

# FACTORED STOCHASTIC TREE MODELING FOR ARTHRITIC JOINT REPLACEMENT DECISIONS

GORDON B. HAZEN, PH.D

IEMS Department, Northwestern University, Evanston IL

JAMES M. PELLISSIER, PH.D

Merck Research Laboratories, Blue Bell PA

ROWLAND W. CHANG, M.D.,M.P.H.

Department of Preventive Medicine, Northwestern University

June 1998<sup>†</sup>

## Abstract

Total hip arthroplasty (THA) and total knee arthroplasty (TKA) have proven to be clinically reliable and durable procedures for the surgical treatment of severe osteoarthritis of the hip and knee. Although THA and TKA seem justified in terms of clinical success,

---

<sup>†</sup> Financial support for this study was provided in part by grants from the National Institute of Arthritis and Musculoskeletal and Skin Diseases; and from the Decision, Risk and Management Science Program at the National Science Foundation. The funding agreement ensured the authors' independence in designing the study, interpreting the data, writing and publishing the report.

they are particularly vulnerable to scrutiny in economic terms. We present here decision-analytic models of the short- and long-term consequences of knee and hip replacement. These models are constructed using factored stochastic trees, which combine features from decision trees and continuous-time Markov chains. A novel feature of the presentation is the use of influence diagrams to portray the relationships among the components (factors) of the model. The authors' software for factored stochastic tree construction is also presented. Although knee and hip replacement do not extend life, when quality of life is included, our results show that these procedures are very cost-effective, comparing favorably with such well-accepted practices as cardiac bypass and renal dialysis.

## Introduction

Total hip arthroplasty (THA) and total knee arthroplasty (TKA) have proven to be clinically reliable and durable procedures for the surgical treatment of severe osteoarthritis of the hip and knee<sup>1,2,3,4,5</sup>. An estimated 120,000 THAs are performed per year in North America,<sup>6</sup> the majority of which were for patients with hip osteoarthritis. In the United States alone, 125,000 to 140,000 TKAs are performed annually<sup>7,8</sup>, with osteoarthritis and rheumatoid arthritis accounting for over 90% of these operations. Because these operations are performed predominantly on the elderly, their frequency is expected to increase as the population ages<sup>9</sup>.

Although THA and TKA seem justified in terms of clinical success, they are particularly vulnerable to scrutiny in economic terms, for several reasons: Their indications are generally elective; their target population is largely geriatric; and they are

high-technology procedures, much more expensive in the short term than simple medical management. From a societal or policy perspective, the cost-effectiveness of these procedures is therefore of particular interest. Moreover, because these procedures do not extend life, their cost-effectiveness must be measured in terms of dollars per *quality-adjusted life year* (QALY).

Although there are a few studies which address cost-effectiveness in the short term, we found no cost-effectiveness analyses for THA or TKA which considered long-term issues such as the need for revision surgeries, or worsening osteoarthritis and its associated custodial care costs. We therefore constructed decision-analytic models of the short- and long-term consequences of THA and TKA. The results of these studies were remarkable<sup>10,11</sup>: When improvements in quality of life are included, THA and TKA can be among the most cost-effective of medical procedures, comparable or superior to well-accepted procedures such as cardiac bypass or renal dialysis. In fact, for some patients, these procedures can be *cost saving*, improving quality of life *and* reducing long-term costs compared to conservative medical management.

The purpose of this paper is to present in detail the decision-analytic models underlying these conclusions. Although these models are summarized in our previous articles, space restrictions and audience background precluded detailed presentations there. Moreover, since the time of our original analyses, we have developed more sophisticated graphical tools for formulating and presenting such models, which we feel are of independent interest.

A major graphical modeling strategy we wish to illustrate here is the use of *influence diagrams* augmented by *stochastic nodes*. Influence diagrams are well-known graphical

tools for formulating and solving decision problems<sup>12,13,14,15</sup>. Chance nodes in an influence diagram represent random variables. As we illustrate below, it is possible to augment influence diagrams by allowing random variables which change state over time. Such variables are usually called *stochastic processes*, and we call the corresponding nodes in an influence diagram *stochastic nodes*. Examples of stochastic processes are discrete-time Markov chains<sup>16,17,18,19</sup>, continuous-time Markov chains<sup>17,18,19</sup>, and *stochastic trees*<sup>20</sup>. The latter, developed recently by one of us, combines features from continuous-time Markov chains and decision trees.

Another tool we wish to demonstrate is the graphical modeling software *StoTree*, which we have implemented as an add-in to Microsoft Excel. *StoTree* enables a user both to formulate *factored* stochastic tree models<sup>21</sup> via a graphical interface, and to calculate mean quality-adjusted durations by rolling back the resulting stochastic tree in a manner analogous to decision tree rollback<sup>20,21</sup>. All stochastic tree diagrams in this article are screen captures from *StoTree*. We will describe this software in more detail below.

Markov chain and stochastic tree models are particularly useful in medical models requiring the treatment of long-term consequences. Our joint replacement models are formulated as factored stochastic trees in which loosely coupled processes such as background mortality, disease progression and prosthesis loosening are modeled separately and subsequently linked. These stochastic factors and corresponding links can be displayed as stochastic nodes and connecting arrows in an influence diagram. Such an influence diagram thereby provides a graphical overview of model structure in a factored stochastic tree.

We discuss influence diagrams augmented by stochastic nodes in the next section. Following that, we give a graphical presentation of our joint replacement models using influence diagrams and stochastic tree diagrams. After giving a short description of our *StoTree* software, we conclude by summarizing the cost-effectiveness results from our joint replacement models.

## Influence Diagrams Augmented by Stochastic Nodes

As we have mentioned, influence diagrams are widely accepted tools for formulating and solving decision problems under uncertainty. In Figure 1 we present an example influence diagram taken from our THA model. In an influence diagram, oval nodes are called *chance nodes* and represent uncertain variables; rectangular nodes are called *decision nodes* and represent decisions; and an arrow between two nodes indicates that the parent node *influences* the child node in a probabilistic sense. If the influence is deterministic, that is, if the child variable is a function of the state of the parent variable, then the child node is given a doubly outlined border. Doubly outlined nodes with no parents are therefore constants.

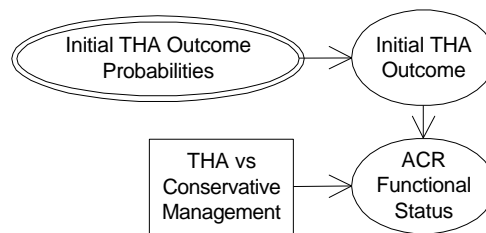


FIGURE 1. An influence diagram involving chance, decision and deterministic nodes.

We introduce a new feature into influence diagrams by allowing designated oval nodes to represent variables which may change state over time. Such variables are also known as *stochastic processes*, and we call the associated oval node a *stochastic node*. We distinguish stochastic nodes from chance nodes by adding a wavy arrow below the node description. Figure 2 portrays an influence diagram containing a stochastic node *Prosthesis Status*.

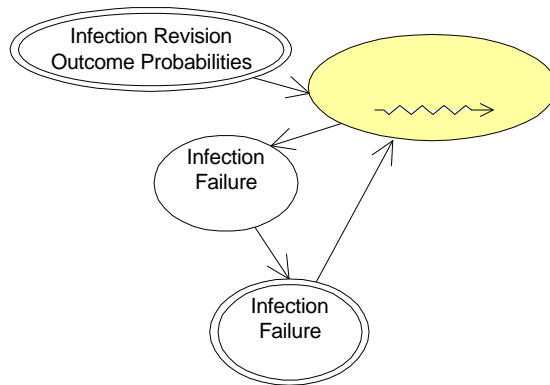


FIGURE 2. An influence diagram containing a node representing a stochastic process. Directed cycles are not permitted in conventional influence diagrams, which do not contain stochastic nodes. However, directed cycles are permitted in the cycle.

The node *Prosthesis Status* in Figure 2 is a stochastic node. In the stochastic tree in Figure 3, an initially functioning prosthesis is subject to  $r$  *Infection* per year. Surgery immediately follows ( $PISucc$ ), the prosthesis is replaced ( $PIFail$ ), it is repeated.  $pIMort$  of surgical mortality.

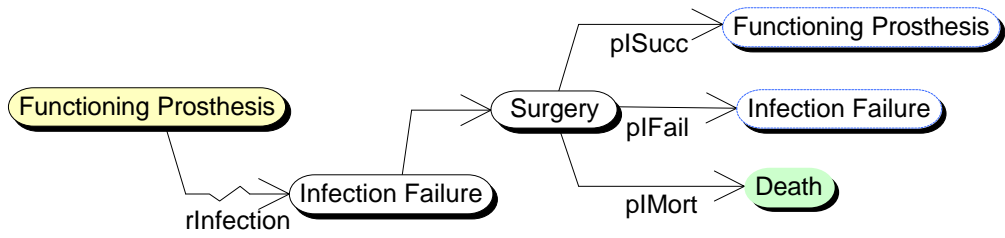


FIGURE 3. A stochastic tree diagram of the stochastic process *Prosthesis Status*.

The node *Infection Failure Count* in the influence diagram of Figure 2 represents the stochastic process depicted in Figure 4. *Infection Failure Count* serves to count the cumulative number of infections (up to three) in the process *Prosthesis Status*. It has no transition arrows – it can change state only when triggered by an infection failure in the latter process. Therefore, we have included no wavy arrow in its node in Figure 2. The process *Infection Failure Count* is required because the infection failure rate  $rInfection$  of Figure 3 is higher when there have been more infection failures. This dependence is indicated in the influence diagram of Figure 2, where the node *Infection Failure Rate* is a deterministic function of *Infection Failure Count*. *Infection Failure Rate* in turn influences *Prosthesis Status*, thereby inducing a directed cycle of influences. Although directed cycles are forbidden in traditional influence diagrams, they are coherent in influence diagrams with stochastic nodes because nodes in a cycle may change state over time.



FIGURE 4. The stochastic process *Infection Failure Count*. This process changes state only when triggered by an infection failure in *Prosthesis Status after THA*. Therefore, no transition arrows are present.

## An

Our stochastic tree models for knee replacement and hip replacement were very similar in structure, although differing in the values of input parameters. Therefore, we begin by

TKA model when appropriate. When we discuss model results, we will present values for both knee and hip replacement.

In Figure 5

between THA and conservative management for hip osteoarthritis. The diagram contains

a factor in our stochastic tree model for THA. For convenience, the nodes enclosed by the dashed line were combined into a single factor in our model.

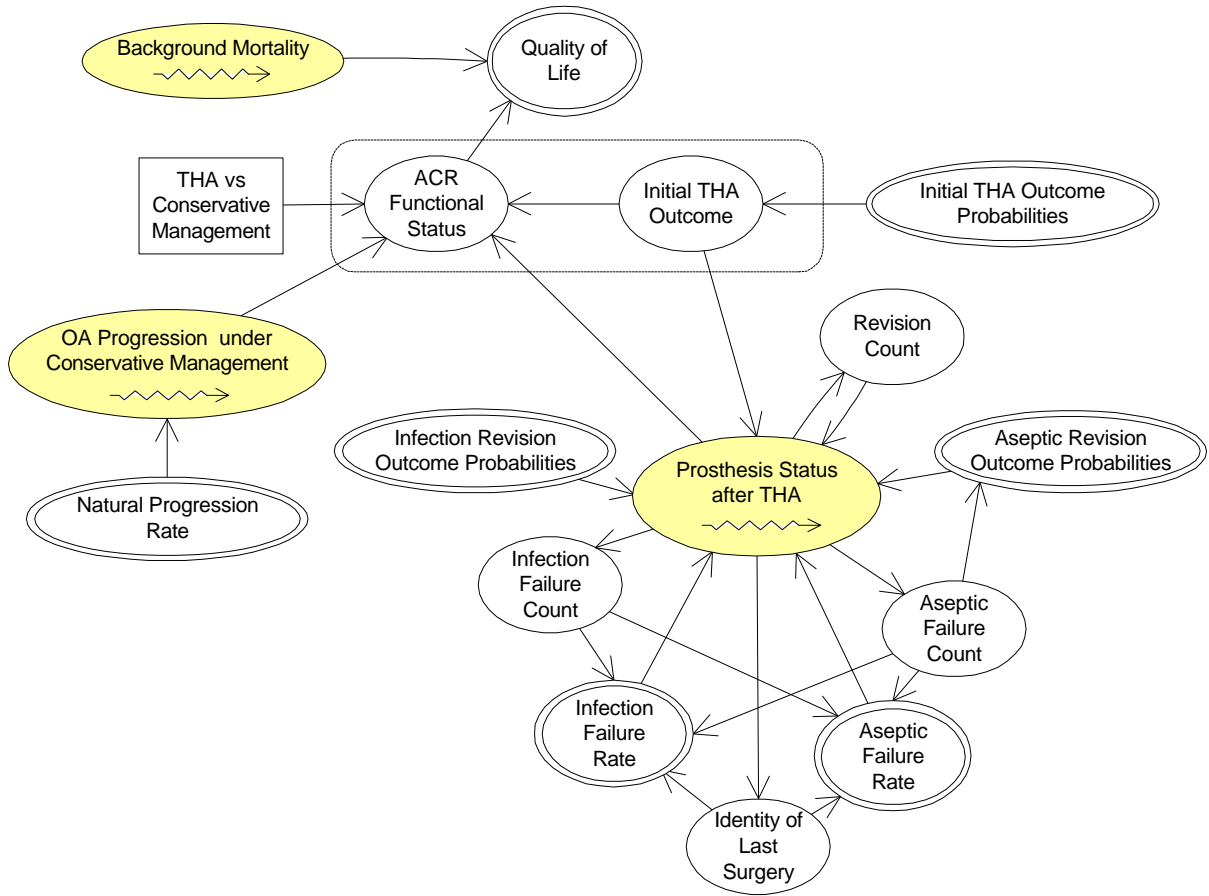


FIGURE 5. An influence diagram for our model of the choice between THA and Conservative Management. The factors in our stochastic tree model were precisely the non-deterministic nodes depicted here. The two nodes surrounded by a the dashed line were grouped into a single factor in our stochastic tree.

We discuss the detailed structure of each factor below. However, first we use Figure 5 to give an intuitive overview of the model. The purpose of the model is to calculate optimal mean quality-adjusted lifetime, and to that end, the node *Quality of Life* is present in the upper portion of Figure 5. *Quality of Life* is a deterministic function of *ACR Functional Status*. The latter is a four-level functional status scale adopted by the American College of Rheumatology (see below). *ACR Functional Status* depends on the decision node *THA versus Conservative Management*. If the choice is THA, then *ACR*

*Functional Status* depends on *Initial THA Outcome*, which indicates the outcome of the initial hip replacement surgery, and *Prosthesis Status after THA*, a stochastic process describing prosthesis failure over time and subsequent revision surgeries. Prosthesis failures can be due to infection (*septic failures*) or mechanical failure (*aseptic failure*). If the choice is conservative management, then *ACR Functional Status* depends on *OA Progression under Conservative Management*, a stochastic process describing the functional deterioration of the hip due to osteoarthritis.

*Prosthesis Status after THA* interacts with a number of stochastic and chance nodes. First, if *Initial THA Outcome* is surgical failure, then an aseptic revision surgery is triggered in *Prosthesis Status after THA*. The stochastic nodes *Infection Failure Count* and *Aseptic Failure Count* record the cumulative number of prosthesis failures due respectively to infection and to mechanical failure. The stochastic node *Identity of Last Surgery* has possible values *Initial THA*, *Aseptic Revision* and *Infection Revision*, corresponding to the possible types of the most recent surgery. The variables *Infection Failure Rate* and *Aseptic Failure Rate* determine the average prosthesis failure rates in *Prosthesis Status after THA*. These rates are deterministic functions of *Infection Failure Count*, *Aseptic Failure Count*, and *Identity of Last Surgery*.

Finally, the stochastic node *Background Mortality* represents mortality due to causes unrelated to THA or conservative management of osteoarthritis. Its only effect on is to reduce *Quality of Life* to zero when mortality occurs.

## STOCHASTIC FACTORS FOR THE INITIAL THA DECISION

The factor tree for the decision node *THA vs. Conservative Management* is depicted in Figure 6. The nodes within the dashed rectangle in Figure 5 depict the outcome of the

initial THA surgery, should THA be chosen, as well as the subsequent functional status under either THA or conservative management. The corresponding stochastic factor is portrayed in Figure 7.

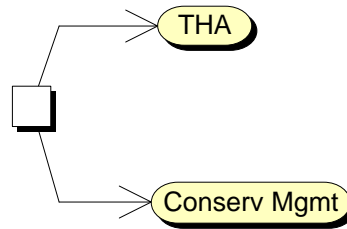


FIGURE 6. The factor tree *THA vs. Conservative Management*

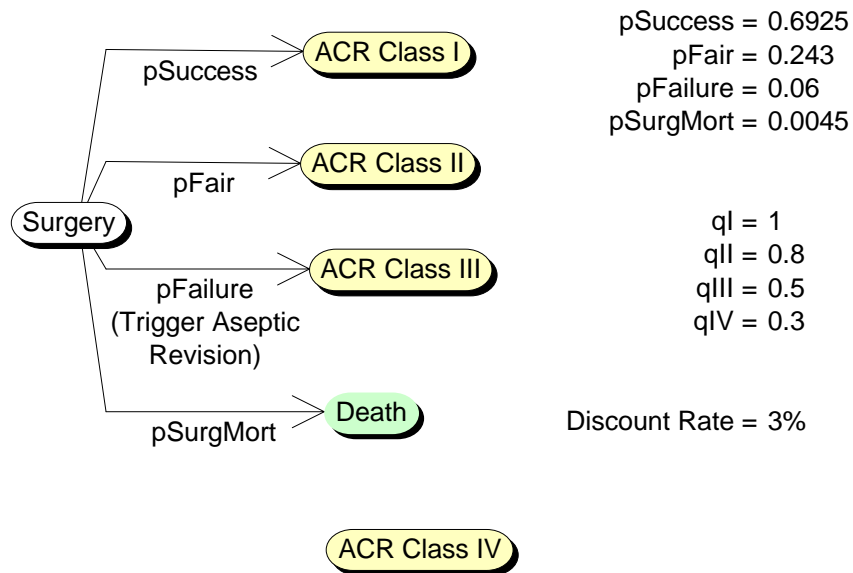


FIGURE 7. Our stochastic factor tree depicting the outcome of initial THA surgery and subsequent ACR functional status.

We chose functional class as our primary measure of effectiveness for THA, adopting the American College of Rheumatology (ACR) functional status classification<sup>22</sup> for use in hip osteoarthritis, described in Table 1. We assumed the patient is initially in functional

class III, a common status of individuals deciding whether to undergo THA. From Figure 7 we see that initial surgical outcome is ACR functional class I, II, or III. Functional class IV cannot be reached initially, but entry into this state can be triggered by progression of osteoarthritis or subsequent prosthesis failure. The quality-of-life values  $qI$ ,  $qII$ ,  $qIII$ ,  $qIV$  we assigned to these functional classes are specified in Figure 7 as well.

Class	Description
I	Complete ability to carry on all usual duties without handicap
II	Adequate for normal activities despite handicap of discomfort or limited motion in the hip
III	Limited only to little or none of duties of usual occupation or self-care
IV	Incapacitated, largely or wholly bedridden or confined to wheelchair, little or no self-care

TABLE 1. American College of Rheumatology (ACR) functional classifications for hip osteoarthritis.

## STOCHASTIC FACTORS FOR PROSTHESIS STATUS AFTER THA

Our stochastic factor tree *Prosthesis Status after THA* is depicted in Figure 8. Following initial THA, the patient occupies the state *Daily Living*, in which the prosthesis is subject to both aseptic and infection failure. Revision surgery is undertaken after prosthesis failure. In practice, available bone stock limits the number of revision surgeries that can be undertaken. We assumed at most three revision surgeries were possible. The stochastic factor *Revision Count* (not shown, but identical to Figure 4) counts the cumulative number of revision surgeries. Should an aseptic or infection failure occur when *Revision Count* is equal to 3, then the *No Revision* branch is taken in Figure 8.

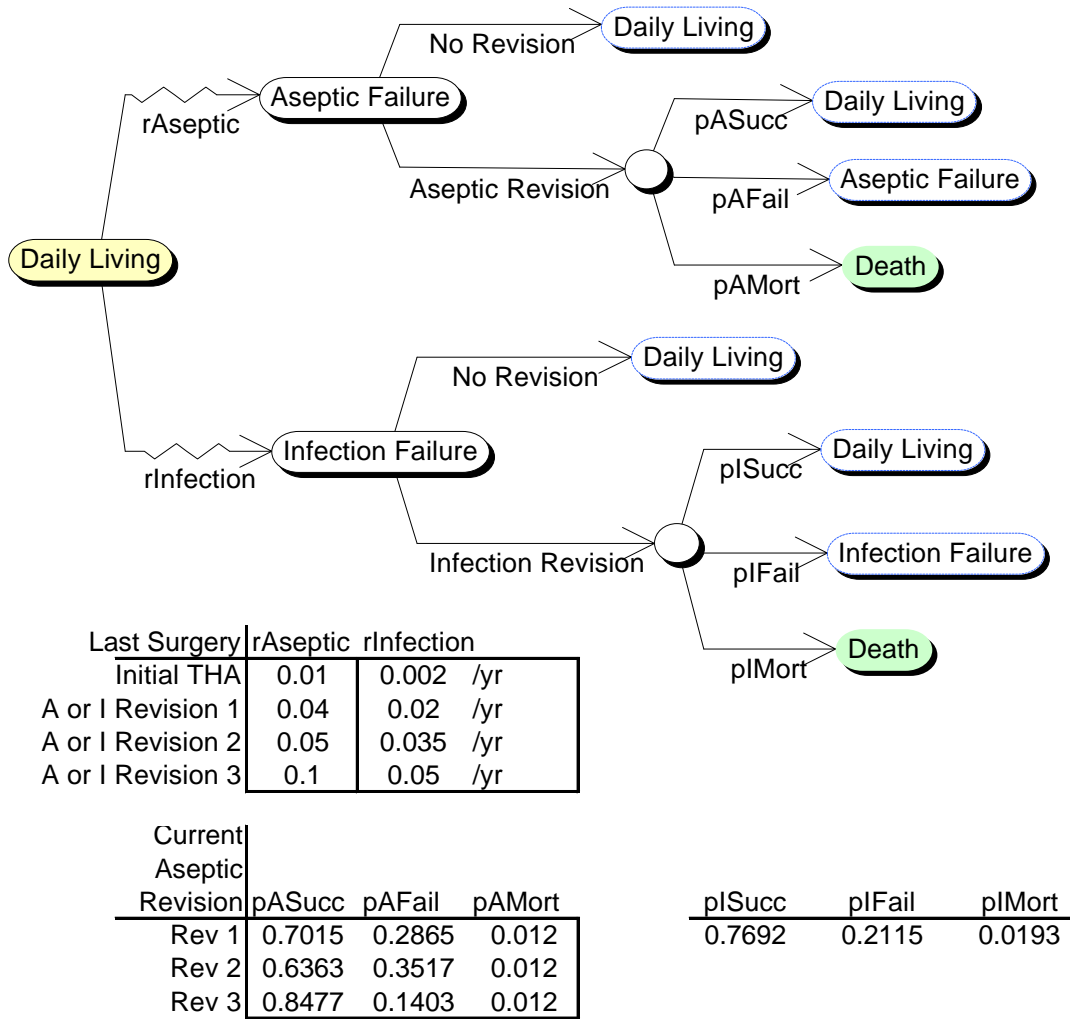


FIGURE 8. Our stochastic factor tree *Prosthesis Status after THA*. Rates of aseptic failure and infection failure are given in the accompanying tables, as are the surgical outcome probabilities. Both rates and probabilities depend on the levels of the auxiliary factors *Aseptic Failure Count*, *Infection Failure Count*, *Revision Count*, and *Last Surgery*.

As our influence diagram in Figure 5 and the tables in Figure 8 indicate, prosthesis failure rates and the outcome probabilities for aseptic revision depend on the type of the most recent surgery, as well as the cumulative number of aseptic revisions and the cumulative number of infection revisions. These cumulative counts are kept by the stochastic factors *Aseptic Failure Count* and *Infection Failure Count* (identical to Figure

4), whose levels are incremented by one, respectively, when an aseptic or infection failure occurs. The type of the last surgery is recorded in the stochastic factor *Last Surgery* shown in Figure 9.

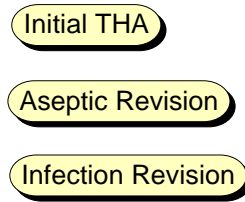


FIGURE 9. The stochastic factor *Last Surgery*. Transitions in this factor can occur only when triggered by revision surgery in the factor *Prosthesis Status after THA*.

#### STOCHASTIC FACTOR FOR CONSERVATIVE MANAGEMENT

The stochastic node *OA Progression under Conservative Management* in Figure 5 represents a two-state stochastic process in which the initial state occupied is functional class III but later transition may occur to functional class IV. Figure 10 depicts this process as a stochastic tree in which the rate of transition from state III to state IV occurs at an average rate of  $rNatural$  per year.

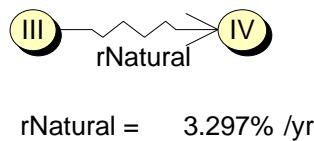


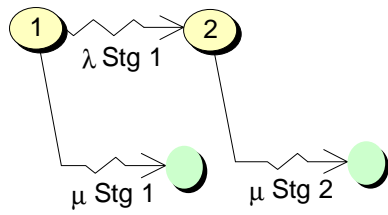
FIGURE 10. The node *OA Progression under Conservative Management* in Figure 5 represents the stochastic process depicted here, in which transition occurs from functional status III to IV at an average rate of  $rNatural = 3.297\%$  per year.

## COXIAN MORTALITY FACTOR

The stochastic node *Background Mortality* in Figure 5 represents mortality due to causes unrelated to hip osteoarthritis or hip replacement. The underlying stochastic process is depicted by one of the *Coxian* stochastic tree models shown in Figure 11. For a particular age and gender, a Coxian model with the appropriate number of stages and parameter values has survival probabilities which quite closely approximate the true ones<sup>23</sup>. For example, Figure 12 compares the true survival probabilities for a 60-year-old white female (ref) with the survival probabilities for the Coxian model of Figure 11.

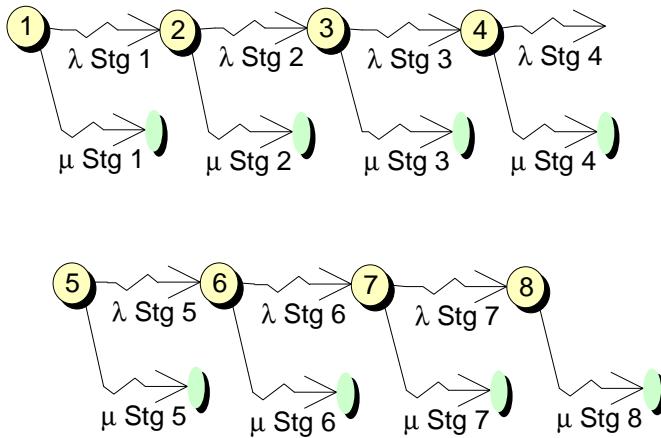
Factoring background mortality from the overall model in this way results in a useful *modularity* property: Should one wish to run the model for a different age or gender, one need not alter any other part of the model. Instead, merely substitute the appropriate age- and gender-specific Coxian mortality factor.

**85-year-old white male**



	$\lambda$	$\mu$
Stg 1	0.253	0.093
Stg 2		0.347

**60-year-old white female**



	$\lambda$	$\mu$
Stg 1	0.288	0.01
Stg 2	0.278	0.02
Stg 3	0.298	0
Stg 4	0.298	0
Stg 5	0.298	0
Stg 6	0.298	0
Stg 7	0.18	0.118
Stg 8		0.298

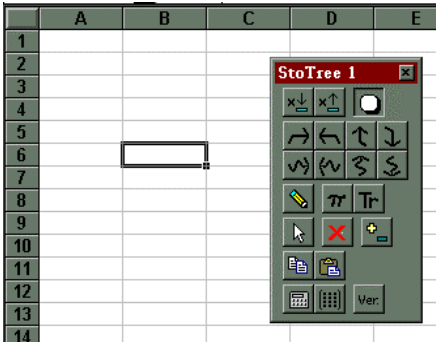
FIGURE 11. Coxian stochastic trees which closely approximate observed mortality for the given ages and genders. Numbered nodes may be thought of as life stages, between which transition occurs at rates  $\lambda$  Stg 1,  $\lambda$  Stg 2, .... Empty nodes represent mortality, to which transition occurs with stage-dependent rates  $\mu$  Stg 1,  $\mu$  Stg 2, ....



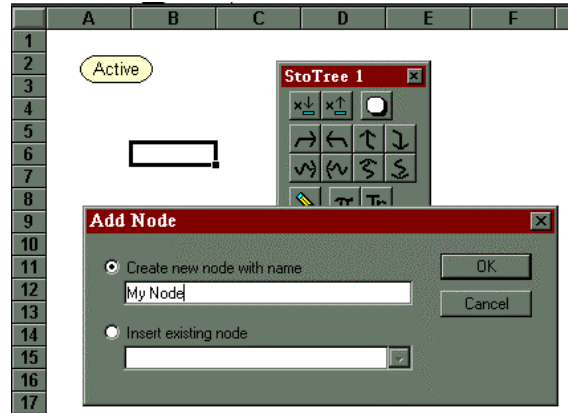
FIGURE 12. Survival probabilities for a white female age 60 (U.S. Center for Health Statistics, 1991). are quite closely approximated by the survival probabilities for the Coxian model of Figure 11.

## Software for Factored Stochastic Tree Modeling

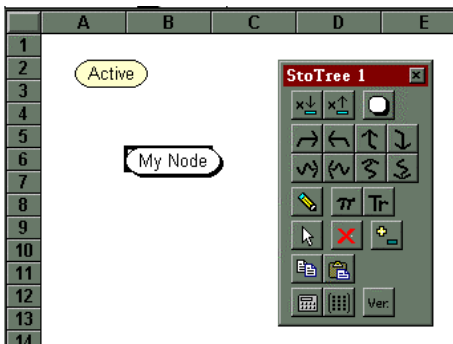
*StoTree* is a graphical interface tool for the formulation and solution of factored stochastic tree models, implemented as a Microsoft Excel add-in. The add-in and supporting documentation are available for downloading at the web site ***Error! Bookmark not defined.*** Operating in a spreadsheet environment, the *StoTree* user can access all the usual features of that environment in addition to those of *StoTree*. The modeling effort begins with graphical model composition and parameter specification. The user concludes by using the rollback features in *StoTree* to calculate average quantities of interest such as mean quality adjusted life years or average costs.



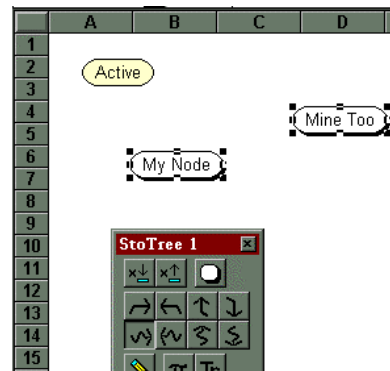
(a) The Excel environment and the *StoTree* toolbar. The user has just clicked on the Add Node button in the upper-right portion of the toolbar. *StoTree* will convert the worksheet to a stochastic factor and label it as active.



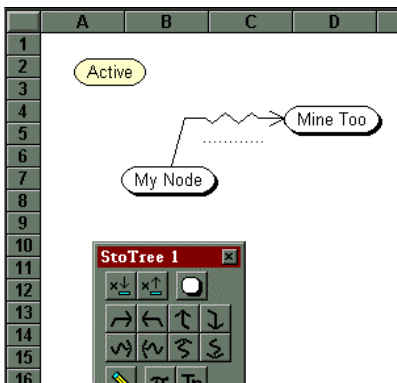
(b) The Add Node dialog appears. The user chooses to create a new node and types the node name "My Node".



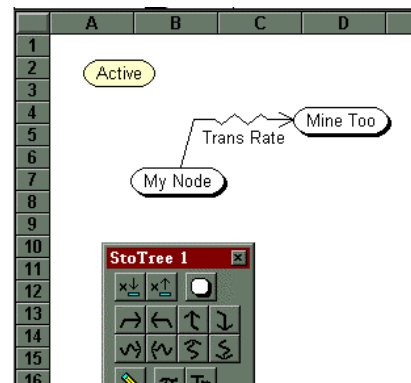
(c) *StoTree* creates the node and gives it the name "My Node". The stochastic factor is now active but the user may de-activate it should the need arise.



(d) The user has created a second node and now wishes to connect the two nodes with a stochastic arc. He first selects the two nodes, and then clicks on one of the Add Arc buttons in the *StoTree* toolbar



(e) *StoTree* connects the two nodes with a stochastic arc and places below the arc a blank textbox into which the user may type whatever label he desires.



(f) The user has typed "Trans Rate" into the arc-labeling textbox.

FIGURE 13. The initial graphical model construction stages in *StoTree*.

## GRAPHICAL MODEL COMPOSITION AND PARAMETER SPECIFICATION

Figure 13 illustrates typical first steps of a user-initiated *StoTree* session. Via point-and-click operations, the user can create nodes, name and position them as he likes, and connect nodes with either stochastic or chance arcs, to which he may attach any desired label. Arcs may be drawn in any of the four compass directions. Each worksheet in the Excel workbook to which nodes have been added is regarded by *StoTree* as a factor in a multi-factor stochastic tree. Capabilities not shown in Figure 13 include subtree copy and paste, node and arc deletion, and tree redraw.

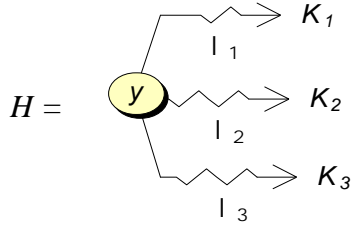
Once the user has specified the graphical structure, he can associate numerical parameters with nodes and arcs. Parameters for arcs consist of probabilities for chance arcs, rates for stochastic arcs, and tolls for either type of arc. Node parameters consist of a quality or cost rate, and a discount rate. These are used in the rollback operation described below. Arc parameters can depend on the state of other factors. The user may also add one or more triggers on any arc, which force transitions in other specified factors when that arc is traversed. All of these features are accessed via dialog boxes called up from the *StoTree* toolbar.

## ROLLBACK ALGORITHM

*StoTree* implements a form of the *Markovian* utility function<sup>24</sup>. Let  $y^t h$  denote a duration  $t$  visit to state  $y$  followed by any other sequence  $h$  of states and durations. If  $x$  is the previously visited state, then the Markovian utility assigned to  $y^t h$  is equal to

$$u(y^t h | x) = w(y | x) + \int_0^t v(y) e^{-a(y)s} ds + e^{-a(y)t} u(h).$$

Here  $w(y/x)$  is a toll associated with the transition from  $x$  to  $y$ ;  $v(y)$  is a quality rate specific to state  $y$ , and  $a(y)$  is a discount factor. At a stochastic fork



in which subtrees  $K_i$  are reached from state  $y$  at competing rates  $l_i$ , the rollback equation for calculating expected utility can be shown<sup>24</sup> to take the simple form

$$E[u(H) | x] = w(y | x) + \frac{v(y) + \sum_i l_i E[u(K_i | y)]}{a(y) + \sum_i l_i}.$$

At a chance fork with associated probabilities  $p_i$ , the usual probability-weighted average is used:

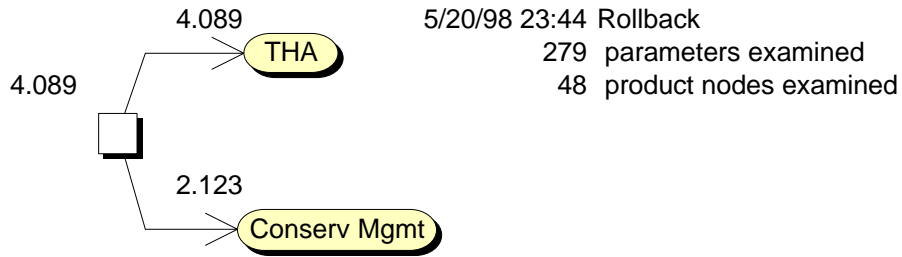
$$E[u(H) | x] = w(y | x) + \sum_i p_i E[u(K_i | y)].$$

*StoTree* repeatedly evaluates these equations for the user-specified multi-factor stochastic tree. In this context, the states  $y$  are vectors  $y = (y_1, \dots, y_m)$ , where  $y_i$  is the state of the  $i^{\text{th}}$  factor. If the  $i^{\text{th}}$  factor contains  $k_i$  non-death states, then in principle there are  $k_1 \times k_2 \times \dots \times k_m$  product states  $y = (y_1, \dots, y_m)$  for which expected utility must be calculated. For example, the full THA model specified above has  $3 \cdot 5 \cdot 2 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 4 \cdot k_8 = 28800 \cdot k_8$  possible product states, where  $k_8$  is the number of stages in the Coxian model (equal above to 2 or 8). In Excel's slow computing environment, such a large number of product states would give unacceptably lengthy rollback times. However, *StoTree* bypasses this problem by calculating expected utility only for those product states which are *reachable* from the

initial combination of states in each factor. For example, due to the many triggers present in the THA model, the number of reachable product states is only  $4 + 22 \cdot k_8$ . *StoTree* identifies reachable product states by performing a breadth-first traversal of the product tree beginning at the combination of user-specified initial states.

*StoTree* displays rollback values next to the corresponding nodes in each factor. For example, Figure 14 displays mean quality-adjusted lifetimes for our THA model, as displayed in two of the model's factors. A useful feature available in *StoTree* is the linking of rollback entries to cell values. For example, at the chance fork in Figure 14b, we as users entered all probability parameters as cell references. *StoTree* then incorporates these cell references into the rollback formulas. The result is that the rollback entries will change when values in referenced cells change. For example, should the cell entry 0.6925 for pSuccess be changed, then the rollback values will also change when the spreadsheet is recalculated. This can be very useful for sensitivity analysis purposes.

(a) *THA vs. Conservative Management*



(b) *ACR Functional Status*

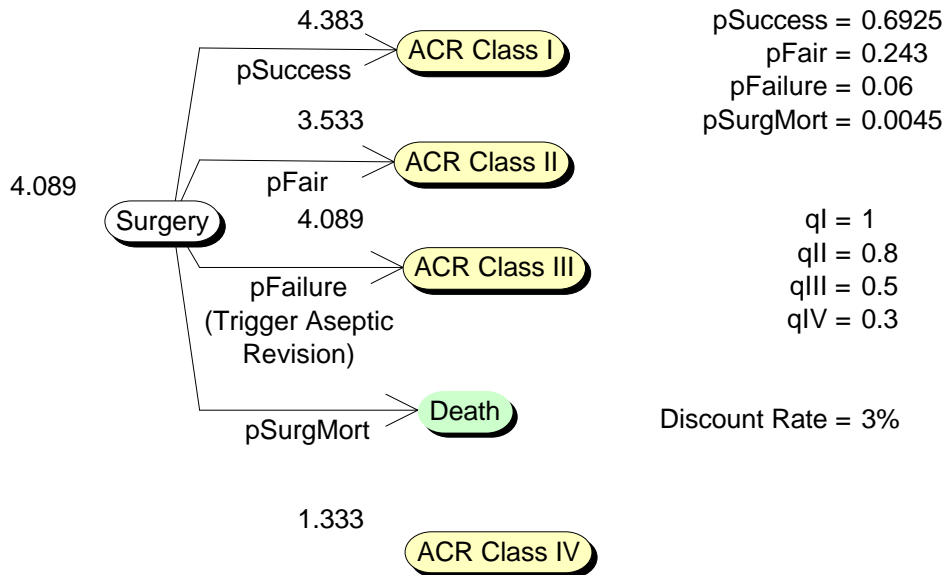


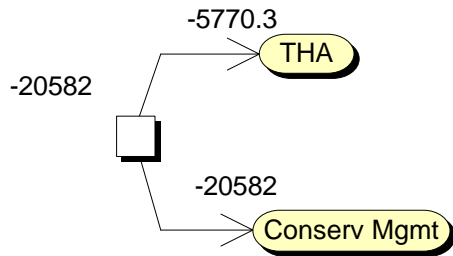
FIGURE 14. Rollback results for the THA model for the case of a white male aged 85, as displayed in the factor trees *THA vs. Conservative Management* and *ACR Functional Status*. Rollback quantities are displayed adjacent to the appropriate nodes, and indicate the mean number of quality-adjusted life years remaining beginning at that state, assuming all other factors occupy their initial states.

## Cost-Effectiveness for Joint Replacement

So far we have only discussed the computation of mean quality-adjusted lifetimes for the hip replacement decision. In order to conduct a cost-effectiveness analysis, one must

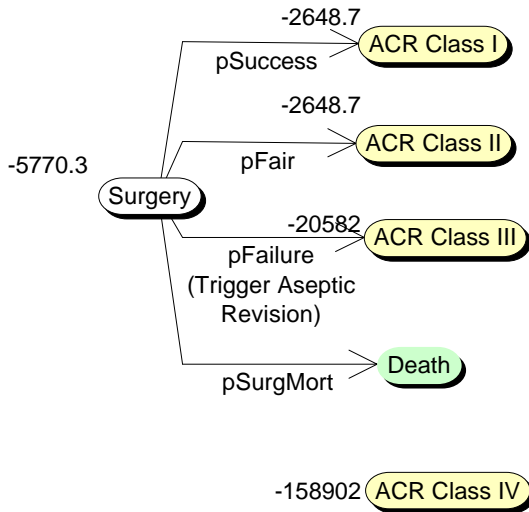
also calculate mean lifetime costs. This is easily accomplished in a stochastic tree model: Simply replace quality rates with ongoing cost rates, and include one-time costs as tolls on the appropriate arcs. Figure 15 displays both ongoing and one-time costs for our THA model, as well as the rollback results using these costs.

(a) THA vs. Conservative Management



5/21/98 0:25 Rollback  
 279 parameters examined  
 48 product nodes examined

(b) ACR Functional Status



pSuccess = 0.6925  
 pFair = 0.243  
 pFailure = 0.06  
 pSurgMort = 0.0045

Costs per case	Primary		
	No THA	THA	Revision
Hospital	\$ -	\$ 17,000	\$ 20,000
Physician	\$ -	\$ 5,000	\$ 5,000
Rehabilitation	\$ -	\$ 3,000	\$ 3,000
Total Per Case	\$ -	\$ 25,000	\$ 28,000

Costs per year	
Medical Per Yr III IV	\$ 775
Custodial Per Yr IV	\$ 35,000

Discount Rate = 3%

FIGURE 15. Calculation of expected costs for a white male aged 85 via rollback in *StoTree*. The cost data used is shown next to the *ACR Functional Status* tree. Total lifetime discounted cost for conservative management is \$20582, and for THA is \$5770.30 + \$25000 = \$30770.30. Conservative management is less costly at this age.

Overall cost-effectiveness results for our THA model are presented in Table 2. For a white male aged 85, the cost-effectiveness ratio is \$5183 per quality-adjusted life year, superior to well accepted procedures such as cardiac bypass and renal dialysis (ref). For a white female aged 60, the procedure both improves quality of life *and* reduces costs. Although THA does not extend life expectancy, the intuitive rationale for its superiority is clear from the table: Compared to conservative management, THA reduces average time spent in functional classes III and IV, which are expensive, low-quality health states.

	White Male Age 85		White Female Age 60	
	THA	Conserv	THA	Conserv
Mean Dctd Years in				
ACR Class I	2.944	0	9.717	0
ACR Class II	1.384	0	5.078	0
ACR Class III	0.063	3.952	0.608	11.39
ACR Class IV	0.022	0.49	0.48	4.634
Mean Dctd Life Expectancy	4.413	4.442	15.883	16.024
Mean Dctd QALY	4.089	2.123	14.23	7.087
Mean Dctd Costs				
Primary Surgery, Rehab	\$25,000	-	\$25,000	-
Revision Surgery, Rehab	\$4,929	-	\$12,284	-
Ongoing Medical	\$66	\$3,442	\$843	\$12,421
Custodial	\$776	\$17,140	\$16,789	\$162,175
Total	\$30,771	\$20,582	\$54,916	\$174,596
Marginal cost	\$10,189		\$(119,680)	
Marginal effectiveness (QALY)	1.966		7.143	
Marginal CE ratio	\$5,183		-	

TABLE 3. The results for THA versus conservative management based on our stochastic tree model. For a white female aged 60, THA saves costs *and* increases quality-adjusted life expectancy. For a white male aged 85, the marginal cost-effectiveness ratio of \$5183 is superior to accepted procedures such as renal dialysis or cardiac bypass.

A similar analysis may be performed for the total knee replacement decision (TKA). We do not present here the TKA stochastic tree model, which is very similar to the THA

model. Our cost-effectiveness results for TKA, taken from Gottlieb et al.<sup>11</sup> are presented in Table 4.

	White Male Age 85		White Female Age 60	
	THA	Conserv	THA	Conserv
Mean Years in				
ACR Class I	4.13	0	17.05	0
ACR Class II	0.90	0	4.60	0
ACR Class III	0.04	4.47	0.61	14.88
ACR Class IV	0.01	0.63	0.19	7.73
Mean Life Expectancy	5.08	5.10	22.45	22.61
Mean Dctd QALY	4.21	2.16	14.35	6.82
Mean Dctd Cost	\$23,641	\$21432	\$30,217	\$165,440
Marginal cost	\$2209		(\$135,223)	
Marginal effectiveness (QALY)	2.05		7.53	
Marginal CE ratio	\$1074		-	

TABLE 4. Cost-effectiveness results from our stochastic tree model of total knee replacement (TKA) versus conservative management. As with hip replacement, knee replacement is very cost effective for a white male aged 85, and is cost saving for a white female aged 60.

## Conclusion

We have presented a factored stochastic tree approach to modeling knee and hip replacement decisions. As we have shown, these models can be used to guide individual decision making, or can be applied at the societal level to determine the cost-effectiveness of these procedures. A novel feature of our presentation was the use of an influence diagram to portray the relationship between model components (factors). The latter can in turn be displayed graphically using stochastic tree diagrams. Alternate stochastic process models such as discrete-time Markov chains could in principle be used as components in

an influence diagram. However, supporting software is available only for multi-factor stochastic trees.

The use of such graphical techniques permits a modular approach to model construction, facilitates the presentation of the model, and opens the model to inspection by other parties. The graphical modeling tool *StoTree* developed by the authors enables users to formulate and solve factored stochastic tree models in a friendly spreadsheet environment.

## References

---

<sup>1</sup> NIH Consensus Development Panel on Total Hip Replacement. Total Hip Replacement. *JAMA* 1995;273:1950–1956

<sup>2</sup> Dennis DA, Clayton ML, O'Donnell S, Mack RP, Stringer EA. Posterior cruciate condylar total knee arthroplasty. Average 11-year follow-up evaluation. *Clin. Orthop.* 1992; 281:168–176.

<sup>3</sup> Ranawat CS, Flynn WF, Saddler S, Hansraj KK, Maynard MJ. Long-term results of the total condylar knee arthroplasty. A 15-year survivorship study. *Clin Orthop* 1993; 286: 94-102.

<sup>4</sup> Scuderi GR, Insall JN, Windsor RE, Moran MC. Survivorship of cemented knee replacements. *J. Bone and Joint Surg.* 1989; 710B(5): 798–803.

<sup>5</sup> Stern SH, Insall JN. Posterior stabilized prosthesis. Results after follow-up of nine to twelve years. *J. Bone and Joint Surg.* 1992; 74-A:980-986.

<sup>6</sup> Harris WH, Sledge CB. Total hip and total knee replacement. *N Engl. J Med.* 1990; 323:725–731.

<sup>7</sup> Hofman AA, Rubash HE, Crofford TW, Fletcher FJ, Crosset LS. Knee and leg: reconstruction. In *Orthopaedic Knowledge Update 4*, pp. 603–624. Edited by JW Frymoyer. Rosemont, American Academy of Orthopaedic Surgeons. 1993.

- 
- <sup>8</sup> Quam JP, Michet CJ, Wilson MG, Rand JA, Ilstrup DM Melton LJ, Wallrichs SL. Total knee arthroplasty: a population based study. *Mayo Clin. Proc.* 1991; 66:589–595.
- <sup>9</sup> Liang MH, Cullen KE, Larson MG, Thompson MS, Schwartz JA, Fossel AH, Roberts WN Sledge CB. Cost-effectiveness of total joint arthroplasty in osteoarthritis. *Arthritis Rheum.* 1986; 29(8):937–943.
- <sup>10</sup> Chang RW, Pellissier JM, Hazen GB, A Cost-Effectiveness Analysis of Total Hip Arthroplasty for Osteoarthritis of the Hip. *JAMA* 1996;275:858–865.
- <sup>11</sup> Gottlob CA, Pellissier JM, Wixson RL, Stern SH, Stulberg SD, Chang, RW. The long-term cost-effectiveness of total knee arthroplasty for osteoarthritis. Department of Preventive Medicine, Northwestern University Medical School. 680 North Lake Shore Drive, Suite 1102. Chicago IL 60611.
- <sup>12</sup> Shachter RD. Evaluating influence diagrams. *Operations Research* 1986; 34(6): 871–882.
- <sup>13</sup> Clemen RL. *Making Hard Decisions: An Introduction to Decision Analysis*. Belmont CA: Duxbury Press, 1996.
- <sup>14</sup> Owens DK, Shachter RD, Nease RF. Representations and analysis of medical decision problems with influence diagrams. *Med Decis Making* 1997; 17(3): 241–262.
- <sup>15</sup> Nease RF, Owens DK. Use of influence diagrams to structure medical decisions. *Med Decis Making* 1997; 17(3):263–275.
- <sup>16</sup> Beck JR, Pauker SG. The Markov process in medical prognosis. *Med Decis Making* 1983; 3:419–458.
- <sup>17</sup> Cinlar E. *Introduction to Stochastic Processes*. Englewood Cliffs NJ: Prentice Hall 1975.
- <sup>18</sup> Ross S. *Stochastic Processes*. New York: Wiley 1983.
- <sup>19</sup> Nelson BL. *Stochastic Modeling: Analysis and Simulation*. New York: McGraw-Hill. 1995.
- <sup>20</sup> Hazen GB. Stochastic trees: A new technique for temporal medical decision modeling. *Med Decis Making* 1992; 12:163-178.
- <sup>21</sup> Hazen GB. Factored stochastic trees: a tool for solving complex temporal medical decision models. *Med Decis Making*. 1993;13:227-236.

---

<sup>22</sup> Steinbrocker O, Traeger CH, Batterman RC. Therapeutic criteria in rheumatoid arthritis. *JAMA*. 1949; 140:659–662.

<sup>23</sup> Pellissier JM, Hazen GB. Coxian mortality factors for stochastic tree modeling. *Med Decis Making*. 1994;14:439. Abstract.

<sup>24</sup> Hazen GB, Pellissier JM. Recursive utility for stochastic trees. *Operations Research* 1996;44:788–809.