A Paradox in Time Preference

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Simple problem: An MBA evaluating a job

Let x_1 be Mr. Miller's first year salary and x_2 his future salary. In the context of a multiattribute analysis, suppose that the other job attributes are priced out in terms of x_1 and x_2.

One may be left with the following evaluation:

$$V(x_1, x_2) = u(x_1) + b\, u(x_2) \qquad (1)$$

where u is the single-period utility function, and b the discount factor.

We consider two cases:

- x_1 and x_2 are uncertain income streams (three models)
- x_1 and x_2 are consumption streams (model 1).

For *uncertain income streams*, consider the following three possible methods of evaluation:

- Discounted utility $\quad V(x_1, x_2) = u(x_1) + b\, u(x_2)$
- Discount of certainty equivalents $\quad V(x_1, x_2) = ce(x_1) + b\, ce(x_2)$
- Utility of Net Present Value $\quad V(x_1, x_2) = u(x_1 + b\, x_2)$

In each case, we want to satisfy the following two assumptions:

A1 The single-period utility function, u, is monotonically increasing and strictly concave (risk aversion).

A2 A shift of payoff from period 2 (later) to period 1 (now) is preferred, i.e.,

$$(x_1 + \Delta, x_2 - \Delta) \bullet (x_1, x_2) \text{ , for all } x_1, x_2, \text{ and } \Delta > 0.$$

A2 is reasonable because one can put the money in the bank.

Discounted utility $V(x_1, x_2) = u(x_1) + b\, u(x_2)$ (1)

Proposition 1

Discounting utility, **A1**, and **A2** are incompatible.

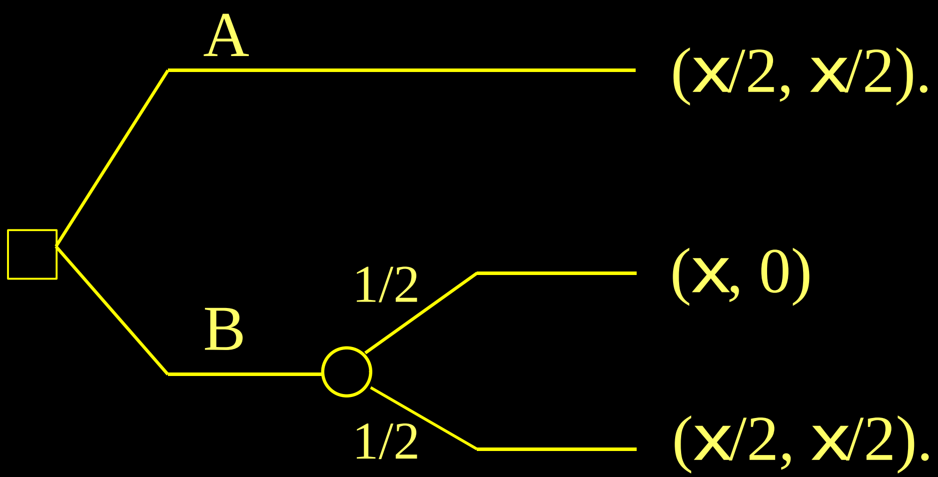
Proof*: If b » 1, then model (1) gives a higher evaluation if* x_1 *is close to* x_2*. Thus, (*x,x*) will dominate (2*x*,0) for reasonable values of b.*

Discount of certainty equivalents $V(x_1,x_2) = ce(x_1) + b\ ce(x_2)$ (2)

Proposition 2

***Discounting certainty equivalents,* A1**, and **A2** are incompatible.

Proof: Consider the following two alternative jobs



By **A2**, B should be preferred to A

By concavity, the expected utilities in alternative B satisfy

Period 1 $\frac{1}{2}u(x) + \frac{1}{2}u(x/2) < u(\frac{1}{2}x + \frac{1}{2}x/2) = u(3x/4)$

Period 2 $\frac{1}{2}u(0) + \frac{1}{2}u(x/2) < u(\frac{1}{2}x/2) = u(x/4)$

so that $ce^B(x_1) < 3x/4$ and $ce^B(x_2) < x/4$.

For b » 1, $ce^B(x_1) + ce^B(x_2) < x$, whereas $ce^A(x_1) + ce^A(x_2) = x$.

Utility of Net Present Value $V(x_1, x_2) = u(x_1 + b\, x_2)$ (3)

Proposition 3

Utility of Net Present Value, **A1**, and **A2** are compatible.

Consumption streams

Recall model (1):

$$V(x_1, x_2) = u(x_1) + b\, u(x_2) \qquad (1)$$

where x_1 is consumption today and x_2 is consumption later.

This model is adequate for evaluating *consumption streams*. However, the time interval separating both periods (today and tomorrow) must be large enough.

If the time interval between periods is short, we have the paradoxical situation illustrated in the following example.

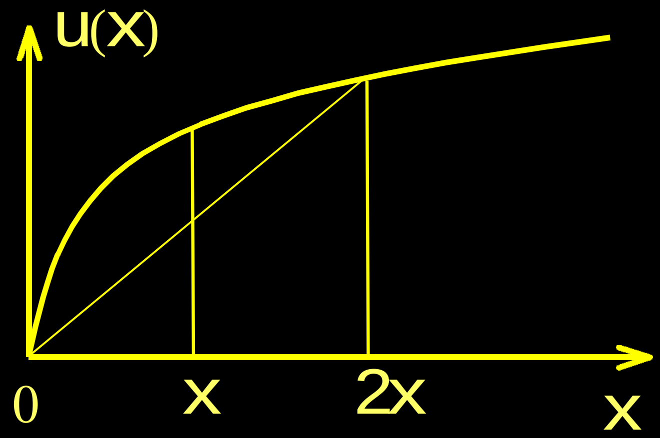
A → Have two pizzas now and none later.

$$V(2x, 0) = u(2x) + u(0)$$

B → Have one pizza now and one pizza an instant later ($b \approx 1$):

$$V(x, x) = u(x) + u(x) = 2u(x)$$

By concavity, $[u(2x) + u(0)]/2 < u(x) \Rightarrow u(2x) + u(0) < 2u(x)$.



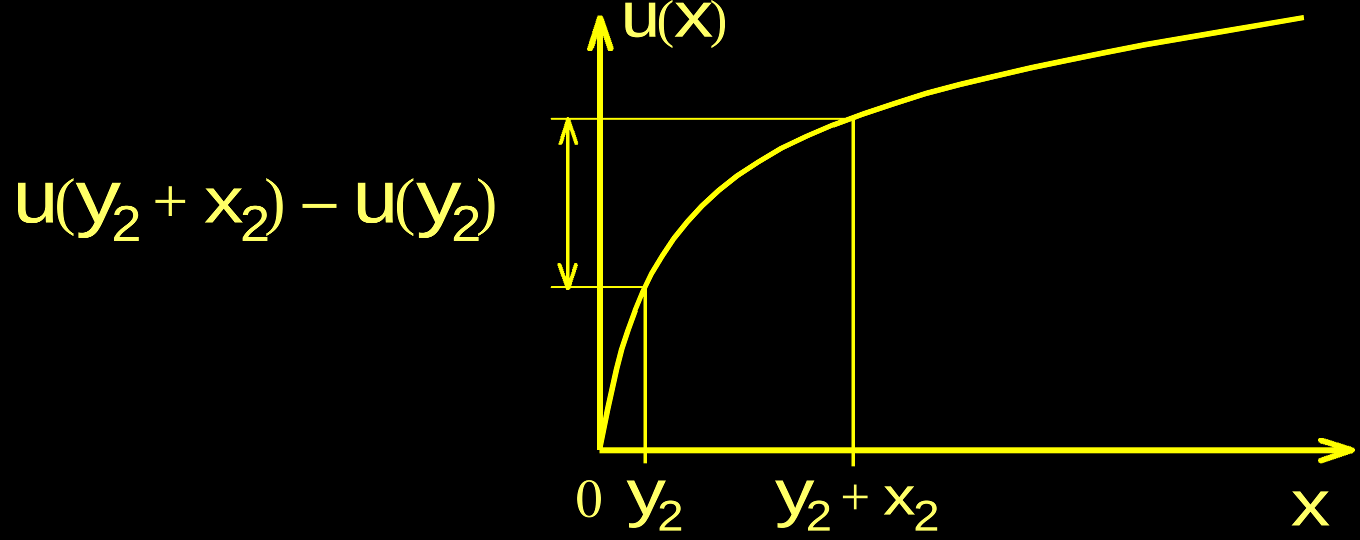
The consumption in each period is evaluated afresh, regardless of the previous consumption and the length of the time interval.

An immediate delay produces a utility jump!

A possible solution is to include *satiation:*

$$V(x_1, x_2) = u(x_1) + b\,[u(y_2 + x_2) - u(y_2)], \qquad (4)$$

where y_2 is the satiation level at period 2 due to previous consumption.



The utility for the second period is really incremental utility.

For example, the satiation level could be $y_2 = x_1 e^{-\sigma t}$, where σ is a parameter of decay and *t* is the time between period 1 and 2.

For *t* large, $y_2 \approx 0$, and

$$V(x_1, x_2) \approx u(x_1) + b\, u(x_2).$$

For *t* small, $y_2 \approx x_1$, $b \approx 1$, and

$$V(x_1, x_2) \approx u(x_1) + u(x_1 + x_2) - u(x_1) = u(x_1 + x_2).$$

Conclusions

For earnings or income streams in Multiattribute Analysis:

a. Do not discount utility

b. Do not discount CE's

c. Take Utility of NPV's

For consumption:

a. Consider the effect of satiation, which depends on the consumption item (for a vacation, the satiation from the previous vacation may last a year or more)

b. Our model of satiation is a tentative suggestion but further research is needed here.