Representing and Solving Asymmetric Bayesian Decision Problems

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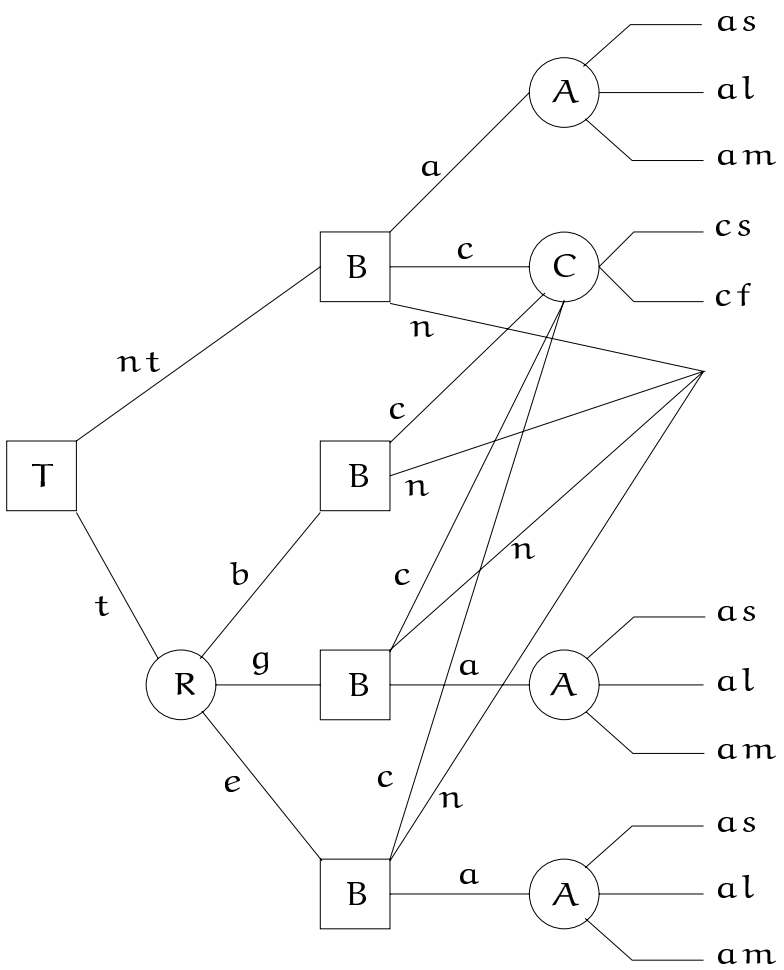
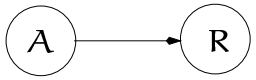
The Reactor Problem



A causal probability model for A and R:



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Asymmetry in Decision Problems

- Structural asymmetry:
  - The variables observed or decided upon may vary in different scenarios.
  - The relative temporal order of two variables may depend on the variables previously observed or decided upon.
- Functional asymmetry:
  - The legitimate decision options of a decision variable may vary depending on the different information states.
  - The possible outcomes of a chance variable may vary depending on the conditioning states.

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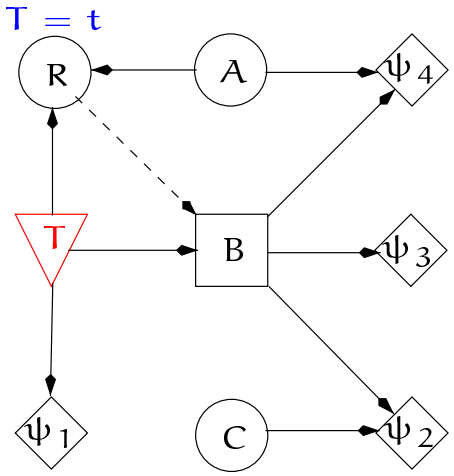
Representing the Reactor Problem



$\gamma_B : sp(R) \hookrightarrow 2^{sp(B)}$:

g, e	{n, c, a}
b	{n, c}

Definition: The test-decision T is a so-called split variable since it influences the structure of the model, i.e., if $T = nt$ then R is never observed.

Property: A test-decision variable has no probabilistic influence on a chance variable. E.g., R is associated with P(R|A).

Constraint: At any point of time there exists at most one split variable with no other split variables as predecessors.

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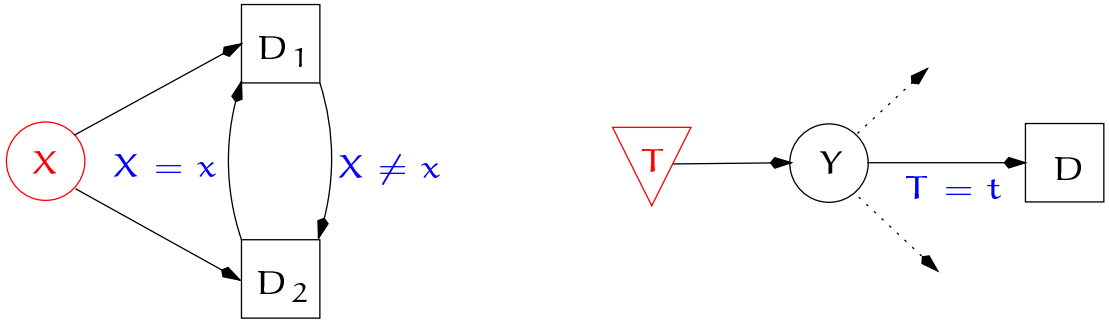
Two Examples of Temporal Dependence



- The temporal ordering of D_1 and D_2 depends on the state of X; $D_1 \prec D_2$ if $X \neq x$ and $D_2 \prec D_1$ if $X = x$.
- Before any of the nodes in a cycle are observed the cycle must be "broken".
- Y is only observed before D if we chose t at T.

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The Asymmetric Influence Diagram

- A representation based on the syntax and semantics of the influence diagram.
- Structural asymmetry is specified by:
  - Labels associated with nodes and informational arcs.
  - Test-decisions (drawn as triangles).
- Functional asymmetry is specified by:
  - Partial probability potentials.
  - Partial utility potentials.
  - Restrictive functions.

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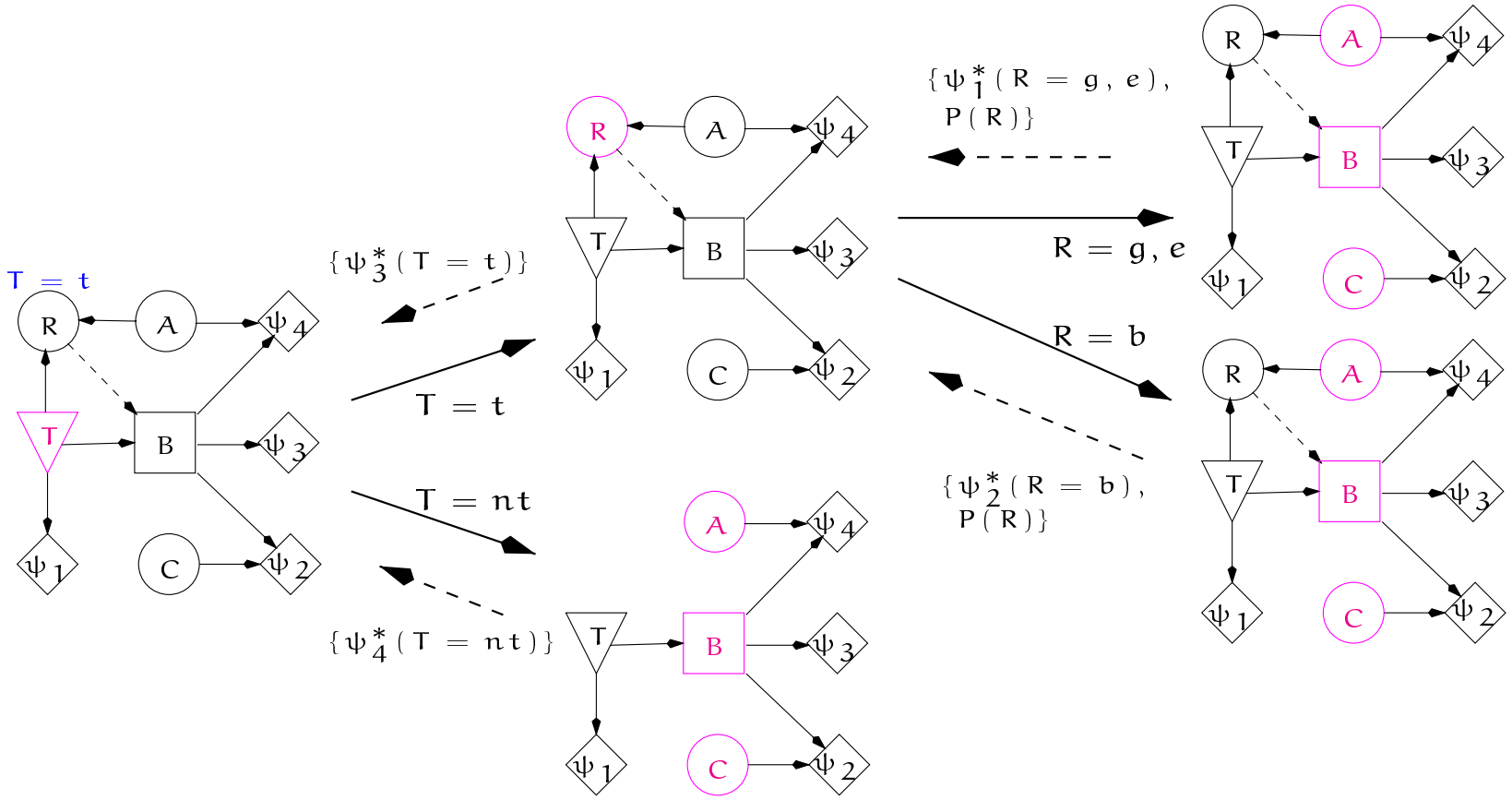
Solving the Reactor Problem



- Decompose the decision problem into a set of symmetric subproblems.
- Recursively eliminate the free variables and send potentials upwards.

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Other frameworks

- Smith, Holtzman and Matheson's extended influence diagrams.
  - Distribution trees are used to encode both numeric information and information about asymmetry, which may produce large conditionals during the evaluation.
- Shenoy's asymmetric valuation networks.
  - Indicator functions are used to encode asymmetry, thereby separating it from the numeric information.
- Demirer and Shenoy's Sequential valuation networks.
  - Hybrid of asymmetric valuation networks and sequential decision diagrams (Covaliu and Oliver).
  - Structural asymmetry is encoded at the qualitative level.

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Results and Conclusion

- The asymmetric influence diagram allows a partial temporal ordering over the free decision variables in the subproblems.
- The requirement of a unique split variable does not seem to leave out any natural decision problems but it can be weakened.
- We conjecture that the asymmetric influence diagram has the same expressive power as the decision tree, however, the class of asymmetric decision problems which can be modelled effectively remains to be determined.

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