A Target-Based Formulation of a Corporate Utility Function

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Target-Based Utilities

- Individual k's utility for payoff x is u(k,x)
- Harsanyi social welfare function is

  $(u(x|1)+u(x|2)+\ldots+u(x|n))/n$
- New Idea

  $u(k,x) = Pr(x>T(k))$: T random

Borch Insurance Model

- Firm wants to stay solvent by end of month
- Cash Reserves are S
- Uncertain expenses, Y arising over month
- If firm gets payoff x, probability of solvency is

  $Pr(x+S>Y)$

- If $T=Y-S$,

  firm's utility for x is $u(x)=Pr(x>T)$

Equivalence

- Castagnoli and LiCalzi:For any utility function u, there exists a target T such that $u(x)=Pr(x>T)$
- For any non-monetary outcome x, there exists a value function V(x) such that $u(x)=Pr(V(x)>T)$

Kahneman & Tversky Result

- Individuals have a reference point
- They take risks if they're below reference point
- They avoid risks if they're avoid reference point

We propose taking the mode of T as the reference point

Our model introduces uncertainty about this reference point

Simon's Bounded Rationality

- Individuals act to achieve goals
- There's environmental uncertainty about what is required to reach the goal
- Treat the target as the mode of T
- This interpretation was supported by work in Cognitive Science(Heath, Yu).

Role of Uncertainty about Target

- Infinite uncertainty implies risk-neutrality
- No uncertainty implies absolute goal-orientation

The Utility Problem

- Harsanyi utility for dividing x into payoffs of x(1)…x(n) for individuals 1….n now becomes (P(x(1)>T(1))+…P(x(n)>T(n)))/n
- How do we define a social welfare function?
- Optimizing the Harsanyi utility---when all utilities are concave---for a fixed x requires making Pr(x(k)=T(k)) equal for all k.

Specifying the Utility Function

- Suppose T(k) is normally distributed with mean m(k) and standard deviation s(k)
- m(k) is individual k's reference point
- If $x(k)<m(k)$ then $Pr(x(k)>T(k))$ is convex
- When $X<m(1)+\ldots+m(n)$, the convexity of the utility function implies we want to be inegalitarian with some individuals getting the minimal amounts so that others can get amounts exceed m(k).

Focus on those exceeding m(k)

- If Utility is concave, then optimal x(k) sets Pr(x=T(k)) equal
- Let t(k) = (T(k)-m(k))/s(k) be standardized unit normal
- Then Pr(x(k)=T(k)) = Pr([x(k)-m(k)]/s(k) = t(k)).
- Making Pr(x(k)=T(k)) equal implies [x(k)-m(k)]/s(k) equal

Optimal Sharing Rule

- Let m=m(1)+….+m(n) be total `reference point' of society
- Let s=[s(1)+…+s(n)] be society's total uncertainty about its reference point
- Then x(k) = m(k) + [X-m][s(k)/s]
- Harsanyi social welfare function is

  (Pr(x(1)>T(1)+…+Pr(x(n)>T(n))]/n = Pr((X-m)/s > t)

Harsanyi Social Welfare Criteria

- Define social target by T= m + s t
- Harsanyi utility: u(X) = Pr(X> T) = Pr(X/n>T/n)

Society acts like individual whose reference point is the sum of everyone's reference point and whose uncertainty is the sum of everyone's uncertainty

Rawlsian Social Welfare Function

Rawls defines utility as the utility of least well off individual.

We interpret this as defining the probability of society meeting its goal as the probability that all individuals meet their goals.

So Pr(Societal Goal Met)=Pr(X(1)>T(1),….X(n)>T(n))

Invoking Previous Results

If individuals are independent, the utility equals

$$\Pr(X(1)>T(1))\ldots.\Pr(X(n)>T(n))$$

If we use previous sharing rule, then it's

$$\Pr(X>T)\ldots.\Pr(X>T)$$

Extreme-Value Distribution

For a large number of individuals, this converges to one of three extreme value distributions:

- Weibull social welfare function
- Double exponential social welfare function

Conclusions

Target-based utility reflects K/T reference point

It provides fairly natural implications for social welfare

- Mandates inequality when society is poor
- Mandates sharing of wealth on the basis of risk-preference when society is affluent
- Provides a natural definition of the representative individual