Coarse Valuation Networks

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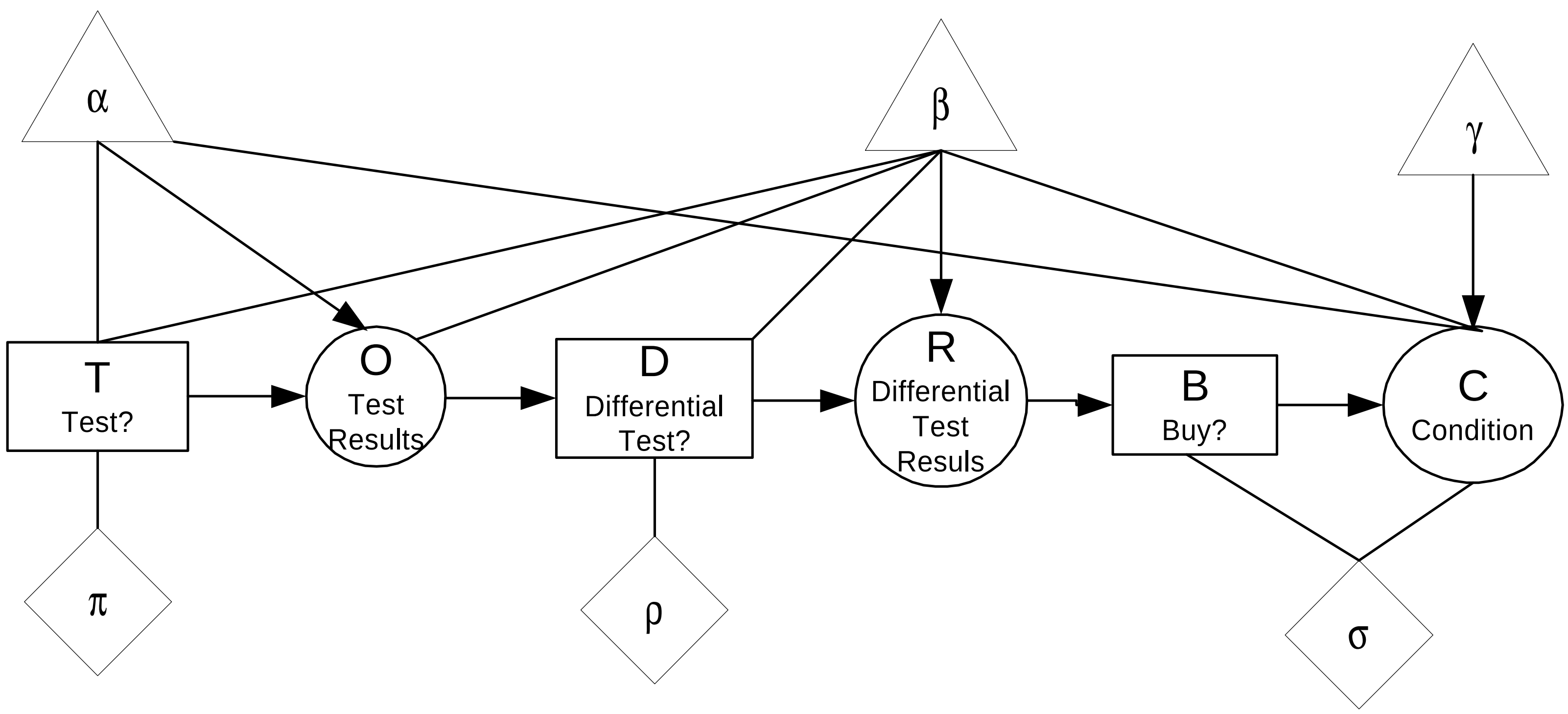
Outline

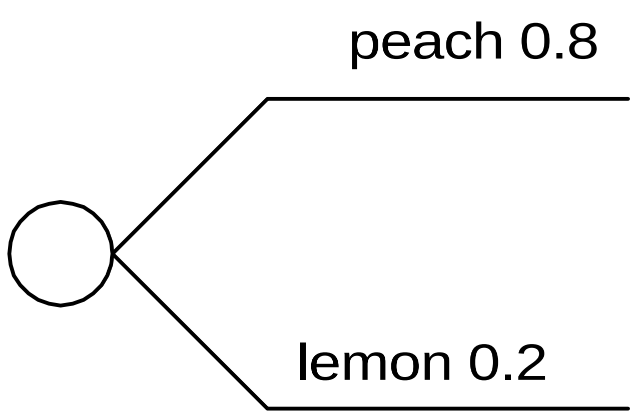
- Coarse Valuations
- Basic Operations on Coarse Valuations
- Fusion Algorithms and Illustration

Coarse Valuation Network

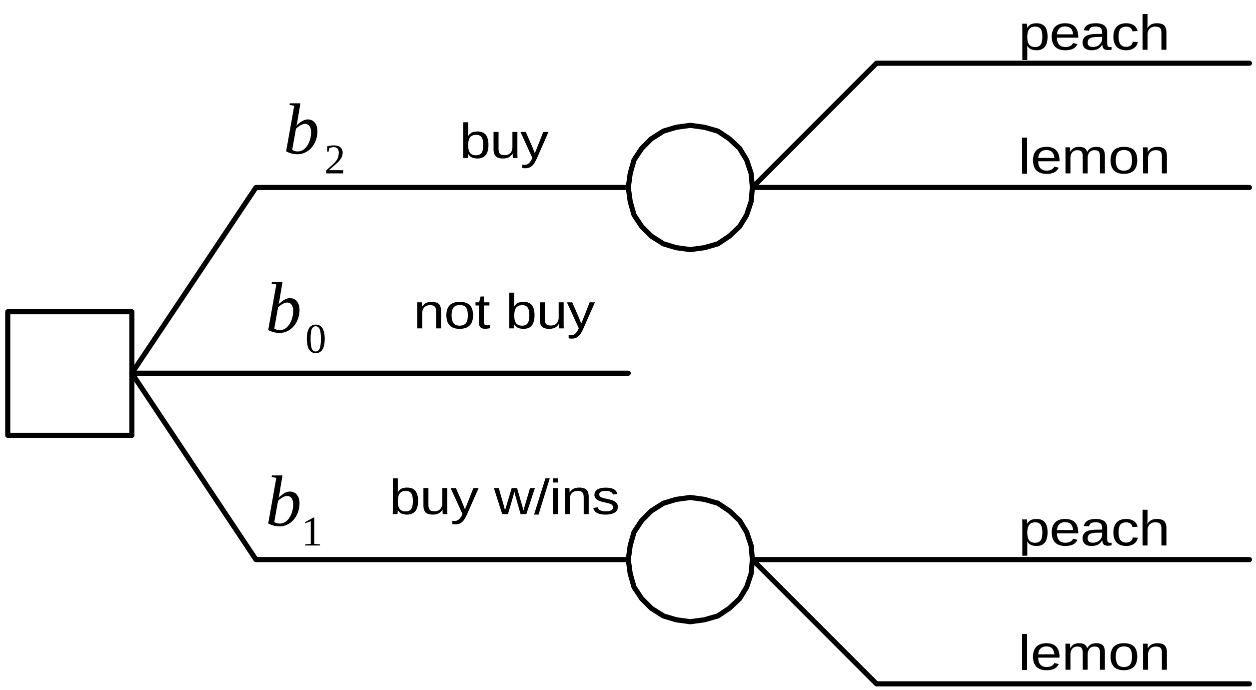


The Used Car Buyer's Problem

- Price $1000
- Market Value $1100.
- From statistics, 20% of the similar cars were lemons and other 80% were very good.



Coarse Payoff Valuations



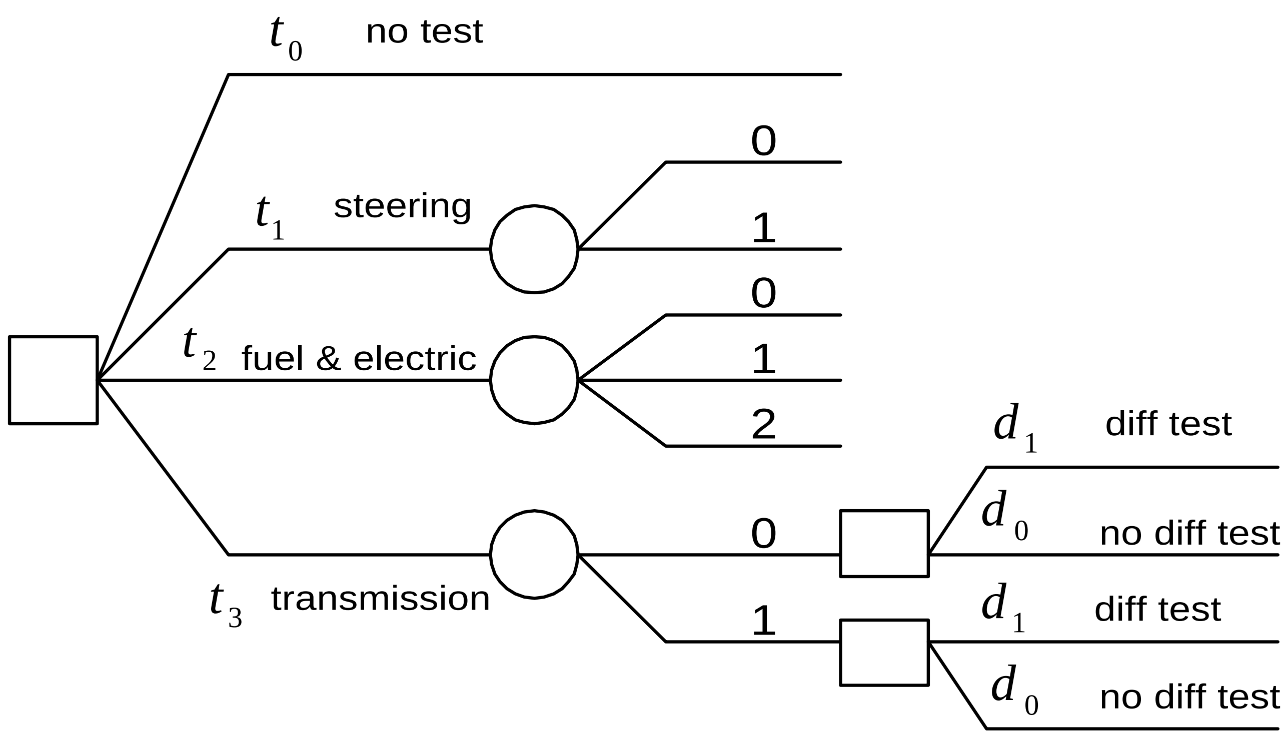
Coarse Payoff Valuations

$\{B, C\}$		σ
b_2	c_p	60
b_2	c_l	-100
b_1	c_p	20
b_1	c_l	40
b_0	c_p	0
b_0	c_p	0

$\{B, C\}$		σ
b_2	c_p	60
b_2	c_l	-100
b_1	c_p	20
b_1	c_l	40
b_0	Ω_C	0

Probability Valuations

- Three diagnostic tests:



Valuation β: Bayesian Approach

- β is a conditional probability function of R given T, O, D, and C.
- Add an artificial state "nr" to the chance variables O and R so that

  $\omega_O = \{0, 1, 2, nr\}$ and $\omega_R = \{0, 1, nr\}$
- The domain for β is

$$\omega_T \times \omega_O \times \omega_D \times \omega_R \times \omega_C$$

has $4 \times 4 \times 2 \times 3 \times 2 = 192$ configurations

Valuation β: Bayesian Approach

- For each configuration in the domain, there must be a specification. So we will have 192 specifications as follows:

$\omega_{\{T,O,D,R,C\}}$	β	$\omega_{\{T,O,D,R,C\}}$	β
t_0 *nr* d_0 *nr* *pe*	1.0	t_0 *nr* d_0 *1* *pe*	0
t_0 *nr* d_0 *nr* *le*	1.0	t_0 *nr* d_0 *1* *le*	0
t_0 *nr* d_0 *0* *pe*	0	t_0 *nr* d_0 *2* *pe*	0
t_0 *nr* d_0 *0* *le*	0	…	…

Valuation β: Bayesian Approach

- Representing any simple fact is not simple: "there is no test result if no differential test (d_0) is done" requires **96** specifications:

$$\boldsymbol{b}(t,o,d_0,nr,c)=1, \boldsymbol{b}(t,o,d_0,0,c)=0, \boldsymbol{b}(t,o,d_0,1,c)=0$$

$$\text{for any } t \in w_T, o \in w_O, \text{ and } c \in w_C$$

Valuation β: Coarse Valuations Approach

- No need for artificial states/acts like “nr”
- β is a conditional belief function of R given T, O, D, and C. Its domain is

  $$\omega_T \times \omega_O \times \omega_D \times \omega_R \times \omega_C$$

  which has $4 \times 3 \times 2 \times 2 \times 2 = 96$ configurations

Valuation β: Coarse Valuations Approach

- There is no need to specify 96 values. Instead, only the following 8 values are required. 96% of dummy ones are avoided

$\omega_{\{T,O,D,R,C\}}$	β	$\omega_{\{T,O,D,R,C\}}$	β
w_T w_O d_0 w_R w_C	1.0	t_3 0 d_1 0 le	3/9
t_3 0 d_1 0 pe	8/9	t_3 0 d_1 1 le	6/9
t_3 0 d_1 1 pe	1/9	t_3 1 d_1 0 le	4/9
t_3 1 d_1 0 pe	1.0	t_3 1 d_1 1 le	5/9

Coarse Valuations

- Assume X contains one or more decision variables, x is a subset of the sample space for X:
  - $\pi(\boldsymbol{x})$ -- the payoff to the DM if $X \in \boldsymbol{x}$
- For any two disjoint sets of random variables H and T, let ***h*** and ***t*** denote respectively the subsets of their sample spaces
  - $\sigma(\boldsymbol{h} \mid \boldsymbol{t})$ – the probability that the true H is in ***h*** if the true T is in ***t***

Combination Operators

- Combination $\otimes$
  - For two payoff functions π and σ:

    $(\pi \otimes \sigma)\ (\mathbf{z}) = \pi(\mathbf{x}) + \sigma(\mathbf{y})$

    where $\mathbf{z}^{\downarrow X} = \mathbf{x}$ and $\mathbf{z}^{\downarrow Y} = \mathbf{y}$
  - For a payoff functions π and a probability valuation α:

    $(\pi \otimes \alpha)\ (\mathbf{z}) = \pi(\mathbf{x})\ \alpha(\mathbf{y})$

    where $\mathbf{z}^{\downarrow X} = \mathbf{x}$ and $\mathbf{z}^{\downarrow Y} = \mathbf{y}$

Combination Operators

- For any two coarse probability valuations α and β:

  $(\alpha \otimes \beta)\ (\mathbf{z}) = \sum\{\alpha(\mathbf{x})\ \beta(\mathbf{y})\ |\ \mathbf{z}^{\downarrow X} = \mathbf{x},\ \mathbf{z}^{\downarrow Y} = \mathbf{y}\}$

Combination Operators

- Example

$\omega_{\{T,O,D,R,C\}}$	β	$\omega_{\{T,O,C\}}$	α
w_T w_O d_0 w_R w_C	1.0	t_0 w_O w_C	1.0
t_3 0 d_1 0 pe	8/9	$\{t_1,t_3\}$ 0 pe	0.9

$\omega_{\{T,O,D,R,C\}}$	$\alpha\otimes\beta$
t_0 w_O d_0 w_R w_C	1.0
t_3 0 d_1 0 pe	0.8
$\{t_1\ t_3\}$ 0 d_0 w_R pe	0.9

Combination Operators

- Marginalization:
  - Marginalizing a decision variable D out of a payoff valuation π:

    $$\pi^{\downarrow Y}(\mathbf{y}) = \max\{\pi(\mathbf{x}) \mid \mathbf{y} \subset \mathbf{x}^{\downarrow Y}\}$$

    where $Y = X - \{D\}$

    I.e., take the maximum among all the values $\pi(\mathbf{x})$ where the projection of the subset **x** contains the subset **y.** This corresponds to the maximization of an expected utility function

Combination Operators

- Example:

$\omega_{\{T,O,D\}}$	π	ψ_D	$\pi^{\downarrow\{T,O\}}$
t_0 w_O d_0	28.00	d_0	28.00
t_3 0 d_0	44.00	d_1	45.38
t_3 0 d_1	45.38		
t_3 1 d_0	32.00	d_0	32.00
t_3 1 d_1	31.96		

Marginalization Operators

- Marginalizing a decision variable D out of a probability function α:

  $$\alpha^{\downarrow Y}(\mathbf{y}) = \alpha(\mathbf{x})$$

  where $\mathbf{y} \subset \mathbf{x}^{\downarrow Y}$ and $Y = X - \{D\}$

  I.e., simply drop D out of X in the table for α.
- Why? When there is need to marginalize D out of α, α must be independent of D according to the semantic condition.

Marginalization Operators

- Example:

$\omega_{\{T,O,D\}}$			α	$\omega_{\{T,O\}}$		$\alpha^{\downarrow\{T,O\}}$
t_0	w_O	d_0	1.0	t_0	w_O	1.0
$\{t_1,t_3\}$	0	d_0	0.8	$\{t_1,t_3\}$	0	0.8
$\{t_1,t_3\}$	1	d_0	0.2	$\{t_1,t_3\}$	1	0.2
t_3	0	d_1	0.8	t_3	0	0.8
t_3	1	d_1	0.2	t_3	1	0.2

Marginalization Operators

- Marginalizing a random variable R out of a payoff valuation π:

  $$\pi^{\downarrow Y}(\mathbf{y}) = \sum\{\pi(\mathbf{x}) \mid \mathbf{y} \subset \mathbf{x}^{\downarrow Y}\}$$

  where Y = X -{R}

  I.e., sum over all the values $\pi(\mathbf{x})$ where the projection of the subset **x** contains the subset **y**. This corresponds to computing an expected value

Marginalization Operators

- Example:

$\omega_{\{T,O,D,R\}}$	π	$\omega_{\{T,O,D\}}$	$\pi^{\downarrow\{T,O,D\}}$
$t_0 \quad w_O \quad d_0 \quad w_R$	28.00	$t_0 \quad w_O \quad d_0$	28.00
$\{t_1\ t_3\} \quad 0 \quad d_0 \quad w_R$	44.00	$\{t_1\ t_3\} \quad 0 \quad d_0$	44.00
$\{t_1\ t_3\} \quad 1 \quad d_0 \quad w_R$	32.00	$\{t_1\ t_3\} \quad 1 \quad d_0$	32.00
$t_3 \quad 0 \quad d_1 \quad 0$	44.62	$t_3 \quad 0 \quad d_1$	49.38
$t_3 \quad 0 \quad d_1 \quad 1$	4.76		

Marginalization Operators

- Assume α is a coarse probability valuation of H given T, and R is a chance variable in H to be deleted. Let X = H$\cup$T , K = H - {R}, and Y =K$\cup$T. Then

  $\alpha^{\downarrow Y}(\mathbf{y}) = \sum\{\alpha(\mathbf{x}) \mid \mathbf{y}^{\downarrow K} = \mathbf{x}^{\downarrow K}, \mathbf{y}^{\downarrow T} \subset \mathbf{x}^{\downarrow T}\}$

  I.e., sum over all $\alpha(\mathbf{x})$ where the head of **x** projects to the head of **y** and the tail of **x** contains to the tail of **y**.

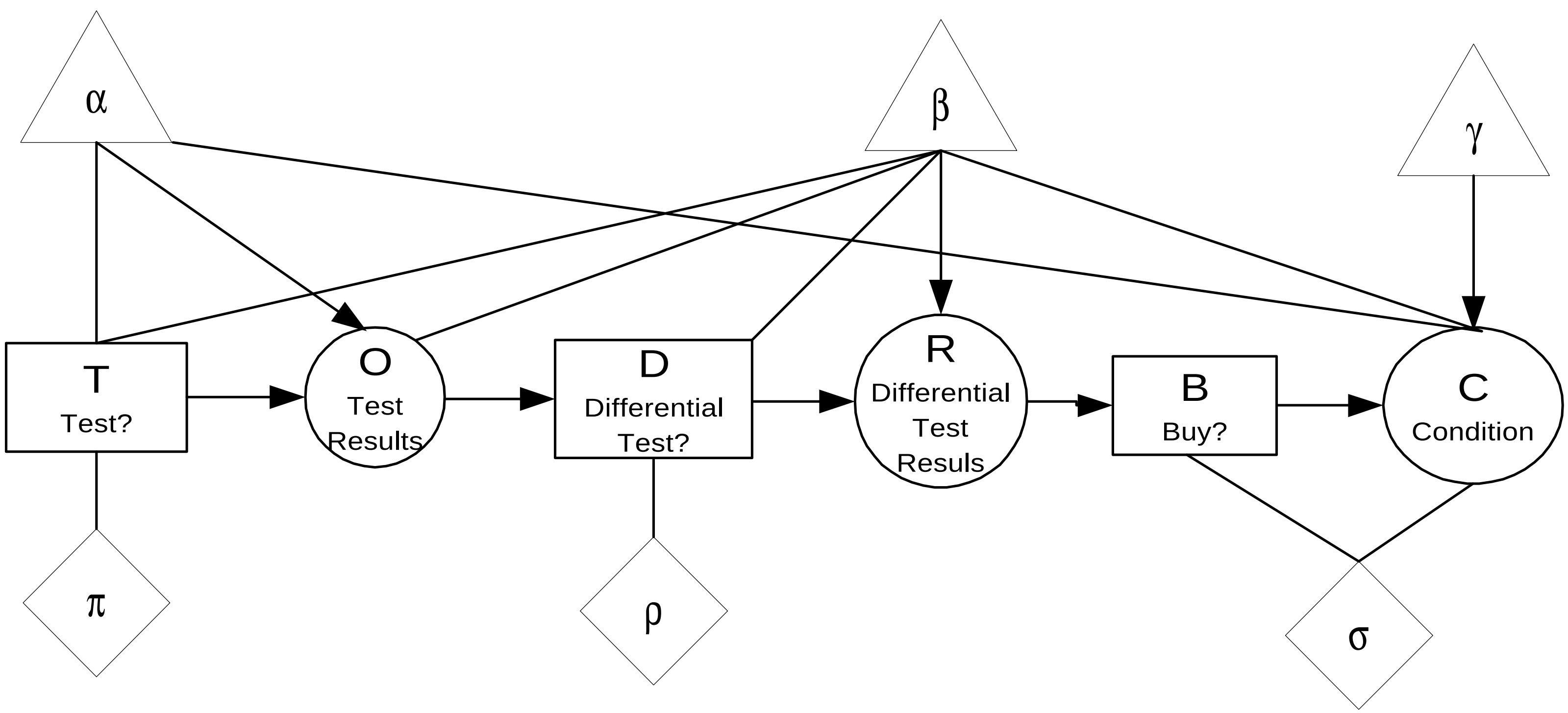
Marginalization Operators

- Example:

$\omega_{\{T,O,D,R,C\}}$	τ	$\omega_{\{T,O,D,R\}}$	$\tau^{\downarrow\{T,O,D,R\}}$
t_0 w_O d_0 w_R pe	0.800	t_0 w_O d_0 w_R	1.000
t_0 w_O d_0 w_R le	0.200		
t_3 1 d_1 0 pe	0.080	t_3 1 d_1 0	0.134
t_3 1 d_1 0 le	0.054		
t_3 1 d_1 1 le	0.066	t_3 1 d_1 1	0.066

Fusion Algorithm

- Overall Control: X is deleted before Y if X is after Y in terms of a precedence order

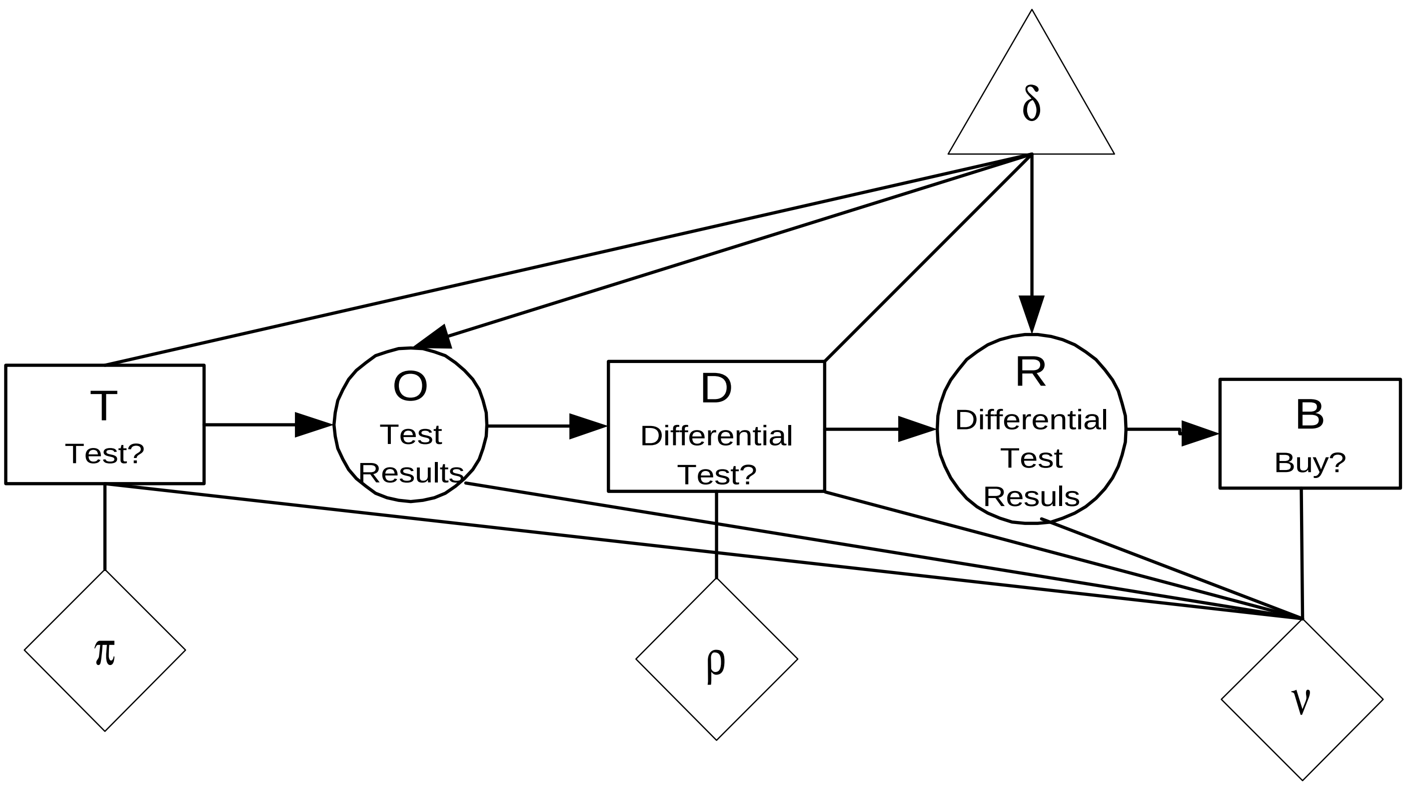


Fusion Algorithm

- Fusion Rule #1:
  - If D is a decision variable and no belief function bears on D, then after fusion, all the payoff valuations that bear on D are combined and marginalized such that D is eliminated. The other payoff valuations and all belief valuations remain unchanged

Fusion Algorithm

- Example for Rule #1:



Fusion Algorithm

- Fusion Rule #2:
  - If R is a chance variable and no payoff valuation bears on R, then after fusion, those belief functions that bear on R are combined and marginalized such that R is eliminated. The other belief functions and all the payoff valuations remain unchanged

Fusion Algorithm

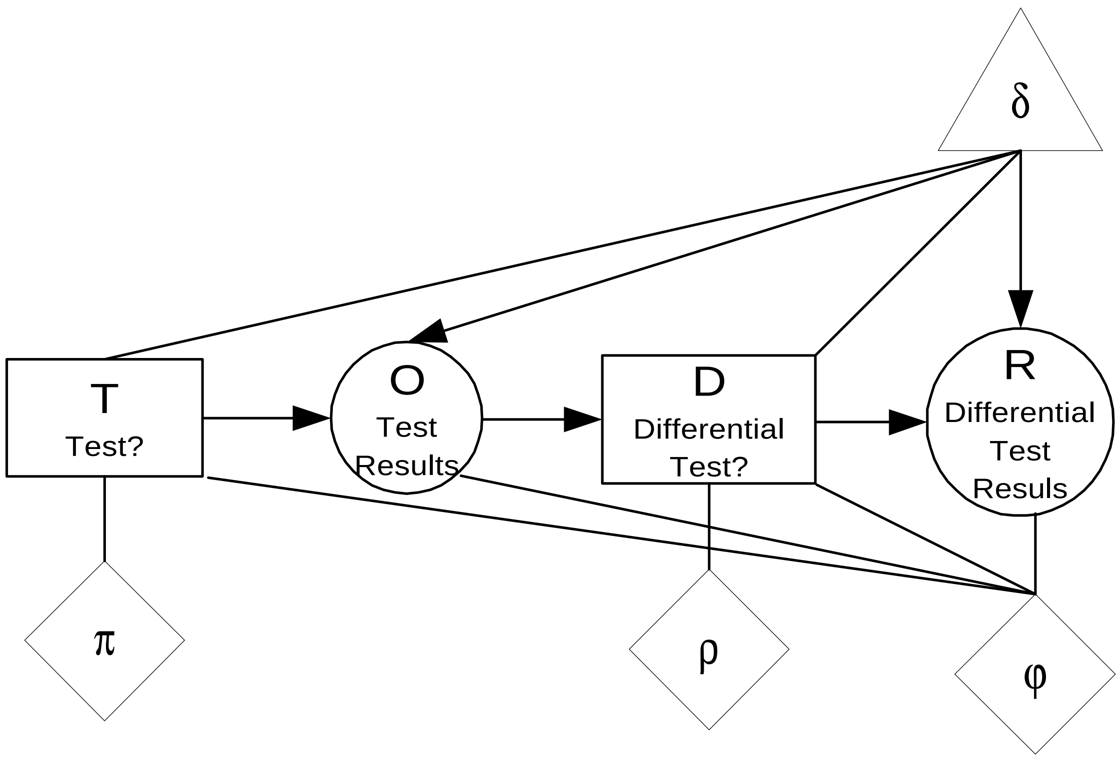
- Fusion Rule #3:
  - If R is a chance variable and all payoff valuations bear on R, then after fusion, all payoff valuations and those belief functions that bear on R are combined and marginalized such that R is eliminated. The other belief functions remain unchanged

Fusion Algorithm

- Fusion Rule #4:
  - If R is a chance variable and only part of the payoff valuations bear on R, then after fusion, the payoff and the belief valuations that do not bear on R remain unchanged. A new belief valuation and a new payoff valuation are created.

Fusion Algorithm

- Example for Rule #4:

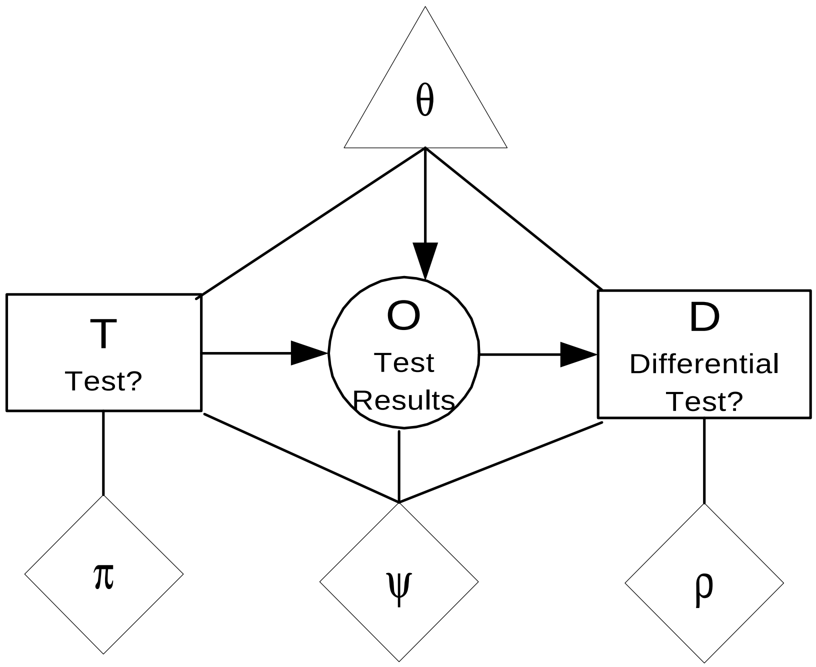


Fusion Algorithm

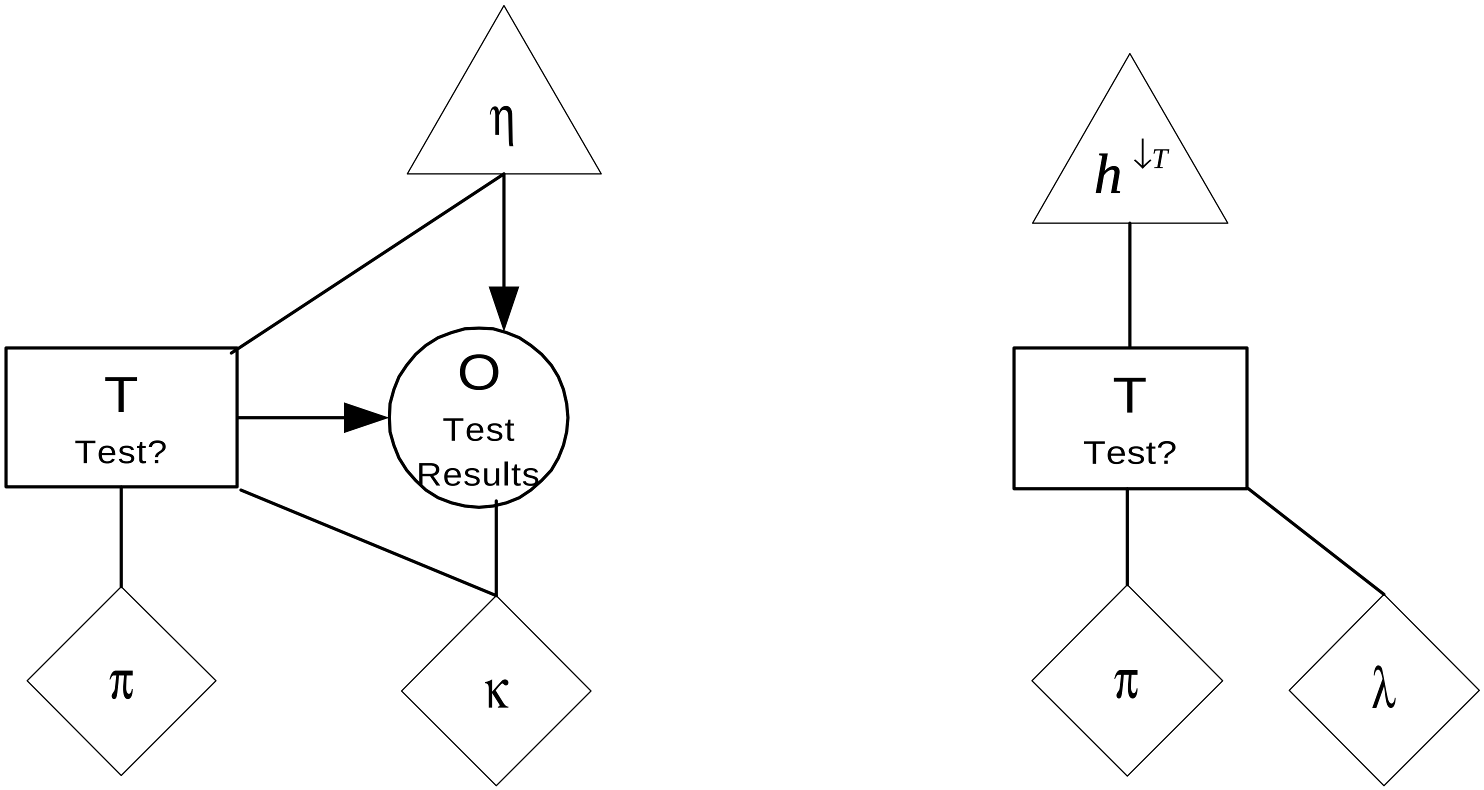
- Fusion Rule #5:
  - If D is a decision variable and there exist belief functions bearing on D, then after its deletion, all the payoff valuations that bear on D are combined and marginalized such that D is deleted. Every belief function that bears on D is marginalized such that D is deleted. All other payoff and belief valuations remain unchanged.

Fusion Algorithm

- Example for Rule #5:



Fusion Algorithm



Summary

- It makes valuation Networks more flexible in representing domain knowledge
- It provides a *most compact representation* of asymmetric decision problems
- It has both pros and cons in computation:
  - Pros: reduce problem spaces
  - Cons: the computation of belief functions is less efficient than that of Bayesian probabilities