Risk Sharing, Fiduciary Duty, and Corporate Risk Attitudes

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Among decision analysts, there is considerable debate about how one should model risk attitudes in corporate applications.

- Model the preferences of the manager responsible for the decision

  Walls and Dyer (1995): Risk tolerance ≈ 1/4 of budget

- Model the preferences of the corporation -- the "subscription model"
  - Spetzler (1968), Howard (1988)
  - Howard's Rule: Risk tolerance ≈ 1/6 of book value of equity

- Model the preferences of shareholders
  - How? Which ones?
  - Finance: Since unsystematic risks can be "diversified away" by shareholders, the firm should be risk-neutral towards these risks.

- Other perspectives: Agency theory, costs of distress (Bickel 1999) …

The goal of this paper is to study how acting in the "best interests" of the shareholders constrains the choice of corporate utility function.

Assumptions about Shareholders:

- **Assumptions:** N expected utility maximizing shareholders

  Single-period model

  Common beliefs about uncertainties

- Notation:

  Amount shareholder i receives from firm: x_i or gamble $\tilde{x}_i$

  Shareholder Utility: $u_i(x_i)$

  (Absolute) risk tolerance: $r_i(x_i) = \; > 0$

  Certainty equivalent: $\text{CE}_i[\tilde{x}_i]$

- If initial wealth is uncertain (i.e., shareholders have other interests), assume independence and replace $u_i(x_i)$ with induced utility $\hat{u}_i(x) \equiv \text{E}[u_i(x + \tilde{e}_i)]$.

  ➔ Focus is on "unsystematic risks"

Assumptions about the Firm (I):

- **Assumption 1:** The firm wants to maximize expected utility, using the same probabilities as the shareholders.

  Notation:

  Amount the firm receives: x or gamble $\tilde{x}$

  Firm Utility: $U(x)$

  Gambles shared among shareholders according to sharing rules $(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N)$ such that:

$$\sum_{i=1}^{n} \tilde{x}_i = \tilde{x}.$$

  Forms of sharing rules will be described later.

- Amounts x should be interpreted as representing the firm's entire portfolio of investments, after paying employees, corporate taxes, debt holders, etc.

Assumptions about the Firm (II):

- **Assumption 2:** The firm wants to make decisions in the "best interests" of the shareholders:

  a) *If any shareholder prefers his share of $\tilde{x}^A$ to $\tilde{x}^B$ and no shareholder prefers his share of $\tilde{x}^B$ to $\tilde{x}^A$, then the firm also prefers $\tilde{x}^A$ to $\tilde{x}^B$.*

  b) *If every shareholder is indifferent between two alternatives, the firm should also be indifferent between them.*

  (Given continuity of *U* and the sharing rules, b follows from a.)

Proposition 0 (Harsanyi 1955): *These assumptions imply that the firm's utility function can be represented as:*

$$U_l(x) = \sum_{i=1}^{N} l_i \, u_i(x_i)$$

for some set of utility weights $l_1, l_2, \ldots, l_N$, $l_i > 0$.

From the firm's perspective, it is natural to assume that the shareholders each receive a fixed proportion of the firm's income.

- Assume *i*'s share of $\tilde{x}$ is given by $\tilde{x}_i = s_i \tilde{x}$ where $s_i > 0$ and $\sum_{i=1}^{N} s_i = 1$.

- Given weights $\boldsymbol{l} = (l_1, l_2, \ldots, l_N)$, the firm's utility function is

$$U_{\boldsymbol{l}}(x) = \sum_{i=1}^{N} l_i \, u_i(s_i x)$$

- The firm's risk tolerance is then given by

$$r_{\boldsymbol{l}}(x) \equiv -\frac{U_{\boldsymbol{l}}'(x)}{U_{\boldsymbol{l}}''(x)} = -\frac{\sum_{i=1}^{N} l_i \, s_i \, u_i'(s_i x)}{\sum_{i=1}^{N} l_i \, s_i^2 \, u_i''(s_i x)}$$

- These utilities and risk tolerances are difficult to characterize analytically, even if all shareholders have an exponential or HARA utility.

"Share Risk Tol." – the risk tolerance shareholder *i* wants the firm to use

Proposition 1 (Fixed-Fraction Risk Sharing)**.** *For any set of utility weights* $l_1, \ldots, l_N > 0$:

a) *If all of the shareholders' utility functions exhibit increasing (or non-decreasing) risk tolerance, then so does the firm's.*

b) *The firm's risk tolerance,* $r_l(x)$, *lies between the following bounds:*

$$\min\left\{\frac{r_1(s_1x)}{s_1}, \ldots, \frac{r_N(s_Nx)}{s_N}\right\} \le r_l(x) \le \max\left\{\frac{r_1(s_1x)}{s_1}, \ldots, \frac{r_N(s_Nx)}{s_N}\right\}$$

c) *For any gamble* $\tilde{x}$, *the firm's certainty equivalent,* $\mathrm{CE}_l[\tilde{x}]$, *lies between the following bounds:*

$$\min\left\{\frac{\mathrm{CE}_1[s_1\tilde{x}]}{s_1}, \ldots, \frac{\mathrm{CE}_N[s_N\tilde{x}]}{s_N}\right\} \le \mathrm{CE}_l[\tilde{x}] \le \max\left\{\frac{\mathrm{CE}_1[s_1\tilde{x}]}{s_1}, \ldots, \frac{\mathrm{CE}_N[s_N\tilde{x}]}{s_N}\right\}$$

The bounds in (b) and (c) cannot be tightened without restricting the allowed utility weights. The inequalities in (b) and (c) will either both hold with equality or will both be strict.

"Share CE" – the CE shareholder *i* wants the firm to assign

Illustrative Calculations:

- Let p_i = relative risk tolerance; w_i = wealth; f_i = fraction of wealth invested in firm. V = Firm value

- Base Share Risk Tolerance for shareholder i = $\dfrac{r_i(0)}{s_i} = \dfrac{(p_i \times w_i)}{((f_i \times w_i)/V)} = \dfrac{p_i V}{f_i}$

- Share-Risk-Tolerance to Firm-Value Ratio = $\dfrac{\text{Share Risk Tolerance}}{\text{Firm Value}} = \dfrac{p_i}{f_i}$

Examples:

Typical, well-diversified: $p_i = 1/6$ and $f_i < 1\%$ ➔ Ratio > 16.6

Typical, not well-diversified: $p_i = 1/6$ and $f_i = 10\%$ ➔ Ratio = 1.66

Manager with most of wealth tied up in company: $p_i = 1/6$ and $f_i = 66\%$ ➔ Ratio = .25

Conflict of interest with shareholders?

Contrast these results with those given by assuming shareholders optimally share risks.

- Given a gamble $\tilde{x}$, the sharing rules ($\tilde{x}_1$, $\tilde{x}_2$, …, $\tilde{x}_N$,) maximize the firm's expected utility:

$$E[U_l(\tilde{x})] = \sum_{i=1}^{N} l_i\, E[u_i(\tilde{x}_i)] \qquad \text{subject to } \sum_{i=1}^{N} \tilde{x}_i = \tilde{x}.$$

- Results (Wilson 1968): Optimal shares x_i^* depend on x and l_i's

Firm utility: $U_l(x) = \sum_{i=1}^{N} l_i\, u_i(x_i^*(x))$

Firm risk tolerance: $r_l(x) = \sum_{i=1}^{N} r_i(x_i^*(x))$

Optimal shares x_i^* satisfy: $\dfrac{\partial x_i^*(x)}{\partial x} = \dfrac{r_i(x_i^*(x))}{r_l(x)}$ = "fractional shares s_i"

Fractional shares are generally not constant, except when shareholders all have exponential utilities or form a "HARA family."

Earlier bounds on firm risk tolerance collapse to $r_l(x)$ if sharing is optimal.

Proposition 2 (Optimal Risk Sharing): *For any set of utility weights* $l_1, \ldots, l_N > 0$:

a) *If all of the shareholders' utility functions exhibit increasing (or non-decreasing) risk tolerance, then so does the firm.*

b) *The firm's risk tolerance,* $r_l(x)$, *lies between the following bounds:*

$$\min_{x_1, x_2, \ldots, x_n} \sum_{i=1}^{N} r_i(x_i) \;\leq\; r_l(x) \;\leq\; \max_{x_1, x_2, \ldots, x_n} \sum_{i=1}^{N} r_i(x_i)$$

c) *For any gamble* $\tilde{x}$, *the firm's certainty equivalent,* $\mathrm{CE}_l[\tilde{x}]$, *satisfies the following:*

$$\mathrm{CE}_l[\tilde{x}] \;\leq\; \max_{\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n} \sum_{i=1}^{N} \mathrm{CE}_i[\tilde{x}_i] \,.$$

The bounds in (b) and (c) cannot be tightened without further restricting the weights or sharing rules. The inequalities in (b) will either both hold with equality or will both be strict.

Notes:

- Bounds on risk tolerances and CEs for optimal sharing schemes may be broader or narrower than the bounds given by assuming fixed fractions.
    - Certainty equivalents may be lower for an optimal sharing scheme than for a non-optimal scheme.

Illustrative Calculations:

Assume that all shareholders have constant proportional risk tolerance p_i

Total shareholder wealth = $W = V/\bar{f}$ where V is the value of the firm and $\bar{f}$ is average fraction of shareholder wealth invested in the firm.

Bounds for optimal schemes:

$$1 = \frac{1/100}{1\%} \approx \frac{\min_i \{ p_i \}}{\bar{f}} \leq \frac{\text{Firm risk tolerance}}{\text{Firm Value}} \leq \frac{\max_i \{ p_i \}}{\bar{f}} \approx \frac{1}{1\%} = 100$$

Bounds are achieved by allocating all shareholder wealth to the shareholder with least/greatest proportional risk tolerance!

Conclusion: *If we want to represent the interest of shareholders, we should use risk tolerances that are much larger than those reported in the DA literature.*

- Results suggest that large firms with reasonably well-diversified shareholders should be essentially risk-neutral.
- Both models of risk sharing lead to this conclusion.
- Why the discrepancy?
  1) Models considered here ignore something important:
     - Costs of "distress" are assumed to be included in the gambles; they are treated implicitly in most utility assessments.
     - Calculations in Bickel (1999) suggest such costs cannot justify the levels of risk aversion observed.
  2) Reported values do not represent the best interests of the shareholders.
     - Reflect personal risk aversion of the managers? Concern for their own reputation, the well-being of employees, etc.
- Is this appropriate?