

# Simple Probabilistic Evaluation of Portfolio Strategies

*Draft Updated 4Oct97*

William B. Poland

---

*Strategic Decisions Group  
2440 Sand Hill Road  
Menlo Park, California 94025  
bpoland@sdg.com*

---

An important part of portfolio strategy evaluation is determining the probability distributions of key value measures, such as the net present value of free cash flow. This task can be computationally daunting when the portfolio consists of many businesses or assets, so we have developed a shortcut method that quickly captures the location, spread, and tilt (mean, variance, and skewness) of the portfolio distribution. We have also developed techniques to help less technical senior management understand portfolio uncertainty. These techniques allowed us to interactively explore portfolio strategies with the executive team of a packaging and plastics company comprising 20 businesses, and to identify strategies with the potential to more than double NPV with reduced risk. We have since applied these techniques successfully in the pharmaceutical, oil and gas, telecommunications, chemical, and entertainment industries.

High-quality decisions about long-term business strategy often require explicit analysis of uncertainty. Base-case returns are not very interesting if plausible undesirable scenarios bankrupt the business. Moreover, clearly defining the plausibility of undesirable and desirable scenarios requires the language of probability.

For portfolios of many businesses or other assets, probabilistic analysis is especially valuable, but poses additional challenges of *communication* and *computation*. The communication challenge is to present a large number of

business and portfolio distributions so that less technical senior executives can absorb the conclusions readily, at a level of detail appropriate for the decisions to be made. Too much detail would distract attention away from strategic insights, toward results that may not be meaningful given the high-level nature of the analysis. On the other hand, too little detail could prevent key insights.

The computational challenge results from the combinatorial explosion of cases to analyze. For example, in the portfolio strategy project described here, our client was a diversified packaging and plastics company with 20 businesses. If we were to discretize the potential return from each of its businesses to just three cases, we would face a probability tree with  $3^{20}$  or about 3.5 *billion* branches.

A traditional way to limit this expansion in our consulting practice starts by sorting the uncertainties by impact on the value measure, as shown in the “tornado” chart of Figure 1. Then we fixate all the lower-impact uncertainties at their means (expected values) and evaluate the remainder probabilistically with a decision tree. To determine which uncertainties to evaluate probabilistically, we can estimate the percent of variance in the value measure explained by each uncertainty. In long-term business strategy models, we often find that the top five or so variables are sufficient to explain at least 90% of the variance.

However, keeping only the relatively high-impact uncertainties may be impractical for large portfolios, because they may have too many such uncertainties (graphically, the sides of the tornado of Figure 1 may slope inward too gently). In the project described here, we especially valued computation speed, because we wanted to interactively explore the implications of alternative portfolios in a workshop with the executive team. We also wanted to avoid the inherent computational waste of a decision tree or Monte Carlo simulation with a branch for every combination of asset outcomes, when the outcomes are always added along the branches. The problem cries out for a more efficient, specialized solution method.

### **Business and Portfolio Evaluation**

Figure 2 gives an overview of our evaluation. We began at the individual business level. Working in parallel, several client/consultant teams developed strategies and associated cash flow models for each of our client’s 20 businesses. Returns were measured in two ways:

1. by NPV of free cash flow to perpetuity, using a simple continuing value model beyond our 15-year planning period, and
2. by NPV over the first six years, to capture short-term impacts of capital investments.

Each business model included two types of input uncertainties: business-specific ones, treated as independent across businesses, and global ones, varied together for all businesses during the portfolio evaluation. We initially considered price,

competitive response, and macroeconomic variables as global uncertainties but found that the key global uncertainties were macroeconomic—near-term and long-term GDP, inflation, and interest rates. We reduced these to a set of representative scenarios for use in each business model. As Figure 1 illustrates, we generated tornado charts from each business model to focus attention on the key business-specific uncertainties and the most promising strategies; decision tree analyses with these yielded probability distributions for return.

The portfolio strategy theme decision at the top of Figure 2 was specified during the final phase of the project, after the business strategies were evaluated. Each portfolio strategy theme selected a specific strategy for each business, as illustrated in Table 1. We stopped short of trying to determine the optimal set of business strategies, leaving our client’s senior management the reduced problem of Figure 2b. This allowed management to include other constraints and goals not reflected in our NPV measures.

This framework led us to the following computational challenge: given the distribution of value for each value measure, for each strategy, for each business, and for each global scenario, how can we approximate the distribution of portfolio value quickly? Our solution required three probability concepts (see Figure 3 and the appendix). First, the mean, variance, and skewness of a specified probability distribution are easy to calculate, and these “cumulants” are additive for independent uncertainties [Howard 1971, Stuart 1987]. Second, we can easily convert between cumulants and raw moments, which can be rolled back in a decision tree to remove conditioning on the global scenario [Smith 1993]. Finally, from a mean, variance, and skewness we can fit an approximate probability distribution, and hence any percentiles. If the skewness, which measures tilt, is zero, our fit reduces to the normal distribution commonly used in simpler probability models. We could have used more cumulants for more accuracy, but we have observed very reasonable accuracy for strategic planning purposes with three cumulants. The more businesses in the portfolio and the more uncertainty is incorporated in each business model, the better our three-cumulant fit, as irregularities in the distributions are smoothed out.

With these probability concepts, we solved the computational challenge in six steps for each portfolio strategy theme and value measure, as illustrated in Figure 4:

1. Calculate the first three cumulants (mean, variance, and skewness) of the return from each business by scenario.
2. Add the corresponding cumulants to get portfolio cumulants by scenario. This works because the returns from each business are assumed to be independent given the scenario.

3. Convert cumulants to raw moments, to allow rolling back the tree across scenarios.
4. Roll back across scenarios: find the overall unconditional raw moments of portfolio return as weighted averages of the overall conditional raw moments with the scenario probabilities as weights. Caveat: although we can also roll back each business's raw moments across scenarios to get moments of unconditional returns by business, these are dependent across businesses, so their cumulants do not sum.
5. Convert from raw moments back to cumulants. (This step can be skipped if you use raw moments in the next step.)
6. Fit a smooth distribution to the final cumulants and note useful percentiles of the distribution, such as the 10<sup>th</sup> and 90<sup>th</sup>.

Note that if we did not need multiple scenarios, we could skip steps 3, 4, and 5, leaving a three-step process. Portfolio variance and skewness arise both from individual scenarios and from differences across scenarios. Thus, the portfolio distribution would still show variance and typically skewness if we did not use multiple scenarios, or at the other extreme, if no uncertainty were modeled within each global scenario.

Our spreadsheet implementation of this process was fast enough for interactive use with senior management. Given previously calculated business-level distributions and their cumulants, we created, evaluated, and displayed results of new portfolio strategy themes in seconds, by filling in a row of the spreadsheet version of Table 1 and recalculating.

### **Portfolio Strategy Exploration**

We chose to summarize the fitted distributions of portfolio return in “flying bar” charts, as illustrated in Figure 5, rather than display the full distributions. Our reasons were:

- The flying bar charts display many distributions compactly, while still capturing important characteristics including skewness.
- Flying bar charts are easier for a less technical audience to understand than probability distributions.
- The additional detail would be hard to justify given the three-parameter approximations we used and the nature of our long-term strategy modeling.

We used a 10<sup>th</sup> to 90<sup>th</sup> percentile range for each bar because the endpoints are easy to explain (“there’s a 10% chance that the outcome will be below the low end and a 10% chance that it will be above the high end”). Also, the endpoints are plausible while spread sufficiently to reflect skewness, and they are consistent with our usual initial 10<sup>th</sup>, 50<sup>th</sup>, and 90<sup>th</sup> percentile assessments of uncertain inputs. We set up our spreadsheet model to generate the flying bar charts automatically, in preparation for a two-day portfolio strategy workshop with the executive team.

During the workshop, the team asked us to explore many new themes as it grappled with capital, human resource, and management attention constraints, as well as other considerations not reflected in our value measures. Together we developed several flexible strategies that satisfied these additional constraints and considerations, while more than doubling the initial NPV of the company. Most of these new strategies probabilistically dominated the company's "momentum" strategy, providing a higher probability of exceeding any given NPV. The almost instantaneous evaluation capability and the flying bar charts proved to be effective in providing and communicating the level of understanding desired by the executive team.

### **Acknowledgments**

The use of portfolio strategy themes and tables, tornado and cumulative distribution charts, and flying bar charts has been refined over many years by Burke Robinson and others in the portfolio strategy practice area at Strategic Decisions Group. Burke provided valuable comments on the draft. Steve Koenig led the project described here, and Todd Flynn and Jay Goldman executed much of the analysis.

### **APPENDIX: Converting Among Three-Parameter Representations of Uncertainties**

Our business evaluations yielded discrete distributions for return, from which cumulants are calculated as follows. If a distribution has probabilities  $p_i$  and values  $x_i$  for  $i = 1, \dots, n$ :

$$\text{mean} = \sum_{i=1}^n p_i x_i$$

$$\text{variance} = \sum_{i=1}^n p_i (x_i - \text{mean})^2$$

$$\text{skewness} = \sum_{i=1}^n p_i (x_i - \text{mean})^3.$$

(However, the fourth cumulant is not simply the expected deviation to the fourth power.)

We summed these cumulants over the portfolio assets for each global scenario. Then we took the expectation over the scenario variable of the corresponding raw moments (moments about zero), which are found as follows:

$$1^{\text{st}} \text{ raw moment} = \text{mean}$$

$$2^{\text{nd}} \text{ raw moment} = \text{variance} + \text{mean}^2$$

$$3^{\text{rd}} \text{ raw moment} = \text{skewness} + 3 \cdot \text{mean} \cdot \text{variance} + \text{mean}^3.$$

The results were the unconditional raw moments for the portfolio, because we can roll back a decision tree by taking expectations of any raw moment, not just the mean [Smith 1993].

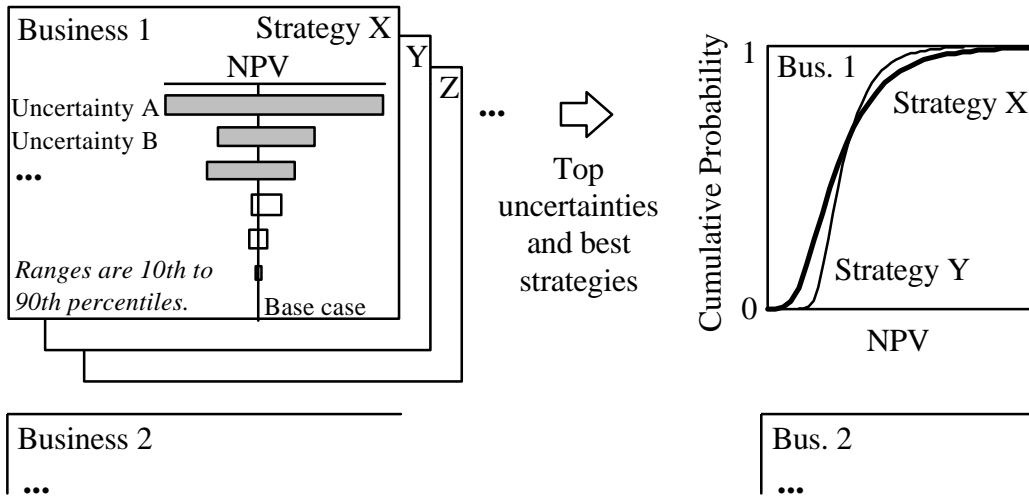
To convert the portfolio raw moments back to a distribution and percentiles, we fitted a shifted lognormal distribution (a lognormal plus a constant determined by

the fit), which approaches a normal as the skewness approaches zero. We reflected the distribution if the skewness was negative, indicating a longer tail on the left side. Many other distributions could have been used, but the reasonable-looking choices that we tested gave similar results. For example, the skew logistic distribution [Lindley 1987] with the same mean, variance, and skewness as our generalized lognormal through 10<sup>th</sup>, 50<sup>th</sup>, and 90<sup>th</sup> percentiles of 100, 200, and 400 respectively, has 10<sup>th</sup>, 50<sup>th</sup>, and 90<sup>th</sup> percentiles of 111, 190, and 404.

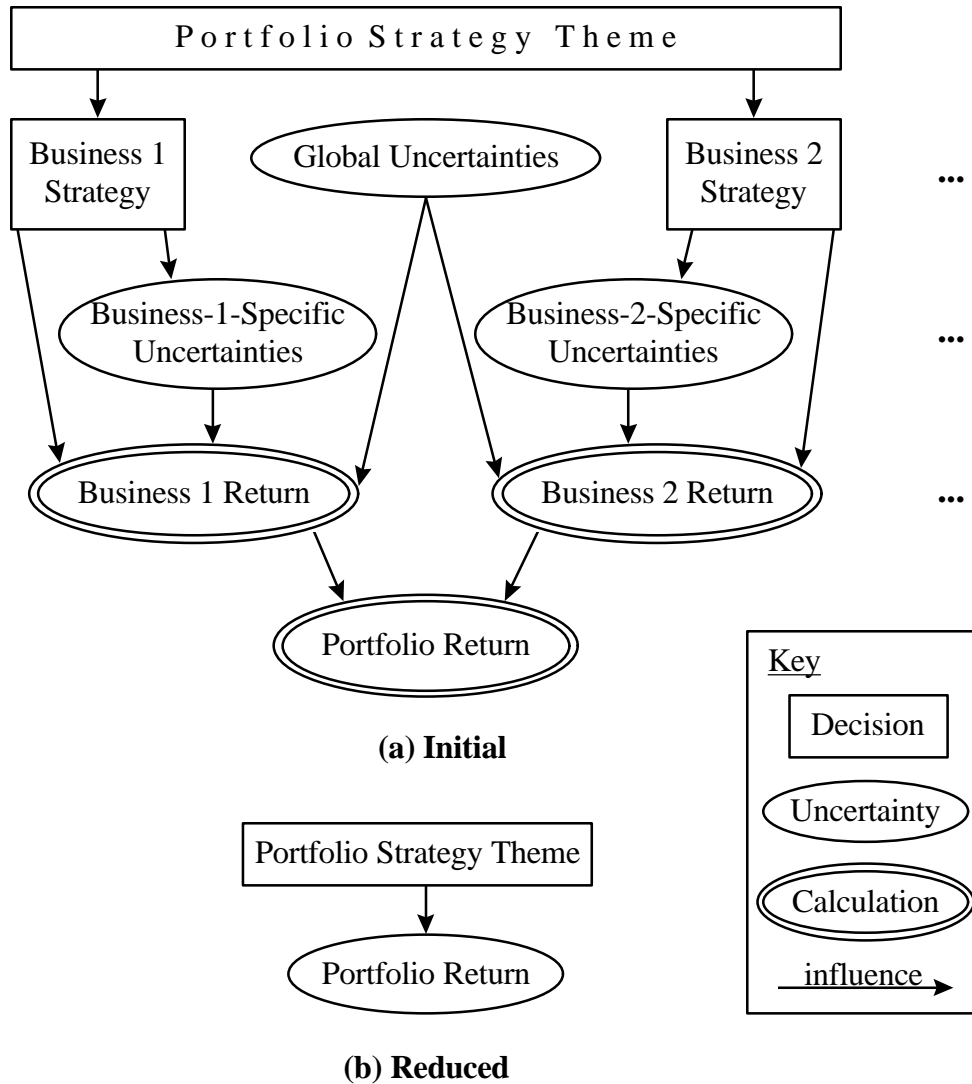
### **References**

- Howard, R. A. 1971, "Proximal Decision Analysis," *Management Science*, Vol. 17, No. 9 (May). Reprinted in Howard, R. A. and Matheson, J. E. (Eds.) 1984, *Readings on the Principles and Applications of Decision Analysis*, Vol. II, Strategic Decisions Group, Menlo Park, CA.
- Lindley, D. V. 1987, "Using Expert Advice on a Skew Judgmental Distribution," *Operations Research*, Vol. 35, pp. 716-721.
- Smith, J. E. 1993, "Moment Methods for Decision Analysis," *Management Science*, Vol. 39, No. 3 (March), pp. 340-358.
- Stuart, A. and Ord, J. K. 1987, *Kendall's Advanced Theory of Statistics, Volume 1: Distribution Theory*, Oxford Pub., New York.

**William B. Poland** is a technical associate at Strategic Decisions Group, a management consulting firm that seeks strategic decision quality. He has a PhD in decision analysis from Stanford University. He never met an uncertainty he didn't like.



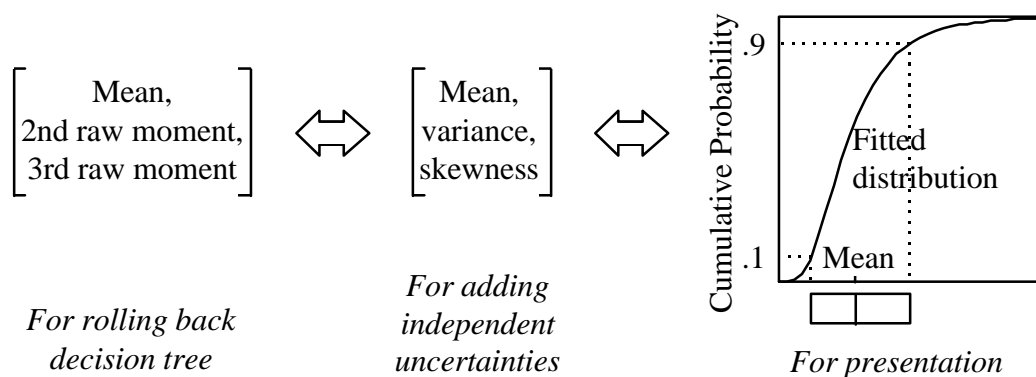
**Figure 1: For each business, tornado charts help us identify the key uncertainties and most promising strategies, and probabilistic analysis of these yields value distributions.**



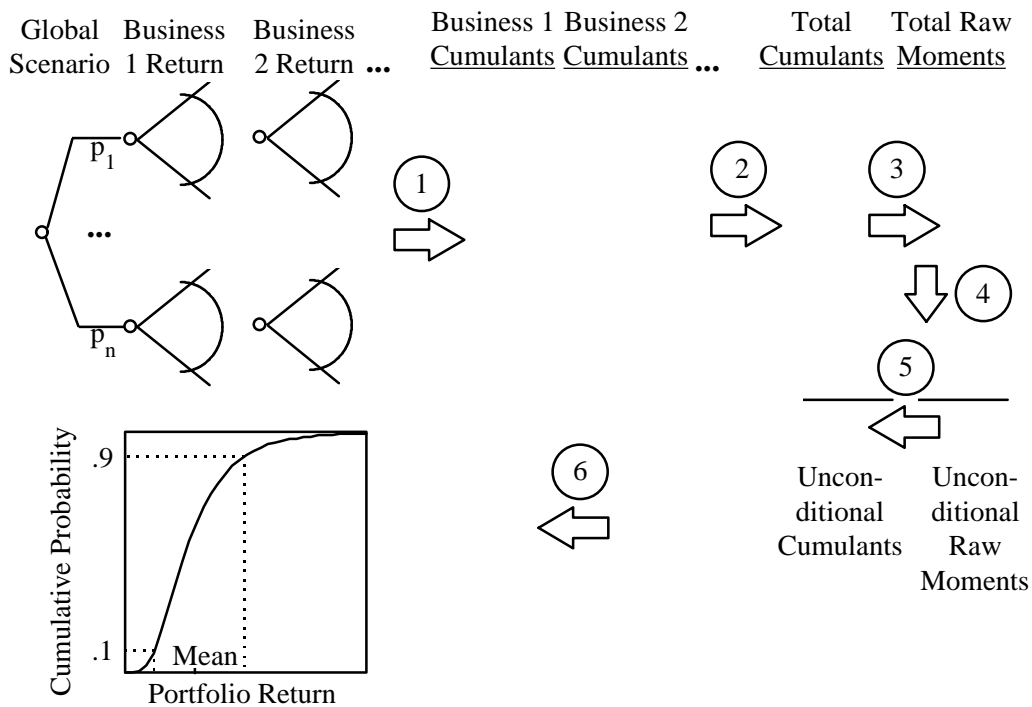
**Figure 2: An influence diagram illustrates the portfolio evaluation problem. The reduced diagram gives the probability distribution of portfolio return for each portfolio strategy theme. In selecting a theme, the decision makers also consider resource constraints and other value measures.**

Portfolio Strategy Theme	Bus. 1 Strategy	Bus. 2 Strategy	...
Momentum	Momentum	Momentum	
Aggressive	Expansion	Acquisition	
Harvest	Contraction	Divestiture	
...			

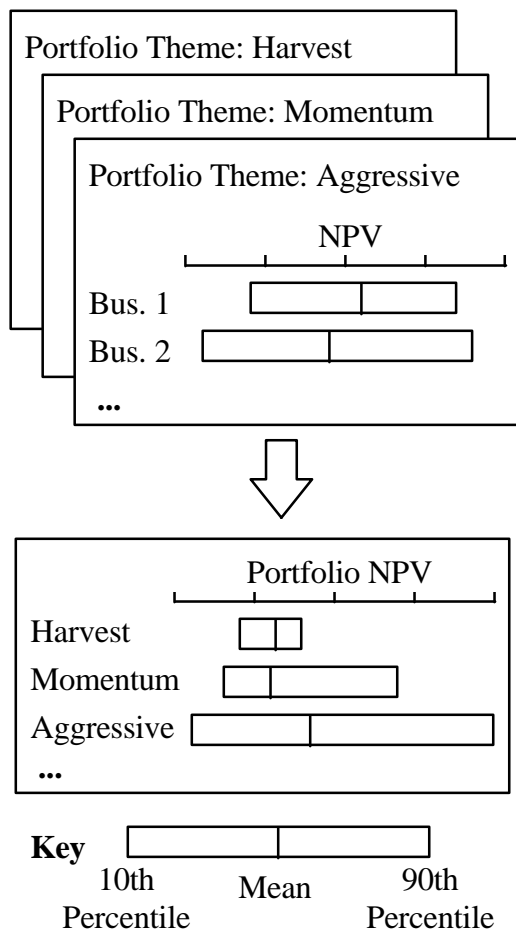
**Table 1: Each portfolio strategy theme corresponds to a set of strategies for all businesses in the portfolio.**



**Figure 3: We can easily convert among three-parameter representations of probability distributions as needed. We could use the 50<sup>th</sup> percentile instead of the mean on the bar, but this is less useful in decision-making.**



**Figure 4:** For each portfolio strategy theme, we combine previously developed distributions for each business by adding cumulants, taking the probability-weighted average over global scenarios of corresponding raw moments, and fitting a distribution to the result.



**Figure 5: Flying bars summarize the distributions of return from each business and the overall portfolio, for each portfolio strategy theme. Similar charts show other value measures such as six-year NPV and show returns conditional on each global scenario.**