Subjective Probability Judgment: Partition Dependence

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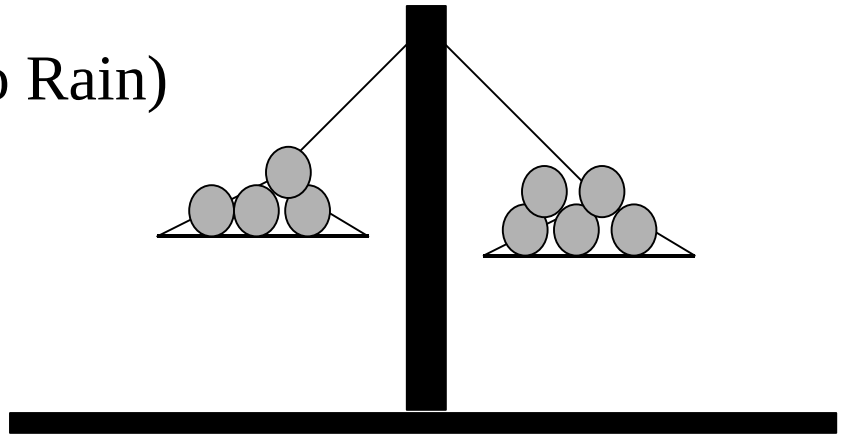
Judgment Under Uncertainty

How do I map my degree of belief to a number onto a number between 0 and 1?

Support theory (TK'94; RT'97): judged probability is interpreted as the balance of evidence for a hypothesis compared to its complement

$$P(\text{Rain tomorrow}) = \frac{s(\text{Rain})}{s(\text{Rain}) + s(\text{No Rain})}$$

$$\text{Odds of rain} = \frac{s(\text{Rain})}{s(\text{No rain})}$$



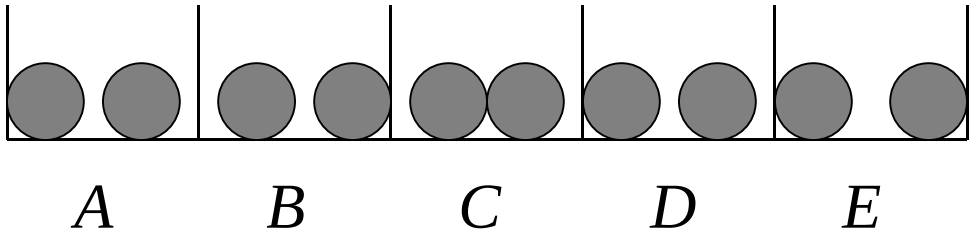
Another Strategy: The Principle of Insufficient Reason

(Leibniz, 1678; Laplace, 1776)

"*If we see no reason why one case should happen more than the other then probability should be defined in terms of a ratio among cases*"

E.g., horse race with five unfamiliar horses

$$P(A) = P(B) = P(C) = P(D) = P(E) = 1/5$$



Note: this scheme yields probabilities that are additive and perfectly calibrated… *assuming that the partition is held constant*

The Problem of Partition Dependence

If people are influenced by the principle of insufficient reason, judged probabilities will depend on the subjective partition that they invoke.

Problem 1: The partition is often not appropriate

e.g. Fischhoff & Bruine de Brun (1999) "blip at 50%"

P(*die of cancer by age 40*) = .50

Problem 2: The partition may be *ad hoc*

P($ in *DA consulting last year* ≤ *$50 million*) = ?
P($ in *DA consulting last year* > *$50 million*) = ?

P($ in *DA consulting last year* ≤ *$200 million*) = ?
P($ in *DA consulting last year* > *$200 million*) = ?

Anchoring on the Ignorance Prior

What is the probability of A?

(1)Partition sample space into elementary hypotheses.
(default = target hypothesis, complement)

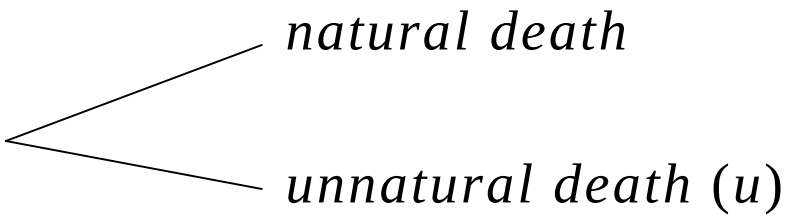
(2)Anchor on uniform distribution across elements.

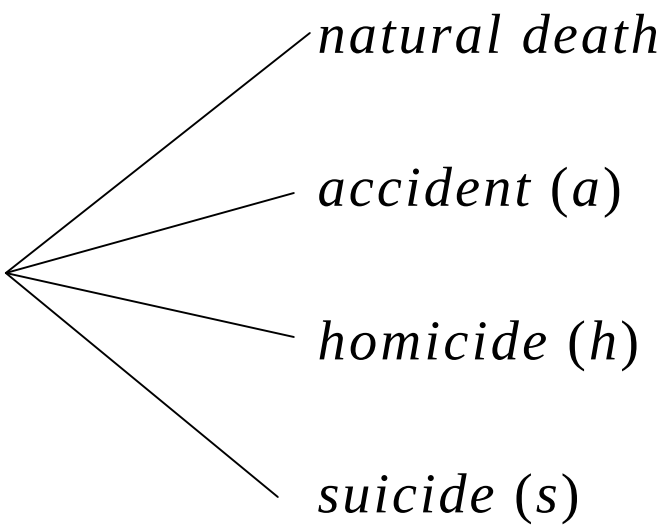
(3)Adjust according to distribution of support.

$$R(A,\overline{A}) = \left[\frac{n_A}{n_{\overline{A}}}\right]^{l}\left[\frac{s(A)}{s(\overline{A})}\right]^{1-l}$$

n_J = *number of elements in event J.* l *is weighting parameter.*

Fault Tree Effect or Pruning Bias





Fischhoff, Slovic, and Lichtenstein, 1978

"Pruning bias": $P(u) < P(a) + P(h) + P(s)$

Explanations for pruning bias

Availability

- Fischhoff, Slovic & Lichtenstein (1978)
- Russo & Kolzow (1994)
- Ofir (2000)

Act of rating

- Van Schie, Ells & Van de Pligt (1994)

Study 1: Judgment Under Ignorance

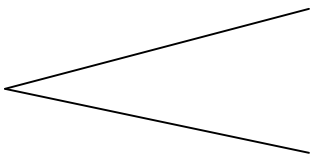
246 Duke MBA students judged future close of JSX

A) less than 500
B) at least 500 but less than 1000
C) at least 1000

a) less than 500
b) at least 500 but less than 1000
c) at least 1000 but less than 2000
d) at least 2000 but less than 4000
e) at least 4000 but less than 8000
f) more than 8000

	median *P*	% take $30
P(*A* or *B*) =	.67	55
P(*a* or *b*) =	.30	31
P(C) =	.25	28
P(*c* or *d* or *e* or *f*) =	.60	58

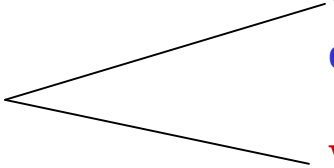
Study 2a: Categorical Partitions (Fault Tree)



A school other than Wharton

Wharton

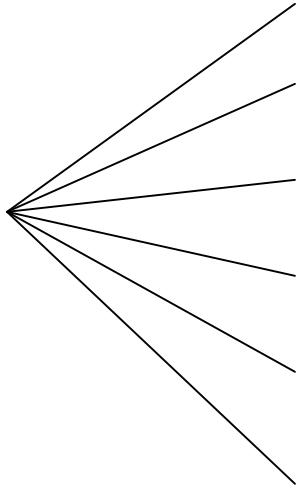
.40 "Packed"



Chicago, Harvard, Kellogg, Stanford, **or** another school other than Wharton

Wharton

.40 "Unpacked"



Chicago

Harvard

Kellogg

Stanford

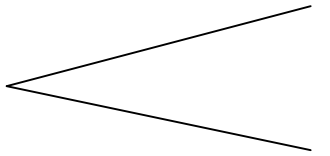
Wharton

None of the above

.70 "Separate Evaluation"

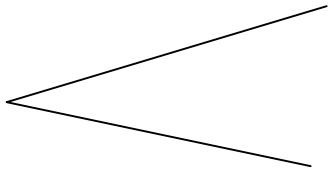
Availability cannot be the whole story.

Study 2b: Dimensional Partitions



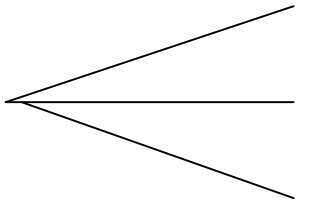
Less than 90 degrees Fahrenheit .30

In the 90's



Less than 80 degrees Fahrenheit **or** in the 80's .20

In the 90's



Less than 80 degrees Fahrenheit .40

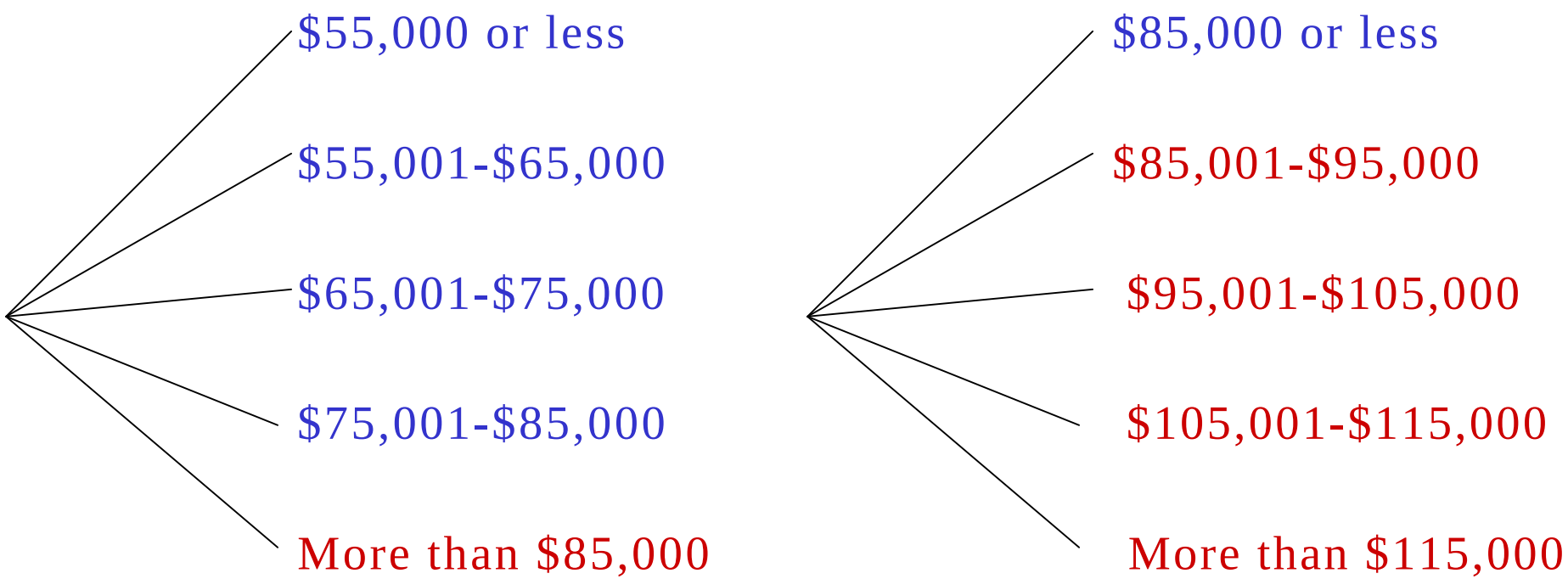
In the 80's

In the 90's

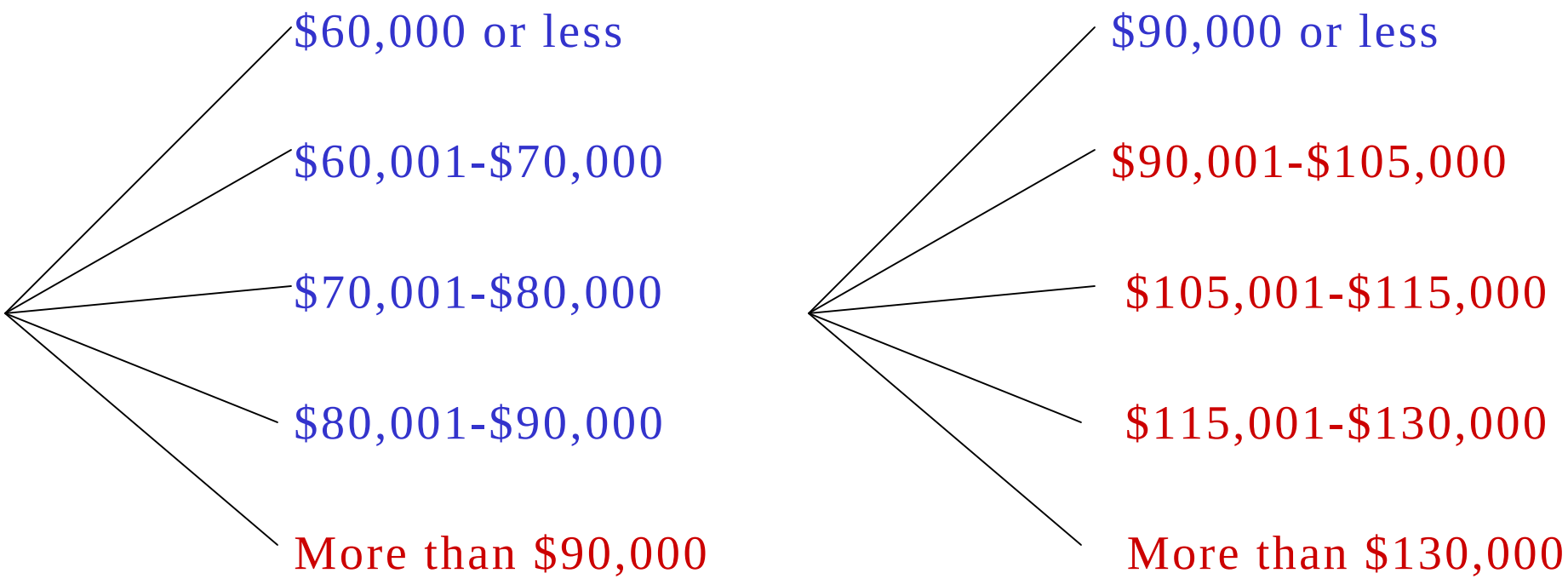
Similar effect for continuous variables

Study 3: Manipulating Knowledge

Duke MBA starting salary



Harvard Law starting salary



Study 3 Results

Duke MBA students in decision analysis elective (n=110)

Knowledge of Duke MBA salaries: *7/10*

Knowledge of Harvard Law salaries: *1/10*

	Duke MBA		Harvard Law	
	< $85K	≥ $85K	< $90 K	≥ $90K
Form 1	**75**	25	**75**	25
Form 2	45	**55**	30	**70**

= packed

= separate evaluation

Less knowledge
➔ Stronger effect

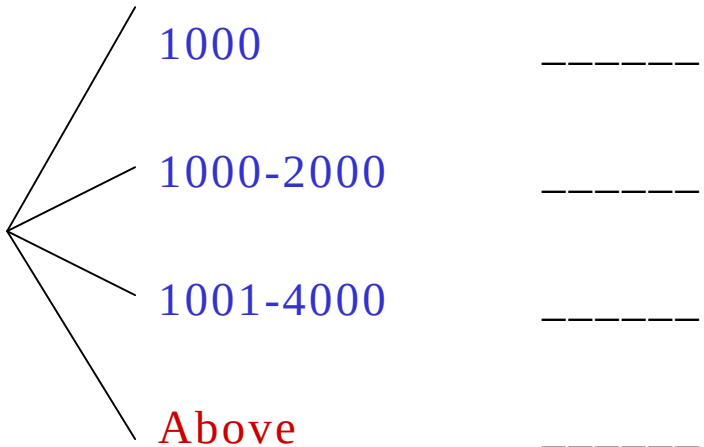
Study 4: Information Control

What is the last digit of your local telephone number?

If this number is *even*, please write "JSX" in the space provided above the tree on the *left* and "NASDAQ" in the space provided above the tree on the *right*.

If this number is *odd*, please write "NASDAQ" in the space provided above the tree on the *left* and "JSX" in the space provided above the tree on the *right*.

Index:_____



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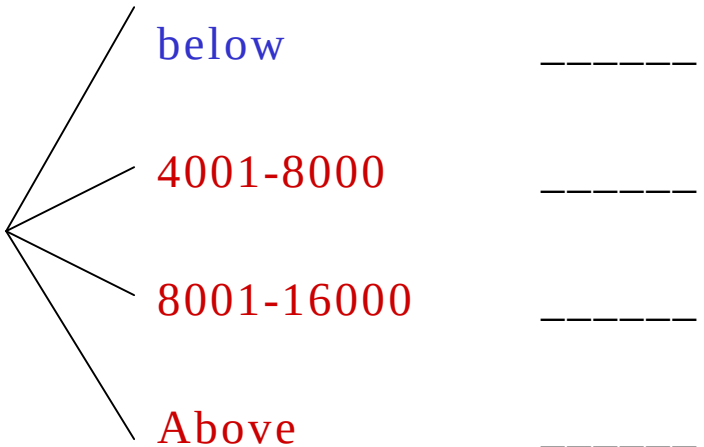
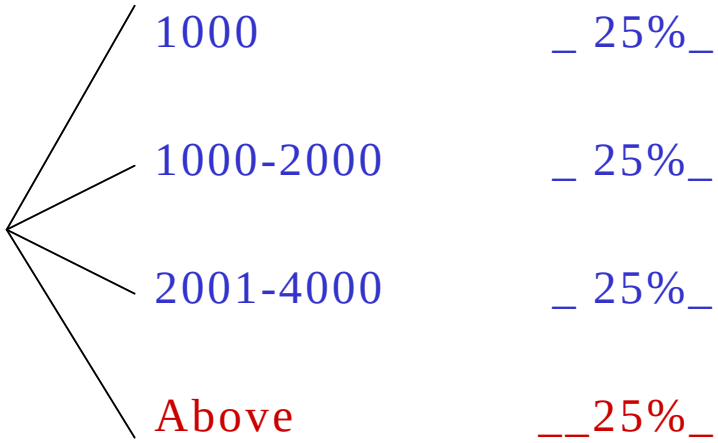
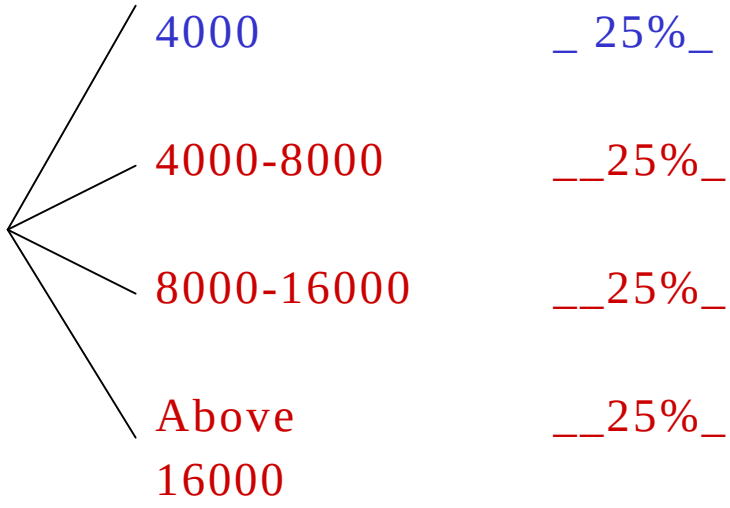


Study 4: Results

INDEX: *JSX*

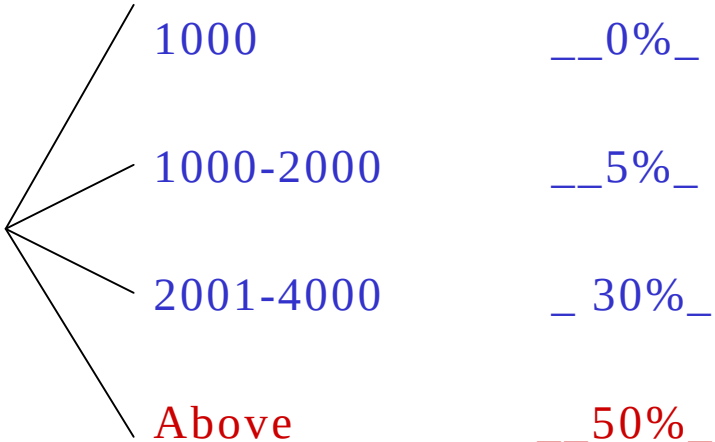


INDEX : *JSX*

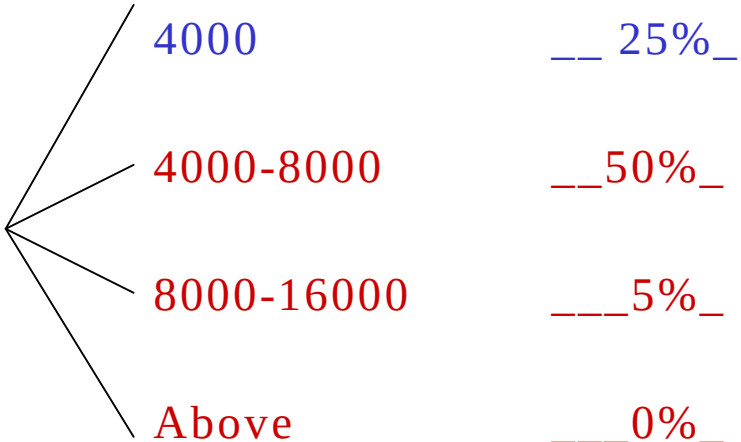


Study 4: Results

INDEX: *NASDAQ*



INDEX: *NASDAQ*



Study 4 Results Summary

Weekend Executive MBA students (n≅100)

	NASDAQ		***JSX***	
	≤4000	*>4000*	*≤ 4000*	*>4000*
Order 1	**25**	**75**	**75**	**25**
Order 2	**50**	**50**	**25**	**75**

= packed

= separate evaluation

Can't attribute effect to information content of partitions (Grice 1975)

Study 5: Probability Assessment Experts

Survey: Members on the Decision Analysis Society email list (n = 57)

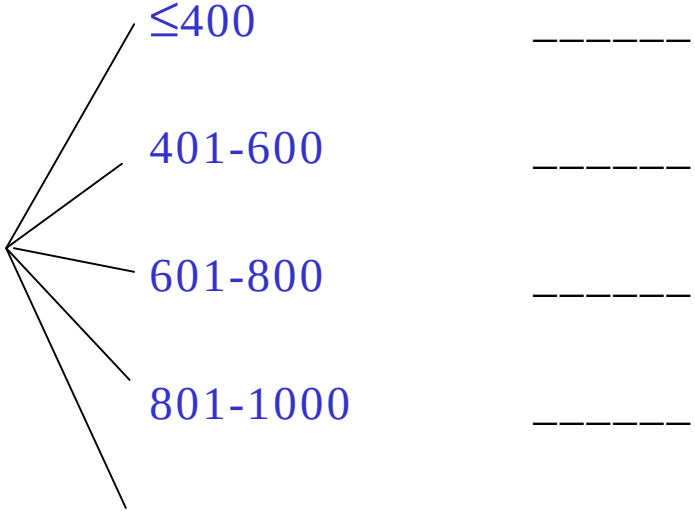
- Approached by e-mail
- 86% have Ph.D.
- 75% have taught course in Decision Analysis
- 63% have elicited probabilities for applied DA project in last 2 years
- Total of 156 projects over last two years

Study 5: Method

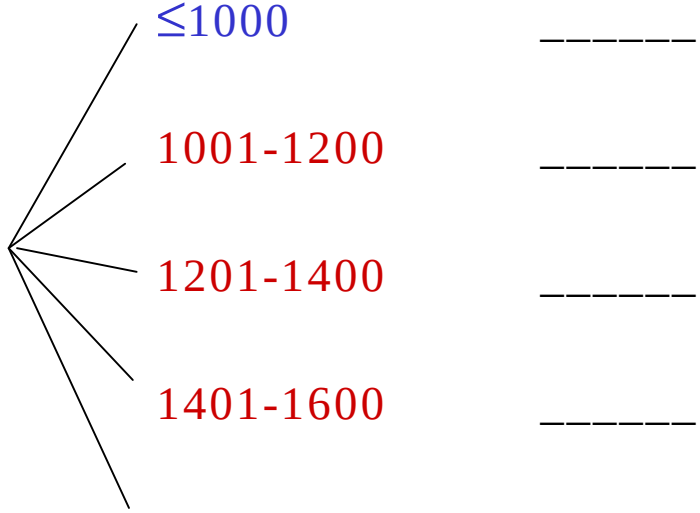
What is the last digit of your local telephone number?
If this number is *even*, please write "DAS" in the space provided above the tree on the *left* and "SQA" in the space provided above the tree on the *right*.

If this number is *odd*, please write "SQA" in the space provided above the tree on the *left* and "DAS" in the space provided above the tree on the *right*.

Society:_____



Society: _____



Study 5 Results

DAS email list (n=57)

	DAS		*SQA*	
	≤1000	*>1000*	*≤ 1000*	*>1000*
Order 1	**65**	**35**	**80**	**20**
Order 2	**90**	**10**	**55**	**45**

= packed

*= separate evaluation*

Even we are susceptible!

Study 5: Super Experts

11 Elder Statespersons

- Ph.D. in 1985 or earlier AND
- At least one applied project in last 2 years AND
- Have taught course in Decision Analysis

	DAS		*SQA*	
	≤1000	*>1000*	*≤ 1000*	*>1000*
Order 1	**75**	**25**	**70**	**30**
Order 2	**93**	**7**	**88**	**13**

= packed

You know who you are!

*= separate evaluation*

How big is the problem?

What methods do we use to assess to assess continuous probability distributions?

Methods used:

- **Ask for probabilities of prespecified intervals**

Medians	Practice	Perception
	35%	50%

- **Ask for percentiles of a distribution**

	60%	60%

Conclusion

- Clear implications for practice!
  - Probability assessments can depend strongly on the partition.
  - Further research to determine
    - Effect on domain experts, real-world tasks
    - Theory and implications when assessing percentiles (10-50-90s)
- Debiasing
  - Multiple assessments, reconciliation
  - Further research to develop and evaluate specific debiasing techniques