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Real Options & the Black Scholes Formula:

What's Wrong with This Picture ?

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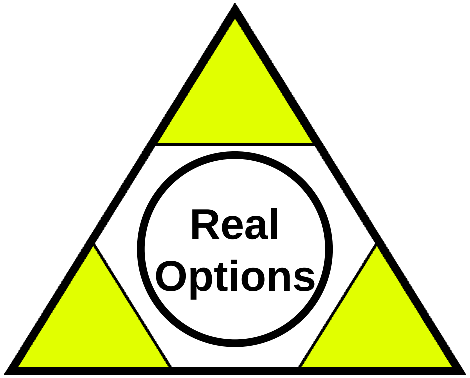
The term “real options analysis” is loosely applied to three different things.

Framing

- Identifying contingent opportunities
- Identifying learning events
- Time sequencing



Calculation Tools

- Equations
- Decision Trees
- Simulation (Monte Carlo)
- Dynamic Programming

Market Discipline

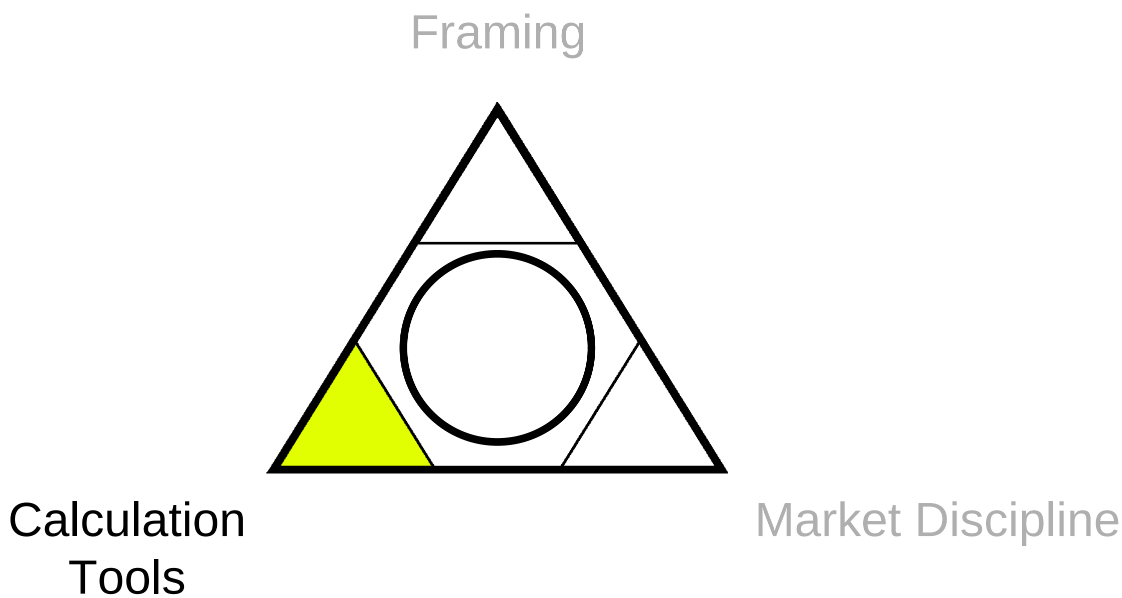
- Price discovery
- Implied volatility
- Historical time-series analysis
- Imperfect proxies

How did this semantic proliferation arise ?

“Real Options Analysis” = Applying the wisdom of **stock option theory** to business strategy and corporate finance

- But - alas - there are many layers and components in stock option theory:
  - The language and structure of contingent contracts
  - The concept of tracking portfolios and “no arbitrage” condition
  - Ito theory of stochastic calculus and partial differential equations
  - The Black Scholes solution formula
  - Dynamic programs and lattice models

Calculation tools translate real options into quantitative dollar value.



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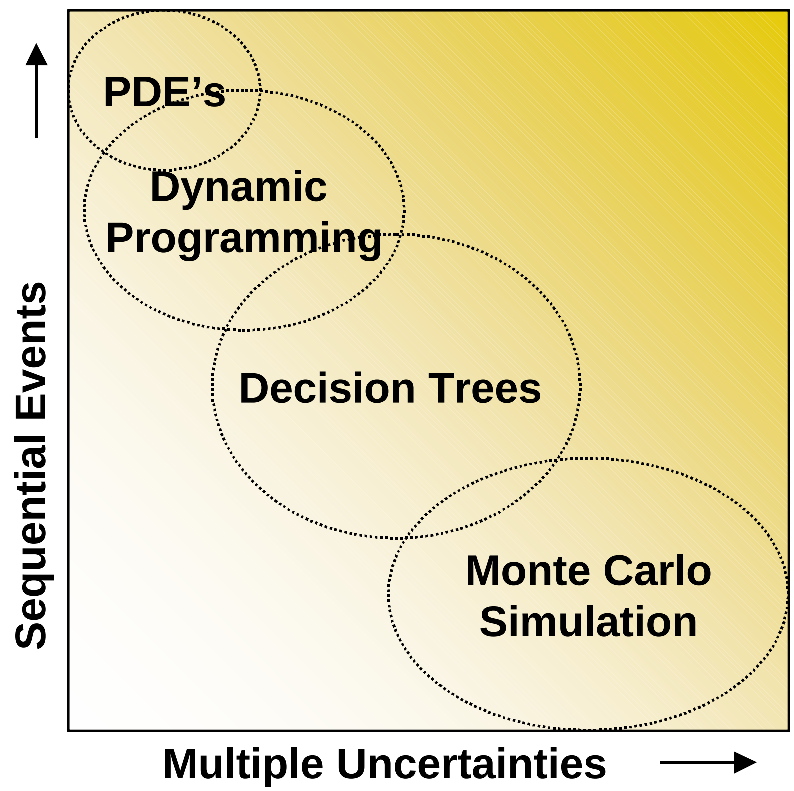
Many tools can be used to evaluate Real Options, each with different scope of applicability.



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How can we analyze real options on the back of the envelope ?

<u>Stock option</u> pricing is invariably associated with names of Black & Scholes.

- Fisher Black, Myron Scholes and Bob Merton (1973) developed a theory for valuation of “contingent assets” (= derivatives) in an efficient market.
  - Nobel prize in 1997
- “Black Scholes” (or Black-Merton-Scholes) is
  - A **theory** of value (Contingent Claims Analysis)
  - A dynamic **equation** indirectly describing the value of any financial derivative (stochastic **P**artial **D**ifferential **E**quations)
  - A **formula** for pricing a simple European call option.

Although its derivation is not trivial, the Black Scholes formula is simple to use.

Option Value = **Probability (1)** X **Asset Value** – **Probability (2)** X **Strike Price**

- The strike price (X) and asset value (S) are present values
  - the strike price needs to be discounted to present value using risk-free borrowing rate.
- The probabilities (1) and (2) are given as cumulative normal distributions of d_1 and d_2, which in turn depend on the riskiness of the stock, measured by the standard deviation, *U*, of the asset's return
  - Invariably expressed in terms of the "volatility", σ
    - Namely, $U = \sigma \times \sqrt{t}$

Several authors have recommended the use of such option pricing formulas to evaluate real options.

- The key is to map the project characteristics into option parameters and plug them into the Black Scholes Formula.

Real Option		Stock Option
Present value of cash flows	⟷	Stock price **S**
Launch costs	⟷	Strike price **X**
Time until the decision must be made	⟷	Time to expiration ***t***
Discount rate	⟷	Risk-free rate ***r***
Risk (probability distribution)	⟷	Cumulative volatility $U = \text{sqrt}(\sigma^2 t)$

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To many this is the essence of real options analysis.

- But there is an aura of mystery as to how this formula magically extracts the real option value

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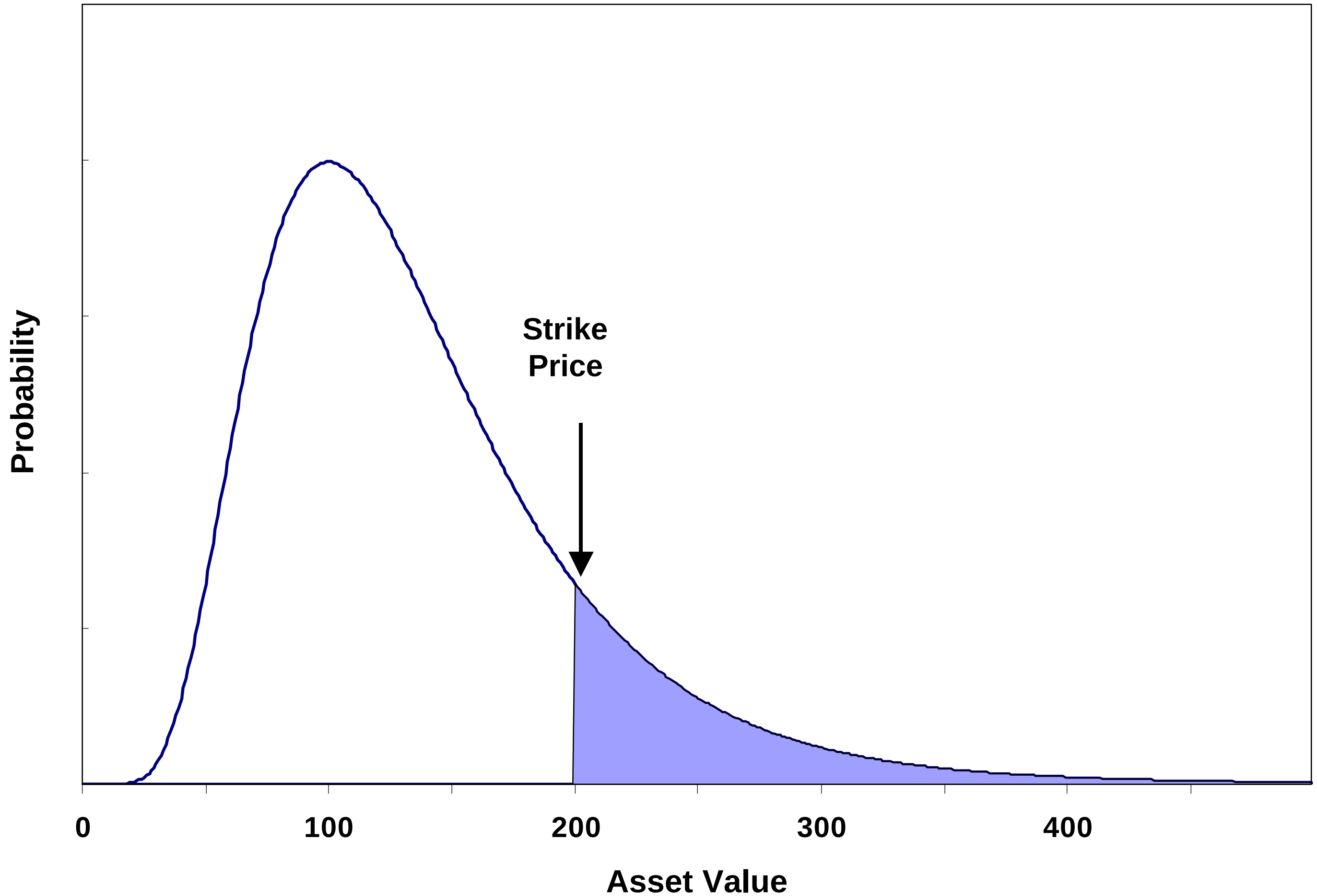
The single biggest source of mystique is the confusion between “what it is” and “how we got there”.

- Historically, the only fundamental difficulty with options valuation was how to account for risk aversion
  - discount rate
- Black, Merton and Scholes showed - with some high flying math - that, amazingly, risk aversion is irrelevant to the value stock options
  - magical cancellation
- Once you know risk aversion is irrelevant, you can derive (and explain) the Black-Scholes formula in very simple probabilistic terms

The Black Scholes formula assumes the future asset value is represented by a ***log-normal distribution****.



* In other words, log(S) has a normal bell-curve distribution.

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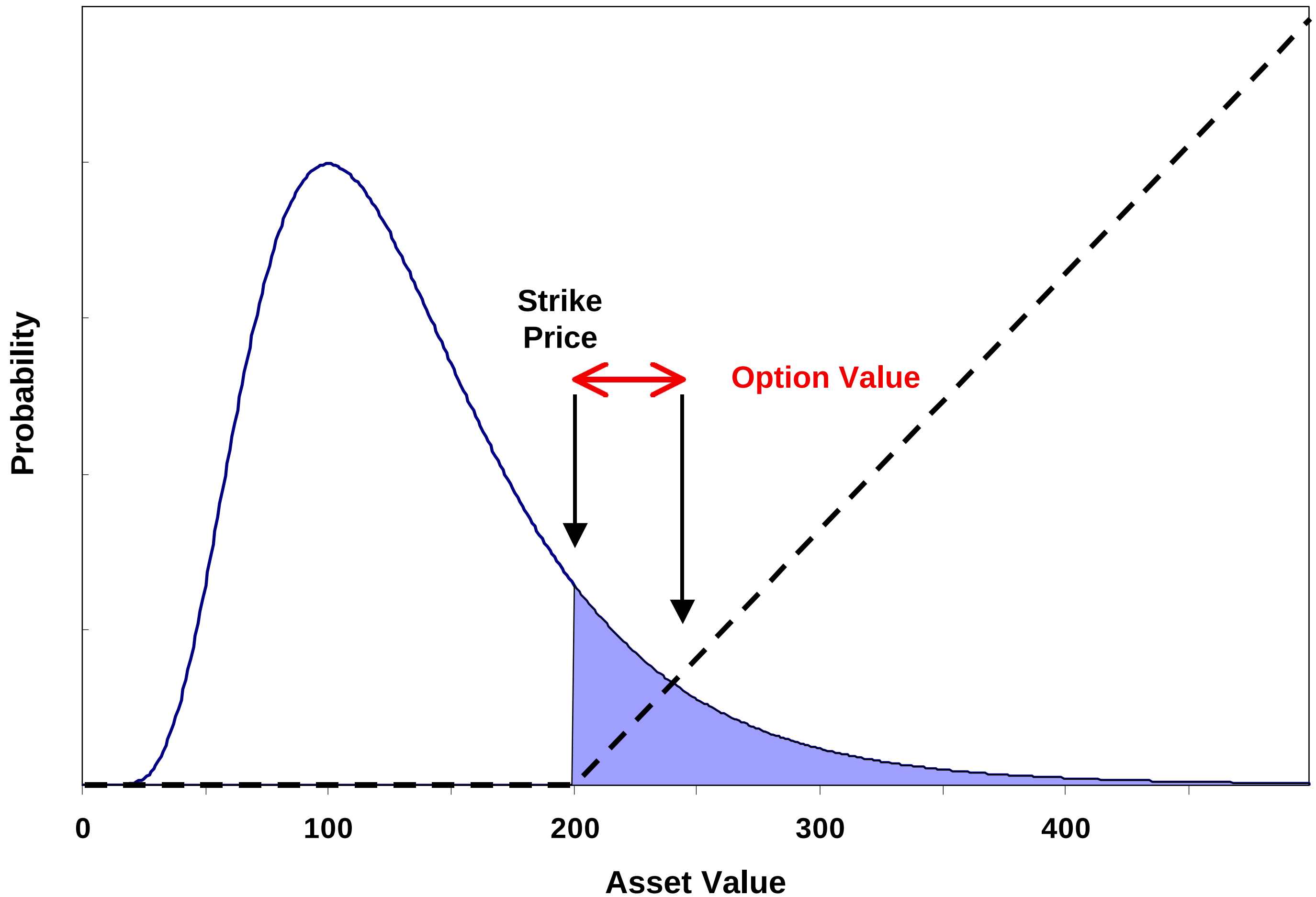
The Black Scholes formula is just the expected value of a hockey-stick payoff with this particular probability curve.



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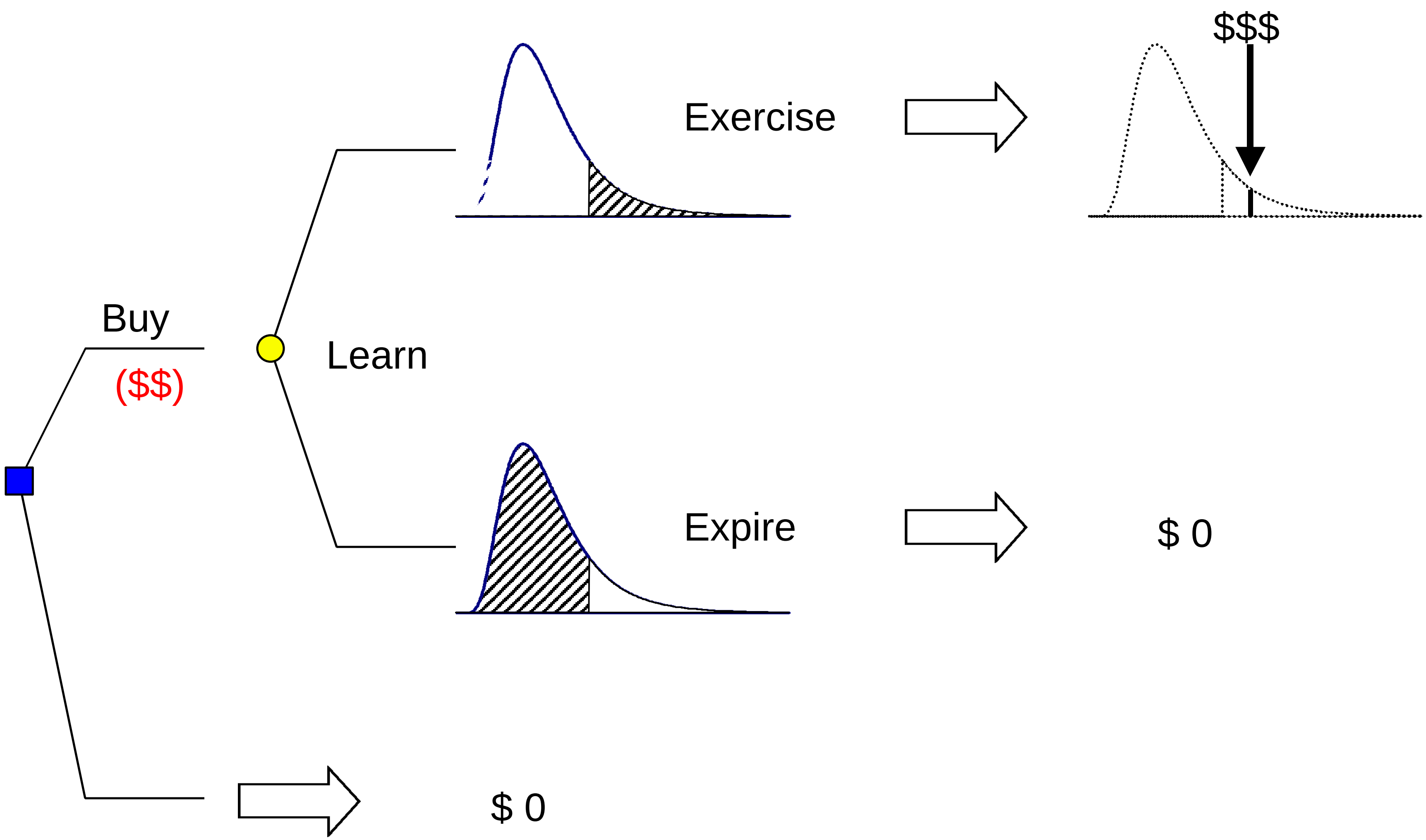
In other words, the Black Scholes formula is mathematically identical to a simple “decision tree”:



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Before applying the Black Scholes formula, we must understand its assumptions and approximations.

- Assumes a **single** exercise date
- Implies that **all the uncertainty is resolved** by the time a decision is made to exercise.
  - Seldom true for real options
- It is based on the assumption that the initial uncertainty in the projected cash flows is represented by a **log-normal distribution**.
  - Usually unrealistic, especially for success-or-failure risks

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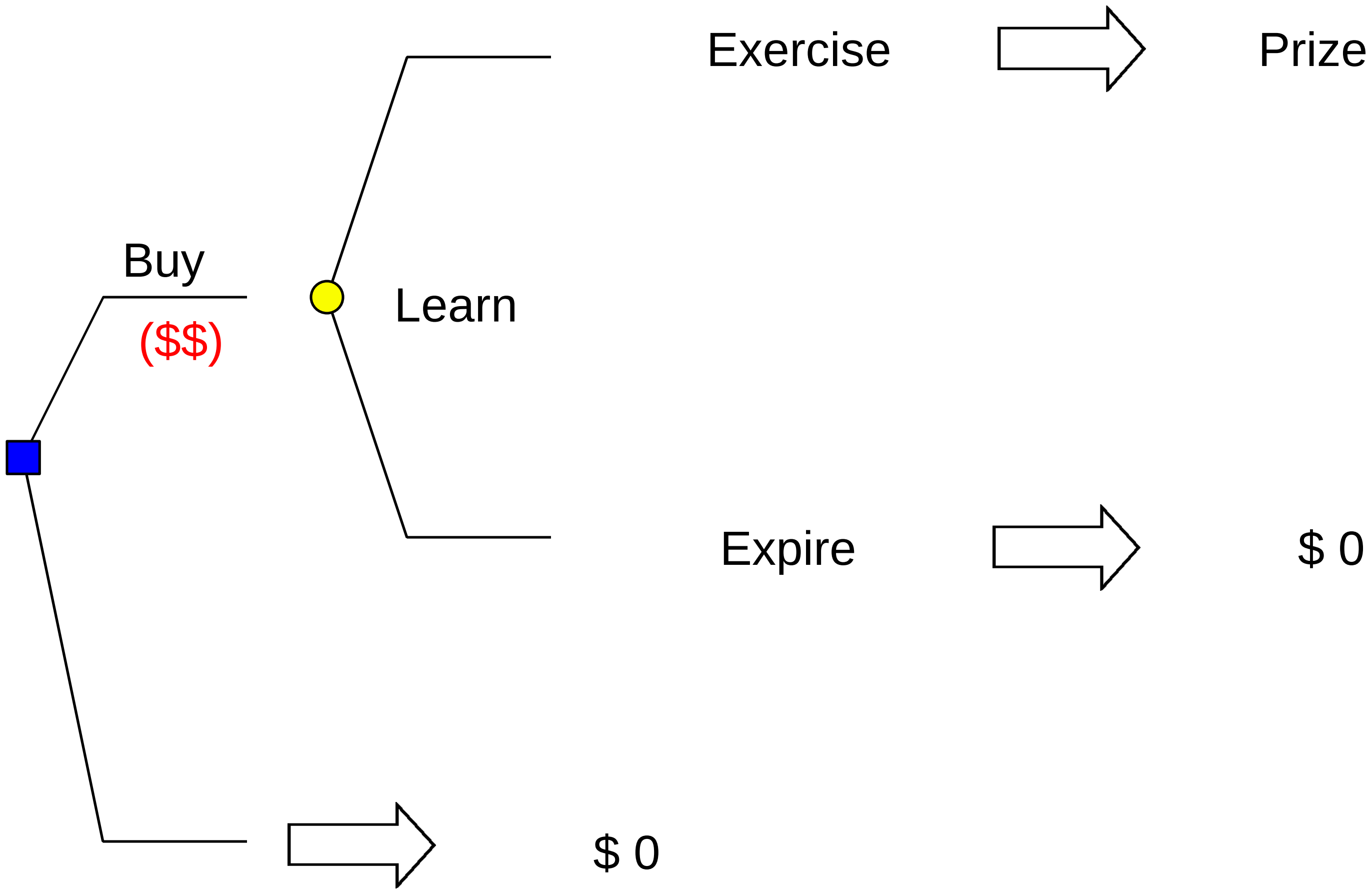
The basic logic of this tree is quite robust and easily generalized.



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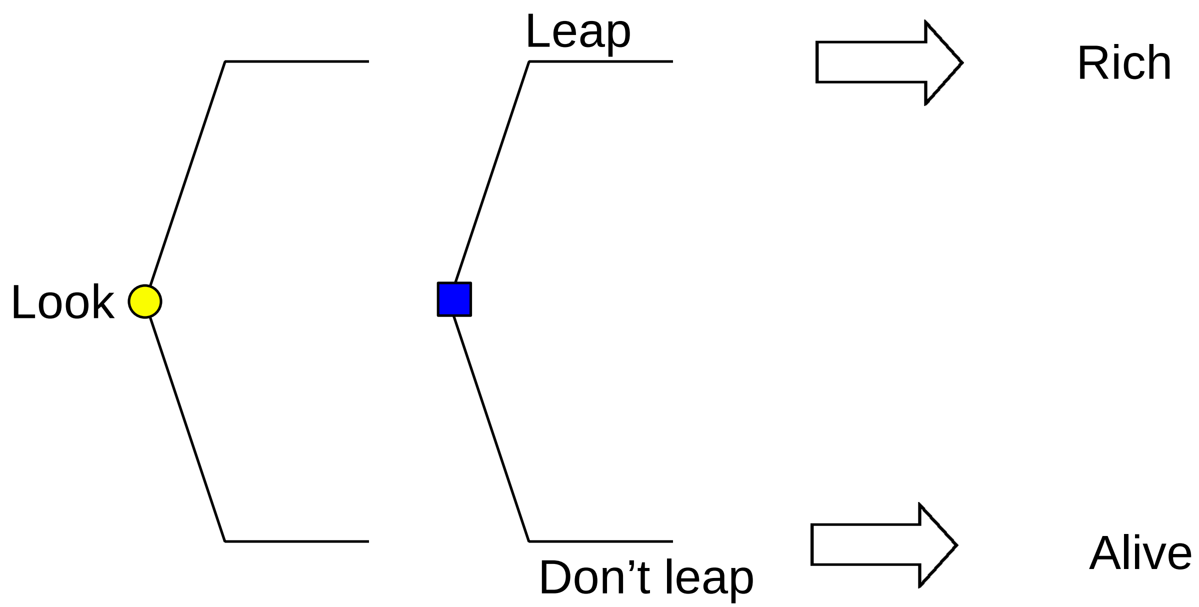
This is the “look before you leap” theory of option value.



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To approximate the value of this tree we simply need to answer two questions.

- The likelihood of ending up on the upper branch ("in the money").
- The typical, or average, size of the prize IF we are on the upper branch.

Option Value = **Probability of being in the money** X **Average net value *assuming* it is in the money**

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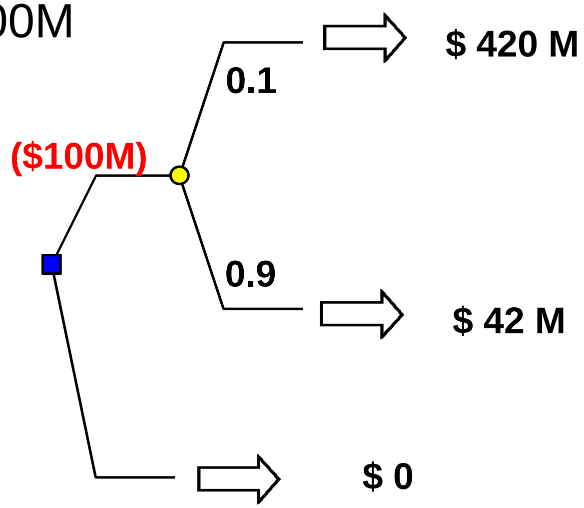
Of course you have to think carefully about these two questions.

- Is the “don’t buy” branch really zero ?
- How do you assess probability ?
- Understanding the exercise trigger for real business options
- Size of Prize is related to exercise trigger.

Back-of-the-envelope example: Launching a new product

- Requires $100M investment in manufacturing plant
- Don't know how successful it will be
  - most likely will be mediocre, generate $10M per year, for 5 years
  - 10% chance for a blockbuster, generating $100M per year
- Probability weighted NPV* is <u>negative $20M</u>.
  - But a 10% chance of an NPV over $300M



* Using 6% discount rate

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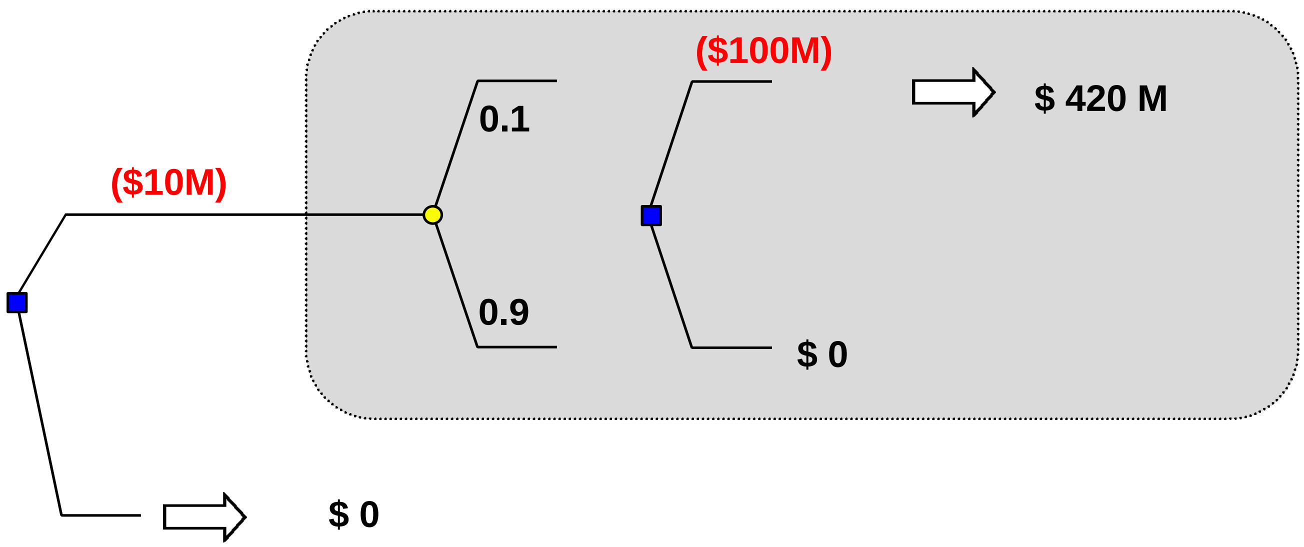
Then we remember real options...

- Build a $10M pilot plant, launch on a small scale
- Build the full size plant for $100 only if looks like a blockbuster
- Is this option worth $10 M ?



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The back-of-the-envelope answer depends...

This is a 10% chance of getting $320M	A one-year call option with • X = $100 M • S = $80M (mean) • Volatility = 50%	A one-year call option with • X = $100 M • S = $40M (median) • Volatility = 50%
Value = $32 M	**Value = $9 M**	**Value = $0.4 M**

Would you pay $10 M for this option (the cost of the pilot plant) ?

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This example is striking because, actually, it is *almost* right.

- Unlike most real options, it is a pure European option
  - The timing of opportunity (strike date) was unusually well defined
  - The uncertainty was 100% resolved by the exercise point
- Still the Black Scholes formula was completely off the mark
  - because of the polarity in the possible outcomes
- The failure can be much more dramatic in more realistic situations where the opportunity window is fuzzy and the uncertainty resolves gradually.

Summary

- The Black Scholes equation is not even approximately right
  - used for the wrong reasons
  - can be off by orders of magnitude
- Back of the envelope is a good idea
  - Based on a simple product: probability X Size of Prize
  - Easier, transparent *and* less likely to generate nonsense
- The Black Scholes formula - for *real* options - may still have several redeeming features:
  - As a framing tool
  - As a tool of market discipline

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