	0.142	1.198	0.244	-0.124	0.464
9	r_M-r_f	1.305	0.057	23.032	0.000	1.187	1.422

What can we infer?

- Can we reject the CAPM?
- Is investing in mid-cap stocks a good strategy?
- What parts of the regression should we be looking at?
- How do we assess statistical significance from the output?

The Security Market Line

Estimating the SML

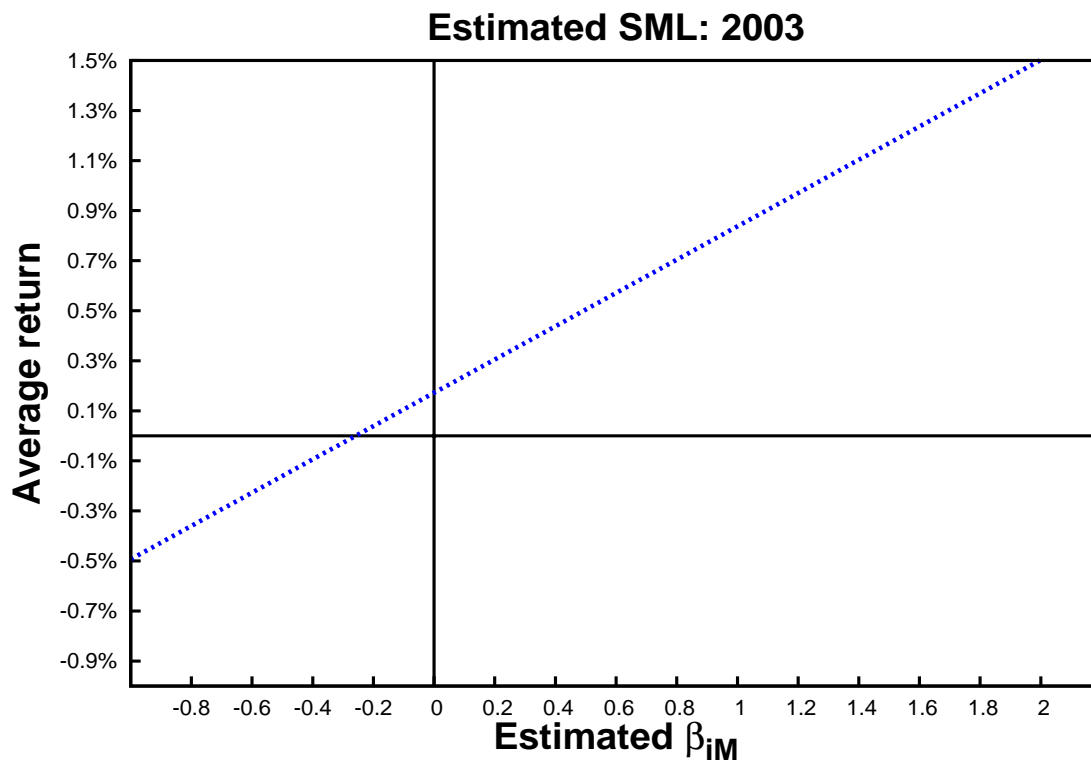
- The SML is just the CAPM equation:

$$E(r_i) = r_f + \beta_{iM}(E(r_M) - r_f)$$

- Specifically the SML is a graph of expected return as a function of beta.
- To estimate we just need estimates of the riskfree rate and the expected return on the market:

$$\begin{aligned}\bar{r}_i &= \bar{r}_f + \hat{\beta}_{im}(\bar{r}_M - \bar{r}_f) \\ &= 0.172 + \hat{\beta}_{im}0.665\end{aligned}$$

The Estimated SML



The SML

Regression and the SML

What's the difference between the estimated SML and the regression line?

The SML and Alpha

- How is the estimated alpha and beta from our regression related to the estimated SML?
- Where would our results from the mid-cap portfolio show up on the graph?

Summary

- The testable implication of the CAPM is that the market portfolio is priced to be the tangency portfolio.
- We can test the CAPM with regression analysis

$$r_{it} - r_{ft} = \alpha_i + \beta_{iM}(r_{Mt} - r_{ft}) + \epsilon_{it}$$

- ▶ If the CAPM holds, alpha will not be statistically different from zero.
- ▶ If the market portfolio is the tangency portfolio, alpha will not be statistically different from zero.
- ▶ If the p-value on the alpha is less than 0.05 then the alpha is statistically different from zero.