# Risk Tolerances for Quasi Syndicates And Publicly Held Firms

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**Abstract**

A financial assets market context is used to present an extension of Wilson's (1968) classic risk tolerance aggregation result. We build upon work by Wilson and Lintner (1965, 1970), and define a *quasi syndicate* as a group of market investors who hold the shares of a risky asset. If investor preferences are modeled using exponential utility functions and percent variance is the measure of risk, we show that Wilson's aggregation result is extended to quasi syndicates when there is more than one portfolio in the financial market. We use this result in the context of firms, and introduce the idea of a firm risk tolerance that is identical to the quasi-syndicate risk tolerance. This work is motivated by the goal of formally establishing a basis for a market-based risk attitude for a publicly held firm. Examples are used to illustrate our points, and we compare our market-based analysis with observed and assessed risk tolerances for fifteen publicly traded energy firms.

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### 1. Overview

Traditional financial theory dictates that managers of publicly traded firms should be responsive only to market information about the price of risk, and otherwise should be risk neutral regarding unsystematic risks. However, empirical evidence regarding the risk attitudes of managers suggests they are often averse to unsystematic risks (MacCrimmon and Wehrung, 1986, Shapira, 1995).

One basis for the conclusion that publicly traded firms should behave in a risk neutral manner is the work of Wilson (1968). He defined a syndicate as a group of individuals who collectively invest in a risky asset, and showed that, if these investors act according to exponential utility functions, ownership shares (and therefore risks and rewards) should be directly proportional to investor risk tolerances. Wilson also showed that the risk tolerance for the syndicate as a whole is the sum of all member risk tolerances.

The notion of risk neutrality for publicly traded firms can be based on an extension of this result; that is, when all investors hold shares of a single portfolio (and therefore shares of the firm in question), the risk tolerance of the portfolio is the sum of all investor risk tolerances (Ekern and Wilson, 1974). As the number of investors increases, the sum of the risk tolerances increases as well. Lintner (1965, 1970) notes that as the number of investors increases in perfect capital markets, the market price of risk will approach zero in the limit, so that certainty equivalents become essentially equal to expected values and risk aversion effectively disappears as far as the market is concerned. However, this result does not address the issue of the risk tolerance for firms with publicly traded equities that are either closely held or have relatively few shareholders.

In this paper, we consider a simple modification of the assumptions underlying the original work by Lintner and Wilson. Rather than assume a single market syndicate (or, the existence of a single market investment portfolio), we allow many market investor *quasi syndicates*, which we define as subsets of investors who hold the shares of the same asset(s). We emphasize that this is a fundamental assumption of our development, which does not follow directly from our remaining assumptions that are otherwise traditional. This is equivalent to Levy's (1978) assumption that each investor sets an upper limit on the number of securities in each market portfolio.

Multiple market portfolios (and therefore multiple market quasi syndicates) do in fact exist for a variety of reasons, even when markets are efficient. Certainly, there are publicly traded firms that are widely held and there are publicly traded firms that have relatively few shareholders. Each publicly traded asset is held by a single quasi syndicate; market investors may belong to one or more quasi syndicates.

We explore the implications of this relaxation of the notion of a syndicate, and illustrate the formal development with a simple example. This leads to new conclusions regarding a world where investors do not hold shares of a market portfolio in proportion to their risk tolerances (as they do in a Wilson syndicate), but instead may hold portfolios that are not diversified and with a limited number of assets. In other words it is the real world, the world of quasi syndicates.

One interesting implication of this work is an argument regarding the existence of an implicit utility function for firms not widely held that might be quite different from a linear model (as implied by the case where the sum of investor risk tolerances is very large). This development may have significance as a basis for synthesizing alternate views of investment strategies, one associated with the field of decision analysis where risky decisions may be based on certainty equivalents determined by a utility function, and the other from finance where risky decisions are based on estimates of the market price of risk.

Luce and Raiffa (1957) discuss the idea of a firm utility function, and Spetzler (1968) describes the assessment and use of a firm utility function in the context of an overall firm risk policy. Howard (1986) uses empirical assessments of risk tolerances and Walls and Dyer (1996) use historical investment decision data to infer firm utility functions. In a recent paper, Smith (1999) derives bounds on corporate risk tolerances and certainty equivalents based on the preferences of the shareholders, which must be either assessed or approximated. However, none of these contributions have addressed the possibility of inferring a firm utility function from market data, which is a novel feature of our development.

## 2. Quasi Syndicates

A quasi syndicate exists any time more than one investor owns shares of a publicly traded risky asset. Investors may belong to as many quasi syndicates as they hold different risky assets; an investor is unlikely to know how many quasi syndicates count her as a member if she owns more than one publicly traded risky asset. The upper limit on the number of quasi syndicates in a financial market is the number of issues traded in that market. The lower limit is, of course, one.

### 2.1 A General Formulation

In order to explore quasi syndicates, we use the general formulation of the investor decision problem due to Lintner (1970) rather than the one proposed by Wilson (1968) because of its intuitive appeal and flexibility. There are i=1,..., I market investors assessing k=1,...,K risky investment opportunities in the context of the risk free rate $r_f$ who seek to maximize the expected utility of their portfolios by choosing optimal $V_{ik}$ asset allocation values subject to their respective budget constraints defined by $V_{P_i} = \sum_{k=1}^{K} V_{ik} + V_{if}$. $V_{if}$ is the amount allocated to the risk free asset by each investor. Risk free borrowing and lending opportunities at the same rate ($r_f$) are presumed to exist in the market.[1]

Investor preferences are modeled with exponential utility functions, so that the utility of any financial outcome *x* is expressed as $u_i(x) \sim \exp(-c_i x)\ \forall\ i$. We assume that all market investors are risk averse, so that the exponential utility function parameter $c_i > 0\ \forall\ i$. An investor's risk tolerance is the inverse of the parameter c. The market is sufficiently large so that no individual investor has the capability to influence market prices single-handedly.

Expected dollar returns for each investor i for each of the k assets are expressed as $V_{ik}(1+ER_{ik})$ and dollar variance is written $\tilde{d}^2_{ik} = V^2_{ik} d^2_{ik}$. $ER_{ik}$ is the expected percentage return for any investor and asset, and $d^2_{ik}$ is the percentage variance contribution of any asset k to any investor's portfolio. We assume markets are efficient, so $ER_{ik} = ER_k\ \forall\ i,k$. Individual $d^2_{ik}$ values will depend on the assets (and correlation matrices) comprising investor portfolios. We also make the standard assumptions that investors have access to information about the market asset percent covariance matrix (Lintner, 1965, 1970; Sharpe, 1970; Fama, 1976), and that all probability assessments are normally distributed.

The single-period investment problem may therefore be written as the maximization of the certainty equivalent of the portfolio, which we denote as $\hat{P}_i \big| V_{P_i}$ :

$$\underset{V_{ik}}{MAX}\ \hat{P}_i \Big| V_{P_i} = \left(\sum_{k=1}^{K} V_{ik}\left(1+ER_k\right) - .5\,c_i\,\tilde{s}^{\,2}_{P_i}\right)\left(1+r_f\right)^{-1} + V_{if}$$

$$\text{ST} \qquad \sum_{k=1}^{K} V_{ik} + V_{if} = V_{P_i}$$

(1)

In (1)[2], $\tilde{s}^{\,2}_{P_i}$ is the investor's dollar portfolio variance, which is written

$$\tilde{s}^{\,2}_{P_i} = \sum_{k=1}^{K}\left(V_{ik}^2 s_k^2 + 2\sum_{h>k} r_{hk} V_{ik} V_{ih} s_k s_h\right)$$

In the above expression, $s_k^2$ is asset k's specific percent risk, and $r_{hk}$ is the percent correlation between any two market assets h and k.

From the formulation above, we may write the expression (for all investors and assets) for the optimal $V_{ik}$ as

$$V_{ik} = \left(ER_k - r_f\right)c_i^{-1} s_k^{-2} - \sum_{h\neq k} r_{hk} V_{ih} s_h s_k^{-1} \qquad (2)$$

The result for optimal $V_{ik}$ s in (2) is identical to that determined by Lintner (1970). Throughout this analysis, we make the strong assumption that all investment portfolios are optimal given the number of risky assets the investor has selected. Thus, any nonzero $V_{ik}$ must be an optimal $V_{ik}$.

This formulation leads to the same results as Wilson's (1968) formulation for syndicates. Wilson notes that the "decision" taken by the syndicate must maximize the expected utility of all

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[1] This is a constraining factor. If borrowing is risky, optimal portfolios for those investors for whom $V_{P_i} < \sum_k V_{ik}\big| r_f$ will be different from optimal portfolios when $V_{P_i} < \sum_k V_{ik}\big| r_f$ but risk free borrowing is available ($V_{ik}\big| r_f$ denotes an optimal asset allocation given the risk free rate).

[2] If borrowing and lending opportunities are available at the same risk free rate, the constraint in (1) will be "loose". While each investor must choose $V_{P_i}$, optimal individual asset valuations are independent of $V_{P_i}$ s for all investors. If $V_{P_i} > \sum_k V_{ik}$, the investor will invest (lend) via the risk free asset. If $V_{P_i} < \sum_k V_{ik}$, the budget constraint is not binding if investors may borrow at the risk free rate and $V_{if} = \sum_k V_{ik} - V_{P_i}$. Although the budget constraint is not strictly binding, choice of $V_{P_i}$ determines the amount borrowed.

syndicate members. In Wilson's work, the syndicate makes a cooperative risky decision, and the output of the static collective maximization problem is an optimal "sharing rule" that is defined by member ownership shares. In this case, ownership shares in a market asset result from independent, static, single-period optimal solutions to the investor decision problem in (1).

## 2.2 A Single Market Syndicate Example

As an example, consider the case where four investors are assessing four risky market assets. The values of $c_i$ (inverse of the investor's risk tolerance) and $V_{P_i}$ (the amount each investor has elected to invest this period) for the four investors, are summarized in Table 1. Table 2 contains asset-specific risk ($s_k^2$) and return ($ER_k$) characteristics, as well as the interasset correlation matrix of the four available assets used in this example.

| | Investor 1 | Investor 2 | Investor 3 | Investor 4 |
|---|---|---|---|---|
| $c_i$ | 1/30,000 | 1/15,000 | 1/7,500 | 1/5,000 |
| $V_{P_i}$ | $40,000 | $25,000 | $10,000 | $15,000 |

*Table 1*. Four hypothetical investors, their risk attitudes and investment wealth.

| $r_{hk}$ | Stock 1 | Stock 2 | Stock 3 | Stock 4 | $ER_k$ | $s_k^2$ |
|---|---|---|---|---|---|---|
| Stock 1 | 1 | 0.20 | -0.05 | 0 | .12 | .1 |
| Stock 2 | 0.20 | 1 | -0.20 | 0.15 | .15 | .25 |
| Stock 3 | -0.05 | -0.20 | 1 | -0.10 | .08 | .15 |
| Stock 4 | 0 | 0.15 | -0.10 | 1 | .1 | .2 |

*Table 2*. Example stock returns, individual variances and correlation matrix.

The risk free rate used in this example is 5 per cent ($r_f = .05$). If all four investors hold shares of all four assets (that is, if there is a single market portfolio implying a single quasi syndicate), $V_{ik}$ s from (1) are shown in Table 3. In Table 3 (and all subsequent work), the m subscript denotes a total, as in a market-wide summation.

| | Investor 1 | Investor 2 | Investor 3 | Investor 4 | Total Values |
|---|---|---|---|---|---|
| $V_{i1}$ | $18345.78 | $9172.89 | $4586.44 | $3057.63 | $35162.74 |
| $V_{i2}$ | 10367.10 | 5183.55 | 2591.78 | 1727.85 | 19870.28 |
| $V_{i3}$ | 10192.93 | 5096.47 | 2548.23 | 1698.82 | 19536.45 |
| $V_{i4}$ | 6644.13 | 3322.07 | 1661.033 | 1107.36 | 12734.59 |
| $V_{if}$ | ($5,550) | $2,225 | ($1,387) | $7,408 | 2695.93 |
| $V_{P_i}$ / $V_m$ | $40,000.00 | $25,000.00 | $10,000.00 | $15,000.00 | $90,000.00 |
| | | | | | |
| $d_{i1}^2$ | .1145 | .1145 | .1145 | .1145 | .1145 |
| $d_{i2}^2$ | .2894 | .2894 | .2894 | .2894 | .2894 |
| $d_{i3}^2$ | .0883 | .0883 | .0883 | .0883 | .0883 |
| $d_{i4}^2$ | .2258 | .2258 | .2258 | .2258 | .2258 |

*Table 3*. Risky asset allocations and portfolio variance contributions in the single syndicate case.

Note that, as with a Wilson syndicate, the single portfolio case results in $V_{ik}$ values for each k that are proportional to investor risk tolerances. This may be seen by noting that $V_{2k}$ = $.5V_{1k}$ for k=1,2,3,4 (Investor 2's risk tolerance is $15,000 and Investor 1's is $30,000), and that the same direct proportionality between investor asset allocations and risk tolerances is true for all four investors. Borrowing and lending levels, determined by the magnitude of $V_{P_i}$ vis-à-vis risky asset allocations, may be seen in the $V_{if}$ row. Note that $V_{2f} > 0$ and $V_{4f} > 0$, while Investors 1 and 3 borrow to achieve optimal investment levels in the four risky assets.

In order to specify syndicate (and quasi-syndicate) risk tolerances, information about investor portfolio variances that are attributable to individual assets is needed. In effect, individual investor portfolio asset variances are aggregated across investors (quasi syndicate members) to determine market variances for all assets. From the formulation in (1), the dollar variance contribution for each investor and asset for which $V_{ik} \neq 0$ is

$$\tilde{d}_{ik}^{\,2} = V_{ik}^2 s_k^{\,2} + \sum_{h \neq k} r_{hk} V_{ih} V_{ik} s_h s_k \tag{3}$$

Percent contributed variance is, as noted above, $d_{ik}^2 = \tilde{d}_{ik}^{\,2} V_{ik}^{-2}$. In this single syndicate example, $d_{ik}^2$ values associated with the solution to the optimization problem are shown in Table 3.

Expression (3) is obtained by multiplication of both sides of the first order optimization conditions from (1) by $V_{ik}$ to obtain a rearrangement of (2) as

$$V_{ik} = \frac{V_{ik}\left(1 + ER_k\right) - c_i\left(V_{ik}^2 s_k^{\,2} + \sum_{h \neq k} r_{hk} V_{ih} V_{ik} s_h s_k\right)}{1 + r_f}$$

The above may be rewritten in dollar terms as

$$V_{ik} = \left(\overline{X}_{ik} - c_i \tilde{d}_{ik}^{\,2}\right)\left(1 + r_f\right)^{-1} \tag{4}$$

where $\overline{X}_{ik}$ is the expected subsequent period dollar return. This expression for an optimal asset allocation $V_{ik}$ differs from the well known exponential utility function/normal distribution certainty equivalent expression by the discounted certainty equivalent risk premium, $.5 c_i \tilde{d}_{ik}^{\,2}\left(1 + r_f\right)^{-1}$.

The key to the aggregation of investor risk tolerances is the specification of the market value of a publicly traded firm as a function of the sum of investor risk tolerances regardless of the number of market quasi syndicates. The form of this expression is the same as the expression for individual optimal asset valuations in (4).

We begin with (2) for any $V_{ik}$ and sum over all investors in the kth asset $\left(i \in I_k\right)$, use the idea that the market value of the firm $V_k = \sum_{i \in I_k} V_{ik}$ and multiply both sides of the equality by $V_k$ to obtain

$$V_k\,(ER_k - r_f)\sum_{i \in I_k} c_i^{-1} = V_k^2 s_k^{\,2} + V_k \sum_{i \in I_k} \sum_{h \neq k} r_{hk} V_{ih} s_h s_k \tag{5}$$

The right hand side of (5) is firm k's contribution to the total dollar market variance, $\tilde{s}_{km}^{\,2}$. The firm contribution to percent market variance $s_{km}^{\,2} = \tilde{s}_{km}^{\,2} V_k^{-2}$.

We may use the fact that $V_k\left(ER_k - r_f\right) = V_k\left(1 + ER_k\right) - V_k\left(1 + r_f\right)$ to write

$$V_k = \left[ V_k \left(1 + ER_k\right) - \left( \sum_{i \in I_k} c_i^{-1} \right)^{-1} \tilde{s}_{km}^{2} \right] \left(1 + r_f\right)^{-1} \qquad (6)$$

In (6), the summation over all $i \in I_k$ signifies the fact that risk tolerance summation is over all quasi-syndicate investors.

From (6), we define the syndicate (or quasi-syndicate) exponential utility function parameter as

$$c_k = \left( \sum_{i \in I_k} c_i^{-1} \right)^{-1} \qquad (7)$$

which (as we show in an example below) is true regardless of the number of portfolios (or quasi-syndicates) in the market. In this single syndicate base case, the risk tolerance for the quasi syndicate $c_k = 1.7391 x 10^{-5} \ \forall \ k$ (=1/57500), which is the well-known single portfolio result.

We also need to link the aggregate optimal allocation to each stock with quasi-syndicate risk tolerances and each firm's contribution to the total market variance. From (6) and (7), the market value of the firm may be written in terms of $s_{km}^{2}$ and the aggregate of quasi-syndicate risk tolerances as

$$V_k = \left(ER_k - r_f\right) s_{km}^{-2} c_k^{-1} \qquad (8)$$

In the next subsection, we present an example illustrating the extension of Lintner's and Wilson's result that is summarized above in expressions (6) - (8).

### 2.3 Example with More than One Market Quasi Syndicate

Instead of investing in all four assets, suppose that each of the four investors in the example in the preceding section invests in only three of the four market assets. If independently acting investors with a static, single period horizon allocate optimal amounts to each asset in which they invest, the risk tolerance of the quasi syndicate is expressed as the sum of all shareholder risk tolerances.

Suppose that there are two portfolios in the market, as shown below. Investors 1 and 2 hold a portfolio that does not include Stock 4, and Investors 3 and 4 hold a portfolio that does not include Stock 1. In this case, Investors 1 and 2 will each solve (1) subject to the additional constraint $V_{i4} = 0, i = 1,2$. Investors 3 and 4 will each solve (1) subject to the additional constraint $V_{i1} = 0, i = 3,4$.

Given the investor and asset parameters from the previous subsection, Table 4 (which is analogous to Table 3) contains optimal $V_{ik}$ values and asset portfolio variance contributions for each investor. Note that as the assets contained in a portfolio vary, optimal allocations (when compared with the single portfolio base case shown above) will vary if asset returns are correlated. Also, $V_{ik}$ values for each k are no longer directly proportional to investor risk tolerances.

| | Investor 1 | Investor 2 | Investor 3 | Investor 4 | Total Values |
|---|---|---|---|---|---|
| $V_{i1}$ | $18,045.07 | $9,022.50 | | | $27,067.57 |
| $V_{i2}$ | 11,209.60 | 5,604.74 | $3,178.26 | $2,118.85 | 22,111.46 |
| $V_{i3}$ | 9,631.00 | 4,815.48 | 2,500.61 | 1,667.05 | 18,614.14 |
| $V_{i4}$ | | | 1,558.52 | 1,039.02 | 2,597.54 |
| $V_{if}$ | 1,114.33 | 5,557.27 | 2,762.60 | 10,175.07 | 19,609.28 |
| $V_{P_i} / V_m$ | $40,000.00 | $25,000.00 | $10,000.00 | $15,000.00 | $90,000.00 |
| | | | | | |
| $d_{i1}^2$ | .116376 | .116376 | | | |
| $d_{i2}^2$ | .267630 | .267630 | .235975 | .235975 | |
| $d_{i3}^2$ | .093448 | .093448 | .089979 | .089779 | |
| $d_{i4}^2$ | | | .240609 | .240609 | |

*Table 4*. Risky asset allocations and portfolio variance contributions in the three quasi syndicates case.

This case is an example of three quasi syndicates, representing ownership of stock 1 (quasi syndicate 1), stocks 2 and 3 (quasi syndicate 2), and stock 4 (quasi syndicate 3). Thus, we will have three quasi-syndicate risk tolerances, and each investor will belong to two quasi syndicates (Investors 1 and 2 in quasi syndicates 1 and 2, Investors 3 and 4 in quasi syndicates 2 and 3).

Percent contributed variances for this example ($d_{ik}^2$) are shown in Table 4 for all four investors and assets. Note that, while $d_{12}^2 = d_{22}^2$ and $d_{32}^2 = d_{42}^2$, $d_{12}^2 \neq d_{32}^2$ because stocks 2 and 3 are held in different portfolios. This may be seen from (3) and the fact that $d_{ik}^2 = \tilde{d}_{ik}^2 V_{ik}^{-2}$. In the single syndicate case, asset allocations for Investor 1 and Investor 3 were proportional to each investor's risk tolerance. More specifically, $V_{1k} = 4V_{3k}\ \forall\ k$ was true. In this multi-market portfolio context, note that the single portfolio relationship between Investor 1's and Investor 3's valuations of their portfolio assets does not hold. Thus, $d_{1k}^2 \neq d_{3k}^2$ for k=2,3.

Within quasi syndicates, $d_{ik}^2$ values are identical for all investors (and assets) only if members of the quasi syndicate associated with the asset (k) hold the same portfolio. Otherwise, if assets are correlated there is no reason to expect $d_{ik}^2$ values to be constant within quasi syndicates. However, the contribution of each firm k to the percent market variance (see (5)), may be expressed as a weighted average using quasi-syndicate member ownership shares.

Contributions to percent market variances and aggregations of quasi-syndicate investor risk tolerances are shown below in Table 5 for each of the four assets in this three quasi-syndicate example. Note that the risk tolerance for quasi syndicate 1 is 45000 ($= c_1^{-1} + c_2^{-1}$) and so on. Also, the relation in (8) may be verified using this example. Thus,

$V_2 = (.1)(57500)/(.260047) = 22111.39$[3].

| Quasi Syndicate | 1 | 2 | | 3 |
|---|---|---|---|---|
| Stock | 1 | 2 | 3 | 4 |
| $s_{km}^2$ | .116376 | .260047 | .092672 | .240609 |
| $\sum_{i \in I_k} c_i^{-1}$ | 45000 | 57500 | 57500 | 12500 |

*Table 5*. Contributions to percent market variances and investor risk tolerance aggregations across quasi syndicates.

## 2.4 Discussion

The above example illustrates how the variation of Lintner's formulation that we use results in optimal asset allocations within quasi syndicates that provide a generalization of Wilson's results for syndicates. This generalization is most obvious in the aggregation of quasi-syndicate investor risk tolerances. The optimal portfolio allocations also result in sharing rules and optimal risk allocations that are either the same or a generalization of Wilson's presentation.

Wilson suggests that a sharing rule, the means by which syndicate shares are divided among members, should be Pareto optimal and should not depend on the payoff or "prize" at stake. The rule should only depend on the agreed-upon return distribution and the allocation of weights, or investment shares. Our arrangement, in the presence of optimal investor portfolios, associated equilibrium share prices, and risk free borrowing/lending opportunities is Pareto optimal. At equilibrium, and prior to the arrival of any new information, it is not only impossible to make any individual investor better off without making another investor worse off, it is also impossible to make any investor better off because an increase in any $V_{ik}$ will result in a decrease of $\hat{P}_i \big| V_{P_i}$ .

The sharing arrangement, as denoted by individual $V_{ik}$ s and $V_{ik} \; / \; V_k$ ratios, is linear and dependent on the agreed upon return distribution in the context of individual investor portfolios. Wilson's requirement that shares determine the allocation among firm investors of outcomes (in this case, the results of managerial decisions and other factors during the investment period reflected in expected percentage returns and variances at the end of the period and any firm dividends) is met in our case. Further, optimal $V_{ik}$ s remove the need for investors to place side bets in order to aggregate quasi-syndicate risk tolerances.

In the Ekern/Wilson (1974) paper, the connection between the group decision and risk tolerance is made via the argument that optimal risk sharing through a market equilibrium based sharing rule leads to the conclusion that all firms should have the same risk attitude (a function of the sum of investor risk tolerances) in the single market portfolio case. The optimal risk sharing argument is summarized in Wilson's Theorem 4, and the accompanying statement that the syndicate risk tolerance is the sum of investor risk tolerances "... means that a compensating risk

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[3] The difference between 22111.39 and 22111.46 - the market value for asset 2 in Table 4 - is due to rounding. We ask the reader to note that, as one works through the calculations in the examples we present, rounding errors are likely to occur.

premium for an infinitesimal risk is distributed among the (syndicate) members in proportion to the variance each undertakes to absorb."  This is the crux of our argument for our more general result.  In our equilibrium valuation model, firm specific risk premiums for each investor are proportional to the dollar amount invested in the firm by the investor, viz.

$$V_{ik}\ V_{jk}^{-1} = c_i \tilde{d}_{ik}^{\,2}\ c_j^{-1} \tilde{d}_{jk}^{\,-2}\ \forall\ i, j \neq i \in I_k \tag{9}$$

The relationship in (9) may be seen from equation (4) in the case of any two firm investors.  In addition, investor ownership shares are directly proportional to investor shares of the total quasi-syndicate risk premium, viz.

$$V_{ik}\ V_k^{-1} = c_i\ \tilde{d}_{ik}^{\,2} \left( \sum_{i \in I_k} c_i\ \tilde{d}_{ik}^{\,2} \right)^{-1} \tag{10}$$

The main departure between our multi-portfolio, quasi-syndicate formulation and Wilson's 1968 discussion is, as noted in the example above, with respect to the syndicate result that investor valuations are directly proportional to risk tolerances.  However, since individual ownership share valuations are proportional to absorbed risks, we argue that a Wilson syndicate is a special case of our more general result.  If investors have a market mechanism to allocate shares in a syndicate and if equilibrium prices for those shares exist, the solution to our problem will be the same as in the case he presents, provided risk free borrowing and lending opportunities exist.

## 3. A Market-based Approach to the Estimation of Firm Risk Tolerances

All firms with publicly held (and traded) ownership shares are owned by a quasi syndicate.  Regardless of the number of investors in the quasi syndicate, the quasi-syndicate risk tolerance is the sum of all investor risk tolerances.  Managers in publicly held firms who are interested in fulfilling their obligations in a principal-agent context should be interested in estimating the sum of the quasi-syndicate investor risk tolerances.  We discuss a method for doing just that in this section.

### 3.1 Estimation of Firm Risk Tolerances

Numerous authors have noted that the estimation of the risk attitude of a firm based on efforts to assess the risk attitudes of shareholders is impractical, and we agree.  Smith (1999) has shown that bounds on corporate risk tolerances can be derived based on estimates of shareholder risk tolerances, although these bounds may be relatively wide, and not directly related to market information.

However, note that the expression for the market value of the firm (8) may be rewritten in the form

$$c_k^{-1} = V_k \, s_{km}^{\,2} \left(ER_k - r_f\right)^{-1} \tag{11}$$

Presumably, the market value of the firm's equity $V_k$ and current risk free rate $r_f$ are easily observed. Observing ex ante risks and returns is another matter. For the purpose of estimating the firm risk tolerance, $c_k^{-1}$, the use of a moving ex post data window to estimate ex ante problem parameters is a reasonable approach, and one that can provide insights regarding implications for the past as well as for the future. This is also the general approach taken in much of the finance and economics literature.

For our market-based risk tolerance estimations, we began with a sixty-month moving return variance for the month z estimate $\hat{s}_{kz}^{\,2}$ of a firm’s a-priori variance. Expected monthly returns were calculated using the tilting method (Luenberger, 1998). The tilted estimate of expected returns in month z, $ER_{kz}$, was found using a combination of a mean return from the sixty preceding months and the Capital Asset Pricing Model (CAPM) – or market model - expected return estimate for that month. As we shall discuss, the estimates of $\hat{s}_{kmz}^{\,2}$ were made based on numerical simulations using randomly generated a priori mean, variance and correlation data. The period studied was January 1983 through December 1995.

**3.2 Risk Tolerances and Market Values for Fifteen Publicly Traded Energy Firms**

In order to explore the implications of the use of (11) to estimate risk tolerances for publicly-held firms, we selected a group of fifteen energy firms that were also the subject of an empirical study on risk tolerances by Walls and Dyer (1996). Although we have suggested that our approach to the estimation of firm risk tolerances may be most appropriate for smaller, more closely held firms, we are not aware of any similar empirical studies of risk tolerances for such firms that could serve as a benchmark for comparison.

Daily return data for the firms were obtained from the CRSP database for the period 1 January 1977 through 29 December 1995. Dividends and stock splits are included in CRSP daily stock return figures. Daily secondary market return data for 3-Month United States Treasury Bills were obtained from the Federal Reserve Bank of St. Louis (www.stls.frb.org/fred/).

Risk free rate ($r_f$) data were calculated from 1 January 1978 using 3-Month Treasury Bond Equivalent Yields (BEY) converted to one month return figures. The converted BEY yield for the last trading day of each month was used for $r_f$ in each computation. Market values were computed for each of the end-of-month trading days as the product of the closing market stock price and the number of common shares outstanding.

Historical moving average monthly returns $ER_{kz}^h$ were computed for a given month z from the returns for the sixty preceding months. The superscript in $ER_{kz}^h$ denotes the expected historical-data based return for firm k in period z. The percent return market variance estimate $\hat{s}_{kz}^2$ was computed from the same 60 months of data. Sixty month moving average CRSP value-weighted market portfolio returns ($ER_{mz}$) were used along with $ER_{kz}^h$ and periodic $r_{fz}$ values to estimate monthly betas ($b_{kz}$) using the Index Model method (Bodie, Kane and Marcus, 1996). These monthly betas were then used to calculate beta-based monthly expected returns and standard errors using the market model

$$ER_{kz}^m = r_{fz} + b_{kz}\left(ER_{mz} - r_{fz}\right)$$

The tilted expected return for each firm for each month in the study period was then calculated as

$$ER_{kz} = \left(\frac{ER_{kz}^h}{s_{hz}^2} + \frac{ER_{kz}^m}{s_{mz}^2}\right)\left(\frac{1}{s_{hz}^2} + \frac{1}{s_{mz}^2}\right)^{-1} \qquad (12)$$

In (12), $s_{hz}^2 = \hat{s}_{kz}^2 / 60$ and $s_{mz}^2 = \hat{s}_{mz}^2 / 60$ are squared standard errors of historical and market model $ER_{kz}$ estimates, respectively. Tilting essentially results in a smoothing of historical 60-month moving average data, and tilted estimates tend to be somewhat more volatile over time than market model estimates of $ER_{kz}^m$. Tilted $ER_{kz}^m$ and $\hat{s}_{kz}$ estimates, as well as $r_{fz}$ figures, trend toward smaller values over the time frame we considered.

The challenge in using (11) is to estimate the values of the firm contributions to percent market variance on a monthly basis, $\hat{s}^2_{kmz}$, since they are not directly observable. Therefore, we explored the relationship between $\hat{s}^2_{kmz}$ and $\hat{s}^2_{kz}$ as suggested by (5) using numerical simulations with randomly-generated a priori mean, variance and correlation data. Interasset correlations were restricted to the range $-.1 \leq r_{hk} \leq .4$. The number of investors in each simulated month was randomly varied, subject to the restriction that are many more (> 500 times as many) investors than market assets. Individual investor risk tolerances were also randomly varied, and were generally restricted to the range [1000, 41000], though the upper bound of this range was allowed to go as high as 4 million in some cases. The probability that any individual investor would choose to hold any asset was varied (sometimes randomly) from .05 to 1. The distribution and confidence intervals reported here are based on the proability of an investor choosing to hold any asset varying between .05 and .5. In other words, less than half of the total (we assumed many - at least 500 times - more investors than assets) market investors hold any single asset. We believe that these assumptions are plausible as a basis for this modeling effort.

Using data from the same 60 month period identified above, our simulation results indicate that the 95% confidence interval for $\hat{s}^2_{km}$ is [$\hat{s}^2_k$, 1.488$\hat{s}^2_k$]. We modeled $\hat{s}^2_{km}$ using a gamma distribution[4], and estimated distribution parameters using $\Pr\left(\hat{s}^2_{km} \leq \hat{s}^2_k = .025\right)$ and $\Pr\left(\hat{s}^2_{km} \leq 1.488\hat{s}^2_k = .975\right)$, where Pr denotes probability. Once we obtained the two gamma distribution parameters $a$ and $b$ (there will be a unique parameter set for each asset in each period), we generated the expected $\hat{s}^2_{km}$ as $E\left(\hat{s}^2_{km}\right) = ab$. This leads to an expected market-based firm risk tolerance whose confidence limits will be a function of the confidence interval for $\hat{s}^2_{km}$.

Once expected $\hat{s}^2_{km}$ values were estimated, the relation in (11) was used to calculate monthly market-based expected risk tolerances for each of the fifteen firms for the period 1/83 through 12/95. Figures 1-4 contain monthly expected risk tolerance estimates for the fifteen firms considered. One outlier, the observation for Pennzoil for 30 November 1992, was omitted from the data. This outlier was the only negative risk tolerance observation.

PLEASE INSERT FIGURES 1-4 ABOUT HERE

The fifteen firms of Figures 1-4 exhibit very similar implied risk tolerance patterns over the period 1/83 through 12/95. Most estimates exhibit volatility over time, though the risk tolerance trends track each other, which might be expected since they are from the same industry. Only 22 out of 105 pairwise linear correlations across firms are less than 0.5, and only one is negative. Major changes in market-based expected risk tolerances happen at about the same time for all firms.

Some monthly firm risk tolerances appear to be centered over the study period, and this is the case for five of the fifteen firms. Simple regression analysis was used to assess the significance and magnitude of the slope of a fitted regression line for each firm's expected risk tolerance over the monthly time frame we considered. Ten of the firms have highly significant, positive linear coefficients associated with their market-based expected risk tolerances over the time frame considered. One, Mobil, is significant at the 90 per cent level. Two, Union Pacific and Texaco, have non-significant negative coefficients, and Phillips and Kerr McGee have relatively small and non-significant positive coefficients. Overall, these firm risk tolerances show an increasing trend from January 1983 through December 1995.

The largest expected market-based risk tolerances over the study period are on average associated with Exxon. Exxon has the largest monthly market-based risk tolerance estimate in 107 of the 156 months. Amoco has the largest estimate for the other 49 months, mostly from earlier in the time frame considered. The smallest on average belong to Consolidated (72 months) and Kerr McGee (84 months). The firm with the largest market value over the study period was Exxon; the smallest were Consolidated, Pennzoil and Kerr McGee. Common stock market value is a variable used to compute risk tolerance, and the total, across-firm Pearson correlation between risk tolerances and market values from 1/84 through 12/95 is .73.[5]

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[4] Distribution fits on our simulation data indicated a relatively high probability (around .7) that a given data set was well modeled with a gamma distribution. Also the distribution of interest is a truncated chi square distribution, which supports the use of a gamma distribution.

[5] The total across firm Pearson correlation between monthly risk tolerance and market value ranks (15 ranks for each of 156 months) is .89, and the Pearson correlation across firms for all thirteen years of monthly data (2340 total ranks) is .85. For each set of monthly observations, Spearman's Rho was calculated for the rank

For the larger firms on the basis of market value (e.g., Exxon, Amoco), the market-based expected risk tolerance estimates are on the order of one to three billion dollars, which would imply essentially risk neutral behavior for typical prospects considered in exploration and production decisions.[6] However, for smaller firms in this study (e.g., Consolidated, Kerr McGee), we obtained risk tolerance estimates on the order of $200 to $300 million, which could lead to certainty equivalent estimates for some of these same prospects significantly different from expected values.

### 3.3 Comparison with Empirically Observed Risk Tolerances

Walls and Dyer (1996) derived the risk tolerances for the exploration and production (E&P) divisions of the fifteen firms in our sample. These estimates were determined by reconstructing lotteries that corresponded to the exploration plays considered by each of the firms in each year, and treating the budgets allocated to these plays as their certainty equivalents. It should be emphasized that the Walls/Dyer estimates were based on observed resource allocation decisions, and were not intended to represent normative results. Therefore, we refer to these estimates as observed risk tolerances.

Table 6 contains a comparison of these figures for 1984-1990. The comparison is simply the market-based expected risk tolerance we calculated (the average of the 12 monthly observations for each year) divided by the Walls/Dyer observed risk tolerance for each of the fifteen firms. Missing data in Table 6 correspond to missing observations in the Walls/Dyer paper.

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correlation between risk tolerances and market values. The range of these correlations is rather small, .74 to .99, and the average monthly Spearman rank correlation between market-based risk tolerances and common stock market values is .89.

[6] Clyman, Walls, and Dyer (1998) report that oil and gas exploration firms often evaluate large drilling prospects with simple, two-state models. The actual expected payoffs and costs associated with a sample of prospects under consideration by a major oil company ranged from $6 million to $315 million.

| | Exxon | Chevrn | Texaco | Mobil | Amoco | ARCO | AmHes | Enron | Phillips | Pennz | Occider | Unocal | Consol | UnionP | KerrM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1983 | 85.57 | 8.19 | 80.06 | 189.78 | 66.06 | 30.53 | 13.36 | 55.38 | 25.65 | 91.22 | 5.43 | | 16.15 | 34.27 | |
| 1984 | 152.8 | 186.85 | 85.14 | 322.17 | 86.13 | 74.25 | 35.19 | 113.14 | 66.62 | 146.99 | 14.65 | | 37 | 51.6 | 64.48 |
| 1985 | 152.73 | 134.6 | 126.5 | 310.28 | 199.71 | 76.45 | 36.71 | 60.27 | 32.53 | 145.27 | 14.91 | | 36.54 | 71.66 | 11.2 |
| 1986 | 107.55 | 107.66 | 86.33 | 167.44 | 166.55 | 36.1 | 30.25 | 136.62 | 13.22 | | 21.89 | | 19.53 | 36.7 | 14.93 |
| 1987 | 107.68 | 84.38 | 88.69 | 122.77 | 55.69 | 39.91 | 40.61 | 112.05 | 10.17 | | 20.98 | 19.54 | 25.92 | | 16.75 |
| 1988 | 139.12 | 117.34 | 207.89 | 150.65 | 64.92 | 75.94 | 68.21 | 173.8 | 19.77 | 163.13 | 37.37 | 35.33 | 176.89 | | |
| 1989 | 136.62 | 79.36 | 6.76 | 125.61 | 127.92 | 60.42 | 25.15 | 266.14 | 33.86 | 400.71 | 32.11 | 29.58 | 182.51 | 44.71 | 13.94 |
| 1990 | 134.72 | 141.36 | 95.84 | 27.41 | 391.42 | 68.04 | 1.36 | 262.71 | 47.52 | 271.17 | 40.15 | 27.72 | 237.6 | 45.68 | 23.1 |

*Table 6*. Expected Market-Based/Observed Risk Tolerances for 15 Publicly Traded Energy Firms

With one exception, every market-based risk tolerance is considerably larger than every observed risk tolerance, sometimes by orders of magnitude. One explanation for this result would be that the Walls/Dyer methodology leads to risk tolerance estimates that are much lower than those that are used, perhaps implicitly, by E&P managers of these firms. An alternative explanation for these differences is that the managers of these firms were much too risk averse with regard to the allocations to their E&P budgets during the study period relative to what their investors would prefer.

We also should emphasize that the Walls/Dyer risk tolerance estimates were established for the E&P divisions of these firms, which typically represent only a fraction of the scope of the operations of a large integrated multi-national firm such as Shell or Exxon. Although we are not aware of any normative explanation for the phenomenon, we have observed through our consulting experiences that the risk tolerances of managers typically decrease as their operating budgets decrease. Therefore, it may be more appropriate to investigate the correlations between these estimates of risk tolerance rather than their absolute numerical ratios.

The Pearson correlation between the across firm aggregate of Walls/Dyer observed and the Anselmo/Dyer market-based risk tolerances is 0.07; and the Pearson rank correlation across firms is 0.36. Although the observed risk tolerances are much smaller than the market-based risk tolerances for these fifteen firms, the respective risk tolerance rankings appear to be somewhat related. This result is not entirely surprising, since both sets of estimates are correlated with measures of firm size (Walls/Dyer used a measure of the firm's oil and gas reserves as a proxy for size rather than market value). The overall, seven year rank correlation between the Walls/Dyer risk tolerances and market values is 0.34.

### 3.4 A Comparison with a Commonly-Used Rule-of-Thumb

Howard (1988) provides some rules-of-thumb for firm risk tolerances based on his practical experience as a consultant. Specifically, he noted that a reasonable estimate of the risk tolerance for a publicly traded firm would be approximately one-fifth of its market value. His observations seem particularly appropriate for a comparison since they were based on assessments of risk attitudes from managers in the oil and chemicals industry. Therefore, we refer to these estimates as assessed risk tolerances.

Table 7 contains a comparison of these figures for 1983-1995. As before, the comparison is simply the annual average market-based risk tolerance we calculated divided by 20 per cent of the annual average market value for each of the fifteen firms based on Howard's rule-of-thumb.

| | Exxon | Chevron | Texaco | Mobil | Amoco | ARCO | AmHess | Enron | Phillips | Pennzoil | Occiden | Unocal | Consol | UnionP | KerMcG |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1983 | 2.76 | 5.14 | 4.54 | 4.99 | 6.32 | 4.65 | 5.68 | 5.87 | 5.55 | 5.82 | 5.23 | 5.94 | 4.33 | 5.55 | 5.5 |
| 1984 | 3.98 | 8.27 | 7.57 | 8.39 | 9.39 | 7.84 | 8.78 | 8.22 | 10.65 | 9.56 | 6.67 | 9.61 | 5.96 | 10.22 | 10.51 |
| 1985 | 3.07 | 7.39 | 7.52 | 6.82 | 7.83 | 7 | 9.15 | 8.11 | 8.66 | 8.13 | 5.96 | 8.67 | 5.63 | 8.46 | 9.4 |
| 1986 | 2.01 | 4.37 | 6.93 | 3.94 | 4.71 | 5.18 | 7.59 | 5.9 | 6.03 | 6.75 | 5.64 | 7.38 | 3.66 | 4.84 | 6.45 |
| 1987 | 1.43 | 2.7 | 5.82 | 2.63 | 2.46 | 3.62 | 5.55 | 4.22 | 4.16 | 6.47 | 4.07 | 4.92 | 2.31 | 2.5 | 4.28 |
| 1988 | 2.34 | 6.29 | 9.99 | 5.28 | 4.24 | 6.47 | 12.88 | 9.05 | 8.18 | 23.16 | 7.73 | 9.02 | 3.69 | 7.03 | 8.96 |
| 1989 | 2.23 | 4.8 | 7.18 | 3.75 | 3.52 | 5.27 | 8.44 | 7.42 | 6.2 | 18.85 | 6.2 | 6.73 | 3.25 | 4.4 | 6.1 |
| 1990 | 2.76 | 4.99 | 6.95 | 3.39 | 3.78 | 5.29 | 8.66 | 7.07 | 6.77 | 21.5 | 9.24 | 7.66 | 3.92 | 5.02 | 6.46 |
| 1991 | 2.45 | 4.82 | 4.4 | 3.12 | 3.42 | 4.32 | 6.34 | 5.64 | 5.44 | 19.79 | 7.98 | 6.47 | 5.43 | 4.5 | 6.04 |
| 1992 | 2.38 | 4.98 | 3.65 | 3.32 | 3.77 | 4.34 | 6.84 | 4.72 | 5.35 | 20.67 | 9.01 | 6.52 | 7.45 | 4.07 | 5.74 |
| 1993 | 1.47 | 3.26 | 2.46 | 1.69 | 3.18 | 3.71 | 6.65 | 5.63 | 3.21 | 6.51 | 3.97 | 4.74 | 4.3 | 1.82 | 3.62 |
| 1994 | 2.21 | 4.72 | 3.37 | 2.64 | 6.02 | 6.64 | 10.79 | 9.15 | 5.46 | 15.6 | 9.32 | 8.74 | 8.97 | 3.26 | 6.69 |
| 1995 | 1.98 | 3.58 | 3.3 | 2.52 | 5.58 | 6.05 | 8.86 | 10.54 | 4.72 | 14.86 | 7.56 | 8.2 | 8.89 | 3.23 | 5.84 |

*Table 7*. Expected Market-Based/Assessed Risk Tolerances for Fifteen Publicly Traded Energy Firms. Annual Average Ratios, 1983-1995.

Once again, every market-based risk tolerance is larger than every assessed risk tolerance, but generally by a smaller amount than was the case for observed risk tolerances. These results also suggest that the assessed risk tolerances that Howard has obtained are lower than those implied from market data as representing the preferences of the stockholders. The correlation between the across firm aggregate of Howard assessed and the market-based risk tolerances is, of course, identical to the correlation between the annual average market-based risk tolerances and the market values reported above.

### 3.5 The Relationship between Risk Tolerance and the Number of Shares and Shareholders

According to 1995 Annual Reports, the numbers of shareholders and shares outstanding in the fifteen energy firms in our sample varied widely (Walker's Western Research, 1996). Table 8 contains the 12/95 market-based expected risk tolerance, market value, number of shares outstanding, and the number of shareholders.

| | Exxon | Chevrn | Texaco | Mobil | Amoco | ARCO | Enron | Phillips | Pennz | Occidt | Unocal | Consol | KerrM | AmHess |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12/95 RT | 2E+09 | 1E+09 | 6E+08 | 1E+09 | 2E+09 | 9E+08 | 3E+08 | 1E+09 | 5E+08 | 2E+08 | 5E+08 | 5E+08 | 3E+08 | 5E+08 |
| 12/95 Val | 1E+10 | 3E+09 | 2E+09 | 4E+09 | 4E+09 | 2E+09 | 5E+08 | 1E+09 | 9E+08 | 2E+08 | 7E+08 | 7E+08 | 4E+08 | 1E+09 |
| Shares | 1E+09 | 7E+08 | 3E+08 | 4E+08 | 5E+08 | 2E+08 | 3E+08 | 3E+08 | 5E+07 | 3E+08 | 2E+08 | 9E+07 | 5E+07 | 9E+07 |
| Holders | 608000 | 141000 | 199998 | 270400 | 134776 | 100000 | 26775 | 69537 | 20025 | 361000 | 35475 | 40828 | NA | NA |
| Avg RT | 4059.6 | 9334.3 | 3248.8 | 4562.3 | 11737 | 8992.3 | 12287 | 14596 | 24436 | 677.13 | 14101 | 11353 | NA | NA |
| ExpMSD | 0.0377 | 0.0513 | 0.0468 | 0.0455 | 0.047 | 0.0545 | 0.07 | 0.0742 | 0.0684 | 0.0681 | 0.0735 | 0.068 | 0.0614 | 0.0632 |

*Table 8*. 12/95 Expected Risk Tolerance (RT), Market Value (Val), Shares of Common Stock Outstanding (Shares), Number of Shareholders (Holders), Average Shareholder Risk Tolerances (Avg RT), and 12/95 Expected Contributions to Percent Market Standard Deviations (ExpMSD) for 12 of the 15 Energy Firms.

As might be expected, firms with higher market values also tend to have more shares outstanding and larger numbers of shareholders. The correlation between 12/95 risk tolerances and the number of shares outstanding is 0.88, and the correlation between risk tolerances and the number of shareholders is 0.64.

We also used the number of shareholders to compute an average (mean) market-based estimate of the shareholder risk tolerance for twelve of the firms. These, along with 12/95 expected firm contributions to percent market standard deviations ($s_{km}$), are also shown in Table 8. These expected contributions to the standard deviation are similar across the energy firms in our sample. This is consistent with the idea that investors in energy firms are reacting to similar market information over time.

The correlation between average shareholder risk tolerance and the expected contributions to market standard deviation across 12 of the 15 firms is .54. These data seem to suggest that, while widely held firms have higher risk tolerances than less widely held firms, less risk tolerant shareholders may prefer to hold stocks with relatively lower market risks.

The average shareholder risk tolerances shown in Table 8 range from under $1000 to slightly more than $24,000. Recall that an individual with a risk tolerance of $5000 would be approximately indifferent between accepting and rejecting a lottery with a 0.5 chance of winning $5000 and a 0.5 chance of losing $5000/2, or $2500. Intuitively, it seems plausible that these averages would be in a reasonable range of estimates of risk tolerances for individual investors in the stock market.

## 4. Conclusion

The above result and discussion represent an extension of Wilson's (1968) result in the case of syndicates and Ekern and Wilson's (1974) observations regarding firm risk tolerances in the case of a single market portfolio. If mean/variance information is efficiently distributed to all market investors, then the risk tolerance for all quasi syndicates - regardless of the number of market assets their investors hold - is the sum of the risk tolerances of their investors. Since all publicly traded assets are held by quasi syndicates, the implication is that corporate managers should be concerned about quasi-syndicate aggregate risk tolerances when making investment decisions.

While the idea that the risk attitude for widely held firms should be nearly risk neutral is widely known, this is the first equity-market-based specification of a quasi syndicate - and therefore firm - utility function parameter of which we are aware. The notion that the firm's risk tolerance is the sum of all investor risk tolerances is in the spirit of Wilson's (1968) result for syndicates, but the conditions under which our result obtains are more general.

These market-based results were compared with the outcomes of the risk tolerance observations made by Walls and Dyer (1996). The firms in this study were selected because the results of the previous study offered a benchmark for comparison. Though all firms in the study might be considered widely held, the number of shareholders in 1995 ranged from 20,025 (Pennzoil) to 608,000 (Exxon). There was also a wide range of risk tolerances, and there is a positive relationship between both risk tolerance and market value and the number of shareholders in these corporations.

The results briefly summarized above suggest that corporate managers in the fifteen energy firms considered for this paper have been too risk averse with respect to implied firm risk tolerances. Further, the rank orderings of market-based risk tolerances were not closely related to the rank orderings of the Walls and Dyer observed risk tolerances for these firms. The relationship between risk tolerance rank orders and market value rank orders is strong (in a correlation sense) from a market-based perspective; it is much less strong in the observed Walls-Dyer data.

Our development was based on the use of the exponential utility function to approximate the risk tolerances of these firms because of its mathematical tractability, the convenient interpretation offered by its corresponding risk tolerance, and its common use in practice. It might be of some interest to explore whether similar results could be obtained for a more general class of utility functions. In addition, our approximations of the corporate risk tolerances were based on estimates of each firm's contribution to percent market variance $\hat{s}^2_{km}$ developed from a simulation model, but there may be other approaches that would refine these estimates in a manner more tailored to each company. We leave these questions for further research. This work represents a synthesis and extension of several ideas that have been around for many years and, to varying degrees, have survived the test of time.

In addition to the potential for managers and decision analysts to specify (or, at least, estimate) a firm risk tolerance for any publicly traded firm, implications of this work lie in the area of corporate governance. The aggregate of quasi-syndicate shareholder preferences may reasonably be assessed over time using historical firm share return data in conjunction with market values and expected periodic returns. If analyzed in conjunction with the market value of a firm's equity, a managerial track record may be established. While maximization of the firm's market-based risk tolerance is an unlikely managerial objective, increases in the quasi-syndicate risk tolerance may be associated with increases in the market value of the firm's equity. If return distributions are stationary, or nearly stationary, increases in the market value of the firm will be directly proportional to increases in the quasi-syndicate risk tolerance.

Results presented in this paper also have potential ramifications in strategic management research. The notion of a market value-based aggregate of investor risk preferences provides a foundation for the analysis of managerial strategic performance in light of historical share price data. Data and information concerning historical strategic decisions such as mergers, cooperative agreements and/or strategic direction changes could be assessed using ex post share price data. Systematic effects, such as changes in the risk free rate or large swings in portfolio or firm share price values could be weighted against the firm risk tolerance and performance. Within and between industry historical analyses could shed light on major events and managerial issues, and might provide insights for managers constantly faced with risky capital investment and valuation decisions.

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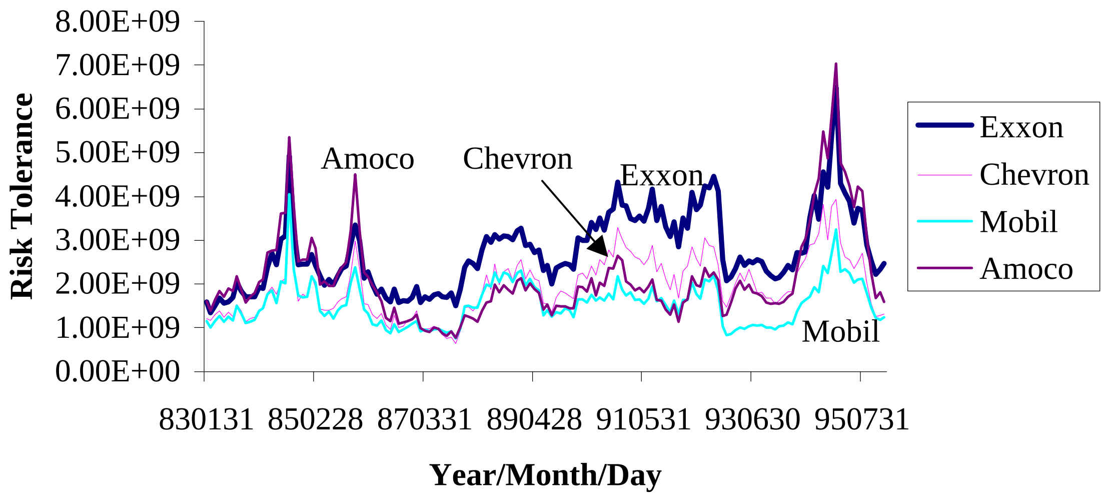


*Figure 1*. Market-based Expected Risk Tolerances for Exxon, Chevron, Amoco and Mobil. Monthly Measures, 1/83 - 12/95.

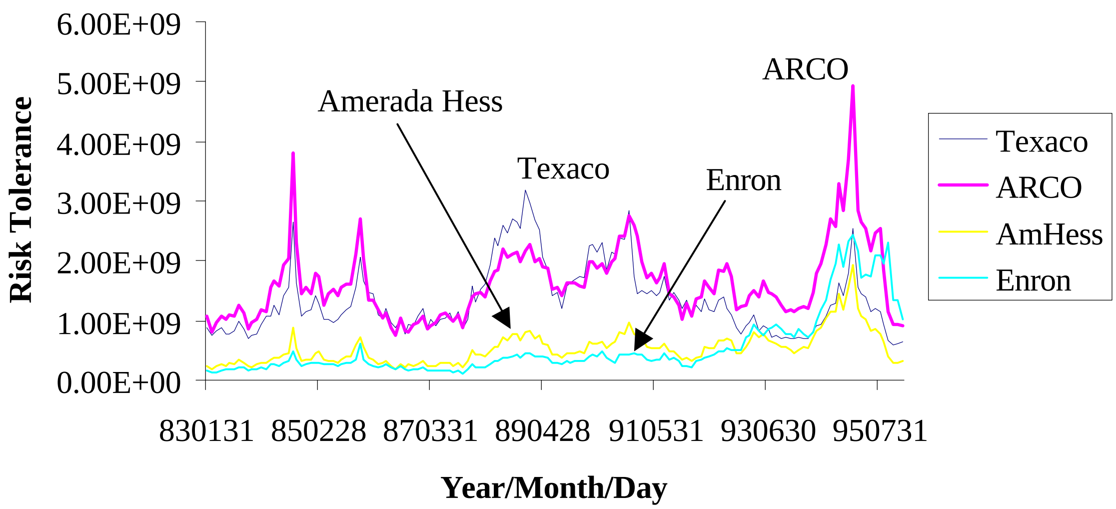


*Figure 2*. Market-based Expected Risk Tolerances for Texaco, ARCO, Amerada Hess and Enron. Monthly Measures, 1/83 - 12/95.

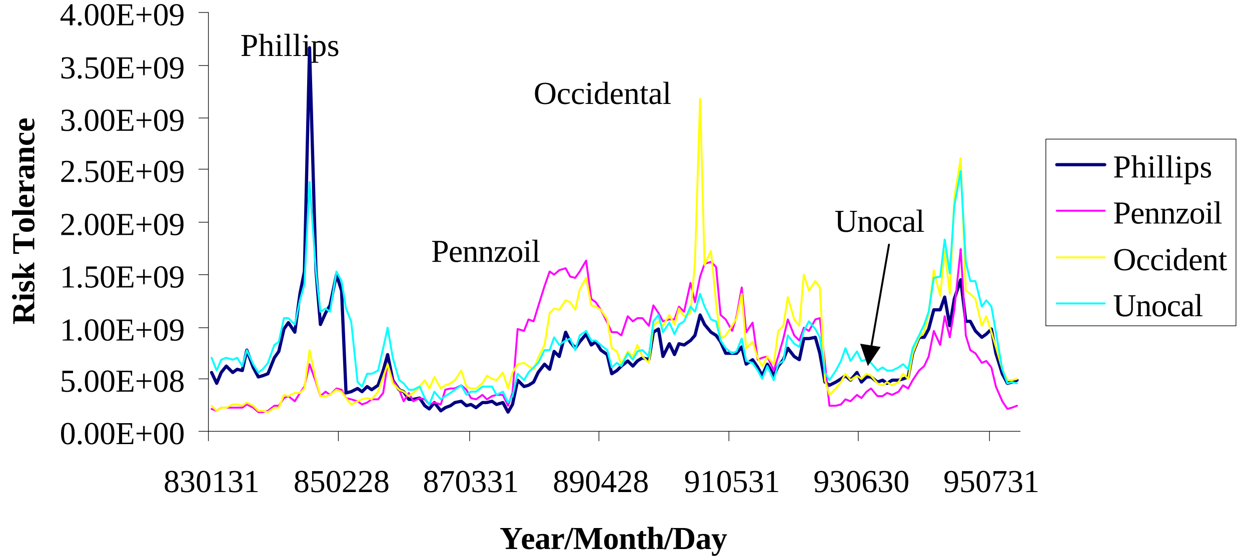


*Figure 3*. Market-based Expected Risk Tolerances for Occidental, Phillips, Pennzoil and Unocal. Monthly Measures, 1/83 - 12/95. One outlier (Pennzoil, 30 November 1992) is omitted.

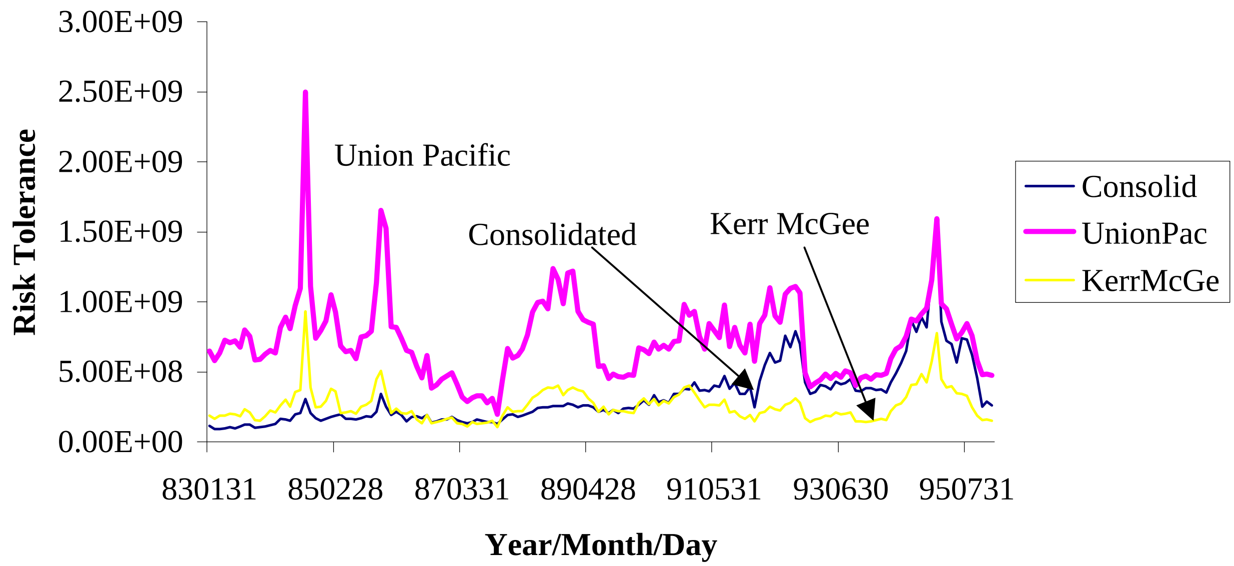


*Figure 4*. Market-based Expected Risk Tolerances for Consolidated, Union Pacific and Kerr McGee. Monthly Measures, 1/83 - 12/95.