Insights for decision analysis from model theory

Jeff Keisler and H.J. Keisler

INFORMS-Philadelphia, 11/7/99

Goal: think about the role of language in decision making.

- Motivating real example:
  - Lack of coordinated strategy between marketing and manufacturing.
- Key features
  - Each knows different things as a result of experience.
  - Each has a formally defined language that is useful for applying what they know.
- How can the decision analyst help?
- How can model theory help the decision analyst help?

Our plan

- Work out a toy model representing organizational decision making that is subject to model theoretic analysis .
- Use this model to:
  - Identify phenomena that occur;
  - Interpret existing theorems from model theory;
  - Suggest behaviors in the real world.

An organization can be represented by a model.

- What could be said about the world (the aggregate language)
  - all (first order) sentences formed from a vocabulary of constants, functions, and relations.
- What's so (the theory of the model)
  - the set of all sentences which are true in the model.
- What a person knows (her tacit theory)
  - a subset of what's so.
- What a person articulates (her explicit theory)
- What a person can articulate (her language)
  - all sentences formed from her vocabulary, which includes at least all symbols used in her explicit theory.

Specifically, sentences you can articulate can be:

- told to others;
- understood if others tell them to you;
- added to your explicit theory from your tacit theory; or
- used to discover new statements through logical deduction.

Example: a consultant has obtained the following information.

- Marketing's theory (T1) includes at least:
  - Trucks have a profit margin of $1000.
  - Cars have a profit margin of $500.
- Manufacturing's theory (T2) includes at least:
  - 150,000 cars could be produced.
  - 100,000 trucks could be produced.
- Neither side understood the other's statements.
- The proposition they need to decide (P): the truck business would be more profitable than the car business.

The consultant concludes something about the two sides' vocabularies.

- Marketing's language (L1) has at least:
  - Profit margin function from vehicle type to R.
  - The relation "x is more profitable than y" on pairs of vehicle types.
- Manufacturing's language (L2) has at least:
  - Capacity function from vehicle type to R.
  - The relation "x is more profitable than y" on pairs of vehicle types.

A useful theorem from model theory

- The Craig Interpolation Theorem states that if sentence A in one language implies sentence B in another language, there is a sentence C in their common language where A implies C, and C implies B.
- The consultant hopes that the two theories combined can decide P, i.e. either (T1 and T2) implies P, or (T1 and T2) implies (not P).
- We rewrite the first of these as T1 implies (T2 implies P).

A useful theorem (continued)

- By Craig's theorem with A=T1, B=(T2 implies P), there must be a sentence C in the common language where T1 implies C, and C implies (T2 implies P).
- A successful scenario: The consultant gets Marketing to deduce C and tell it to Manufacturing. Now Manufacturing can deduce P!

We should look for an intermediate sentence C that both sides understand.

- Example: (1) The function “profit ratio of trucks to cars” belongs to both languages L1 and L2.
  - (2) Manufacturing knows that “Profit ratio of trucks to cars >production ratio of cars to trucks implies P”.
  - A key sentence C in the common language is “The profit ratio of trucks to cars is 2”.
- Exercise: What if the function “production ratio of cars to trucks” is in the common language.

Another application of Craig's Theorem

- If two theories, T1 and T2, are inconsistent, there is a sentence C in their common language where T1 implies not C and T2 implies C.
- This could be useful if the theories contained possibly false beliefs.

If we could not decide, there would be several ways to expand the theories.

- Add the missing concept to the vocabularies of both languages (e.g., profit ratio) and then proceed as before.
- Have one of the sides learn the other's language and then import the theory we already heard.
- Either side could just collect more information.
- Bring in someone who knows both languages but neither theory to learn from both sides and decide the proposition for them.

We can think of the decision analyst as another character in the toy world.

- Has a theory (decision theory axioms, etc.) and vocabulary.
- Knows some of the facts and vocabularies that others in the organization know.

This concept can help us figure out what the consultant's role should be

Does consultant know axioms (other than DA) needed to decide propositions

Does consultant know client's language for theory needed to decide propositions	Yes	No
Yes	Decision maker	Analyst
No	Expert	Educator

It is known that there are cases where an arbitrary proposition or its negation may be:

- an element of a class that can be proven or disproven systematically (decidable);
- provable or not provable within a theory;
- proven more efficiently with some languages than with others;
- equivalent to some other statements;
- expressible in some languages but not in others.

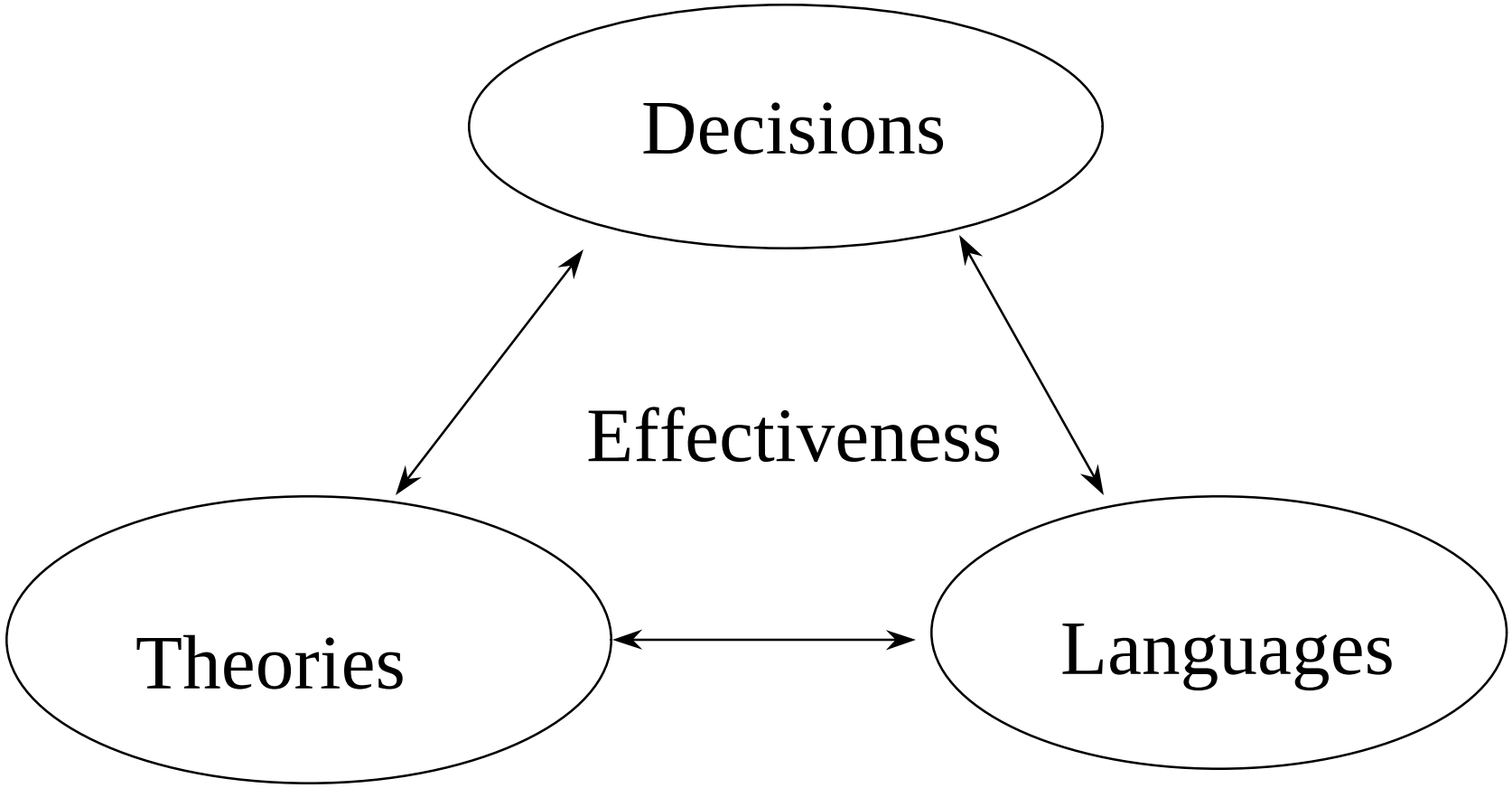
The choice of intervention itself is in principle a decision problem.

- Interventions such as communicating, changing a language, obtaining new information, or performing logical deduction, each have a cost.
- Deciding any proposition has some benefit.
- Knowing what propositions can be decided with which theories gives us a way to tradeoff the costs of an intervention against the benefits of the theory it leads to.

Some other theorems in model theory have similar applications

- When can implicit terms be defined explicitly?
- What is decidable?
- When can an infinite theory be replaced by a finite theory?
- How efficiently can propositions be proven or disproven?

Takeaway 1: Language is important, in a way that depends subtly on context.



Takeaway 2: Model theory seems applicable to knowledge management.

- Some references if you're interested:
  - Model Theory (Chang & Keisler, 1990);
  - Basic Model Theory (Doets, 1996);
  - A Shorter Model Theory (Hodges, 1997);
  - Situations and Attitudes (Barwise & Perry, 1998).

A motivating case

- Manufacturing capacity planning process
- Terms were
  - Ambiguous (flexible)
  - Not shared (planned production)
  - Conflicting (demand)
  - Missing (maximum production possible)
- Implications
  - Inconsistent action (marketing what could not be produced)
  - Indecision (late decisions lead to higher cost)
  - Blunders (
  - Missed opportunities (unprofitable products)