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RISK-VALUE THEORY

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Abstract

In this paper, we propose a new descriptive theory of decision making under risk, called "risk-value theory," which leads to decision making by explicitly trading off between risk and value (i.e., a subjective value of mean). Our risk-value theory can be either consistent with expected utility theory or represent preferences consistent with non-expected utility theory. Introducing a risk independence condition and other preference conditions, we propose several explicit forms of risk-value models and discuss their dominance and risk aversion properties. Specifically, we develop three useful classes of decision models: exponential risk-value models, generalized disappointment models, and moments risk-value models. These models provide new resolutions based on the intuitively appealing idea of risk-value tradeoffs for observed risky choice behavior and the decision paradoxes that violate the independence axiom of expected utility theory. In particular, this development unifies two streams of research: one in developing preference models and the other in modeling risk judgments. Because of the new perspective of our risk-value theory, it also provides fresh insights into many practical problems and new tools for resolving them.

*(**Keywords**: Risk, risk-value tradeoffs, expected utility theory, non-expected utility theory, decision paradoxes)*

1. Introduction

Risk has been a subject of interest and study in many fields including decision science, finance, insurance, economics, and psychology. The role of risk in decision making under uncertainty has been investigated extensively in these fields for many years. In the area of finance, investors are assumed to make their choices among financial alternatives by intuitively trading off risk and mean return. Markowitz (1959, 1987, 1991) proposed variance as a measure of risk, and a mean-variance model for portfolio selection based on minimizing variance subject to a given level of mean return. The mean-variance approach has been widely used in financial decision making; it has also been the basis of mean-variance capital-market equilibrium models (e.g., see Sharpe, 1964, 1970, 1991). But arguments have been made that mean-variance models are appropriate only if the investor's utility function is quadratic or the joint distribution of returns is normal. However, these conditions are rarely satisfied in practice. In fact, even expected utility theory, used as the foundation of mean-variance models and other mean-risk studies (e.g., Stone, 1970; Fishburn, 1977; Meyer, 1987), has been called into question by empirical studies of risky choice (e.g., Allais, 1953, 1979; Kahneman and Tversky, 1979; Machina, 1987; Fishburn, 1988). A better model and a general theory regarding the paradigm of risk and return tradeoffs are needed.

In the area of psychology and decision making, Coombs (1969, 1975) proposed a Portfolio theory in which a choice among risky alternatives (hereafter, "lotteries") is a compromise between maximizing expected value and "optimizing" the level of risk. This framework has been axiomatized first by van Santen (1978), and then by Suck and Getta (1994) with some refinement. The basic assumption of Portfolio theory, that preference can be determined by expected value and perceived risk, was supported by several experimental studies (e.g., Coombs and Huang, 1970; Nygren, 1977; Lehner, 1980; Wilm and Wendt, 1995). However, the measure of risk in Portfolio theory has been left essentially undefined, and an explicit model regarding this theory has never been developed.

In recent years, there have been several studies on measures of risk and risk-value (or risk-return) tradeoffs within the expected utility framework (Bell, 1988, 1995; Sarin and Weber, 1993; Jia and Dyer, 1996; Dyer and Jia, 1997a, 1997b). In our own study, we proposed a standard measure of risk based on the converse expected utility of normalized lotteries with zero-expected values. This measure of risk has many desirable properties that capture intuitively appealing notions of risk. It includes many previously proposed measures of risk as special cases, and provides a preference-based and unified approach for risk studies. We showed that under a condition called risk independence, an expected utility model can be represented as a mean-risk form, called the standard risk-value model. This model leads to decision making by explicitly trading off between risk and value, where value is defined as the utility value of mean. However, because this study is based on the expected utility theory that is in a state of flux, the standard risk-value model may not be valid for describing actual choice behavior based on risk-value tradeoffs. The purpose of the present study is to extend our risk-value framework for non-expected utility preferences and to develop a new descriptive theory of decision making under risk, called the “risk-value theory."

The rest of this paper is organized as follows. In the following section, we develop a general theory of risky decision making based on risk-value tradeoffs, and propose several basic forms of risk-value models. In Section 3, we discuss dominance conditions and risk propensities in our risk-value models. In Section 4, we further develop three useful classes of risk-value models: generalized exponential risk-value models, generalized disappointment models and moments risk-value models. We demonstrate that our risk-value models are very flexible in modeling people's choice behavior under risk, and show how those models can resolve some decision paradoxes that violate the independence axiom of expected utility theory. Finally in Section 5, we summarize this study and discuss some implications of our risk-value theory. All proofs in this paper are provided in the appendix.

2. Development of risk-value models

In the following, we present a basic framework of decision making by risk-value tradeoffs, and propose several general forms of risk-value models based on two-attribute utility axioms.

2.1. A Two-Attribute Structure for Risk-Value Tradeoffs

In our previous study (Jia and Dyer, 1996), we considered decomposing a lottery (or a random variable) X into its mean $\bar{X}$ and its standard risk X' defined by $X' = X - \bar{X}$. Thus, a lottery can be represented by a "two-attribute" structure, i.e., $X = (\bar{X} + X')$, and an expected utility model can be decomposed into a risk-value form under a risk independence condition. However, the notion of risk-value tradeoffs within the expected utility framework is very restrictive, and only consistent with a small set of utility functions (i.e., quadratic, exponential, and linear plus exponential models). In fact, as we mentioned, the traditional expected utility theory is not appropriate to be used as a foundation for a descriptive decision theory .

In order to be more realistic and flexible in the framework of risk-value tradeoffs, it is natural to extend the basic idea above and consider a general two-attribute structure ($\bar{X}$, X'). In this way we can truly separate the evaluation of lotteries into two attributes, mean and risk, so that the mean-risk (or risk-value) tradeoffs are not necessarily consistent with the traditional expected utility decisions. Note that we only use the structure ($\bar{X}$, X') for the ex ante evaluation of a lottery X. After the realization of a lottery, its ex post performance or actual outcome should be ($\bar{X} + x'$), where x' is the realization of the standard risk X'. Further, for ease of exposition, we will refer to the realization of a lottery as though it is measured in dollars, although this development applies generally to an outcome measured on any real-valued scale.

Let P be a convex set of all simple probability distributions or lotteries $\{X, Y, Z, ...\}$ on a nonempty set $\mathbf{X}$ of outcomes,[1] and P^o be a subset of P, a set of normalized probability distributions defined by $P^o = \{X¢ \mid X¢ = X - \bar{X},\ X \quad P\}$, where $\bar{X}$ is the mean of X. Similarly, let $\mathbf{P}$ be a convex set of the corresponding two-attribute probability distributions

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[1] In this paper, we use X, Y and Z (or $(\bar{X}, X')$, $(\bar{Y}, Y')$ and $(\bar{Z}, Z')$) to refer to either probability distributions or random variables interchangeably.

$\{(\bar{X}, X'), (\bar{Y}, Y'), (\bar{Z}, Z'), ...\}$on a product set $\mathbf{X}_1 \times \mathbf{X}_2$ of outcomes. For this special case, the outcome of a lottery on $\mathbf{X}_1$ is fixed, which is its mean; thus, the marginal distribution on $\mathbf{X}_1$ is a degenerate one with a singleton outcome $\bar{X} \in \mathbf{X}_1$. In this study, we assume $|\bar{X}| < \infty$; that is, the mean of a lottery is bounded. For the second attribute, the marginal distribution on $\mathbf{X}_2$ is X' $\mathbf{P}^{o}$, which is a normalized probability distribution. Therefore, $(\bar{X}, X')$ denotes the distribution in $\mathbf{P}$ that yields $\bar{X} \in \mathbf{X}_1$ with probability 1 coupled with $x' \in \mathbf{X}_2$ with probability X', where x' is a realization of X'. Since $\bar{X}$ is a constant, the two marginal distributions $\bar{X}$ and X' are sufficient to determine a unique distribution in $\mathbf{P}$.

Let $>_{\tilde{\mathbf{p}}}$ be a strict binary preference relation on $\mathbf{P}$; $\sim_{\tilde{\mathbf{p}}}$ an indifference relation; and $\geq_{\tilde{\mathbf{p}}}$ a weak preference relation. We shall assume throughout that the von Neumann and Morgenstern utility axioms, or an equivalent set of axioms (e.g., Fishburn, 1970), hold for the two-attribute relation $>_{\tilde{\mathbf{p}}}$ on $\mathbf{P}$. Thus, based on the expected utility theory, there exists a function U: $\mathbf{X}_1 \times \mathbf{X}_2 \to$ Re, such that for all $(\bar{X}, X'), (\bar{Y}, Y') \in \mathbf{P}$,

$$(\bar{X}, X') >_{\tilde{\mathbf{p}}} (\bar{Y}, Y') \iff \mathrm{E}[U(\bar{X}, X')] > \mathrm{E}[U(\bar{Y}, Y')]$$

where the symbol E is an expectation operator. Moreover, such a U is unique up to a positive linear transformation. For the two-attribute expected utility model or risk-value model $\mathrm{E}[U(\bar{X}, X')]$, because the first attribute is certain, the expectation only needs to be taken over the marginal distribution for the second attribute, which, in fact, is the original distribution of a lottery X over the standard risk $X' = X - \bar{X}$. As a special case when the relationship between $\bar{X}$ and X' is a simple addition, the risk-value model $\mathrm{E}[U(\bar{X}, X')]$ reduces to a traditional expected utility model, i.e., $\mathrm{E}[U(\bar{X}, X')] = \mathrm{E}[U(\bar{X} + X')] = \mathrm{E}[U(X)] = a\mathrm{E}[u(X)] + b$, where a > 0 and b are constants, and u is a single-attribute von Neumann-Morgenstern utility function.

In the past, the independence axiom of (single-attribute) expected utility theory has been challenged as a reasonable assumption for risky choice behavior (e.g., Allais, 1979; Kahneman and Tversky, 1979; Machina, 1982, 1987). In this development, the traditional independence axiom is replaced by a two-attribute independence axiom; i.e., for all $0 < \mathbf{l} < 1$ and $(\bar{X}, X')$, $(\bar{Y},$

$Y')\in\mathbf{P}$, if $(\bar{X},\ X')>_{\tilde{p}}(\bar{Y},\ Y')$, then $\{\lambda,(\bar{X},\ X');(1-\lambda),(\bar{Z},Z')\}>_{\tilde{p}}\{\lambda,(\bar{Y},\ Y');(1-\lambda),(\bar{Z},Z')\}$ for all $(\bar{Z},Z')\in\mathbf{P}$. Thus, the preference function U also has a linear property in the sense that a convex combination of two lotteries $\{\lambda,(\bar{X},\ X');(1-\lambda),(\bar{Y},\ Y')\}$ can be evaluated by $\lambda\,\mathrm{E}[U(\bar{X},\ X')]+(1-\lambda)\mathrm{E}[U(\bar{Y},\ Y')]$, where $0<\lambda<1$.[2] If $(\bar{X},\ X')\equiv(\bar{X}+X')=(X)$ and if the lotteries considered are not all degenerate (i.e., $X'=0$) or are not all pure risk (i.e., $\bar{X}=0$), then the two-attribute independence axiom will not generally result in "linearity in probability" in the risk-value model. This is because the underlying probabilities of lotteries are also implied in the means, which can enter both the value attribute and the risk attribute in some nonlinear and asymmetric fashions. Thus, our treatment is different from other non-expected utility models; we attribute "non-linearity in probability" to some "inconsistent" tradeoffs between risk and value.

2.2. Risk Independence and Risk-Value Models

To obtain some separable forms of the risk-value model $\mathrm{E}[U(\bar{X},\ X')]$, we assume a risk independence condition similar to the utility independence condition of multiattribute utility theory (Keeney and Raiffa, 1976).

DEFINITION 1. X' is risk independent of $\bar{X}$ if there exists a $\bar{w}_o\in\mathbf{X}_1$ for which $(\bar{w}_o,\ X')>_{\tilde{p}}(\bar{w}_o,\ Y')$, where $X',\ Y'\in P^o$, then $(\bar{w},\ X')>_{\tilde{p}}(\bar{w},\ Y')$ for all $\bar{w}\in\mathbf{X}_1$.

Intuitively, this risk independence condition states that for lotteries with a common mean, the choice between them will be determined by the ordering of the risk associated with their normalized probability distributions. In an early experimental work, Edwards (1954) studied preferences over pairs of lotteries involving different fixed probabilities and a common (though

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[2] Here λ can be viewed as a marginal probability. Thus, in this two-attribute structure, we can treat the marginal probability and the intrinsic probabilities of lotteries (i.e., $(\bar{X},X')$ and $(\bar{Y},Y')$) differently, which allows the violation of the compound probability principle. This has an important implication in multi-stage gambles, and can provide an interpretation for both the isolation effect and the decision paradoxes that violate the independence axiom of expected utility theory (see Jia, 1995).

variable) mean, and found that the choice patterns at different levels of common means are substantially the same, i.e., independent of the mean. His study directly supports our assumption of risk independence. Other relevant supporting evidence is Markowitz's (1952) observation that an individual's risky choice behavior is essentially the same whether he or she is poor or rich. Machina (1982) also assumed the relative invariance of risky preferences to initial wealth in his local utility theory. Kahneman and Tversky (1979) made the even stronger assumption that people tend to discard components that are shared by the offered lotteries, such as a common bonus to all lotteries.

Note that our notion of risk independence only concerns lotteries that have the same expected value (including initial wealth). For general lotteries involving different means, the preference ranking of these lotteries may shift when their means change in a common amount (or wealth level alters).

In the risk independence condition, since we could choose $\overline{w}$ = 0, the same ordering for lotteries with zero-expected values would be required for lotteries with equal, non-zero expected values, but with the same normalized probability distributions. For lotteries with zero-expected values, it seems natural to require a consistent condition between $>_{\tilde{\mathbf{p}}}$ for the risk-value preference and $>_p$ for the traditional utility preference, where $>_p$ is a strict preference relation defined on P (or its subset P^o).

DEFINITION 2. Decisions are consistent if $(0, X') >_{\tilde{\mathbf{p}}} (0, Y') \Leftrightarrow X' >_p Y'$, where $X', Y' \in P^o$.

Together with the risk independence condition, the consistency condition implies that for lotteries with the same expected values, the two-attribute (risk and value) decision problem can be reduced to a traditional expected utility problem. Thus, for $\overline{X}$ = 0, $\mathrm{E}[U(0,X')] = a\mathrm{E}[u(X')] + b$ or $\mathrm{E}[U(0,X')] = -aR(X') + b$, where $a > 0$ and b are constant, u is the traditional utility function, and $R(X') = -\,\mathrm{E}[u(X')] = -\,\mathrm{E}[u(X - \overline{X})]$ is the standard measure of risk (Jia and Dyer, 1996); and for $\overline{X} = \overline{Y}$, then $\mathrm{E}[U(\overline{X}, X')] > \mathrm{E}[U(\overline{Y}, Y')]$ if and only if $\mathrm{E}[u(X')] > \mathrm{E}[u(Y')]$ or $R(Y') > R(X')$. In fact, for lotteries with zero-expected values, the only choice attribute of relevance for them is risk. A riskier lottery would be less preferable and vice versa, assuming one is risk averse. (A similar argument can be made for a risk seeker.) Therefore, the riskiness ordering of these lotteries should simply be the inverse of the preference ordering. This provides a connection between the risk-value model and the standard measure of risk.

Even though expected utility theory (mainly the independence axiom) has been challenged by some empirical studies for general lotteries, we believe that it should be appropriate for describing risky choice behavior within a special set of normalized probability distributions with the same expected values. For general lotteries with different means, however, our two-attribute risk-value model can deviate from the traditional expected utility preference. This consideration leads to a great simplification for our risk-value model.

THEOREM 1. The risk-value model can be decomposed into

$$\mathrm{E}[u(\bar{X}, X')] = V(\bar{X}) - \varphi(\bar{X})[R(X') - R(0)] \qquad (1)$$

if and only if the risk independence and consistency conditions hold, where $\varphi(\bar{X}) > 0$. Furthermore, three other functions $F(\bar{X})$, $\psi(\bar{X})$ and $G(X')$, corresponding to $V(\bar{X})$, $\varphi(\bar{X})$ and $R(X')$ respectively, also satisfy (1), if and only if there exist some constants $a, c > 0$ and b, d such that $F(\bar{X}) = aV(\bar{X}) + b$, $G(X') = cR(X') + d$, and $\psi(\bar{X}) = a/c\, \varphi(\bar{X})$.

In model (1), $V(\bar{X})$ is a subjective value measure for the expected value of a lottery, $R(X')$ is a standard measure of risk, and $\varphi(\bar{X}) > 0$ is a tradeoff factor between risk and value. Since $R(0) = -u(0)$ is a constant, $R(X') - R(0)$ is also a standard measure of risk, where $R(0)$ serves as an anchor for this measure of risk. If the utility function u is strictly concave, then $R(X') - R(0) > 0$ and model (1) will imply risk aversion. This will be discussed in Section 3.3.

In general, these three functions, $V(\bar{X})$, $R(X')$ and $\varphi(\bar{X})$ in model (1), can be considered independently, which thus leads to a very flexible structure for risk-value tradeoffs. As a special case when the risk-value model (1) is consistent with an expected utility model, the value measure

$V(\bar{X}) = u(\bar{X})$ and the tradeoff factor $f(\bar{X}) = u''(\bar{X})/u''(0)$ (Jia and Dyer, 1996). Thus, the traditional expected utility preference requires very restrictive conditions regarding risk-value tradeoffs.

The joint factor $f(\bar{X})[R(X') - R(0)]$ in model (1) can be viewed as a measure of the intensity of the risk effect on preference, which is also an appropriate measure of perceived riskiness if $f(\bar{X})$ is a decreasing function of $\bar{X}$.[3] Thus, according to the structure of model (1), preference can be determined simply by a value measure less the perceived riskiness. A decreasing tradeoff factor $f(\bar{X})$ also indicates that the intensity of the risk effect on preference decreases as a positive constant amount is added to all outcomes of a lottery.

In the risk-value model (1), the standard measure of risk is also based on a utility model. But the choice of a value measure for the certain attribute should be different from that of a utility measure for the uncertain attribute because the latter involves risk (e.g., see Dyer and Sarin, 1982; von Winterfeldt and Edwards, 1986). Our risk-value model (1) actually provides an explicit way to separate an individual's risk attitude from the measure of value. Especially when there is no risk involved, then $U(\bar{X}, 0) = V(\bar{X})$ and the risk-value decision problem reduces to a single-attribute problem concerning only the certain consequences. In general, the value measure $V(\bar{X})$ should be an increasing function; that is, more money is preferred to less.

The basic form of risk-value model (1) may be further simplified if some stronger preference conditions are satisfied (see Jia, 1995). When $f(\bar{X}) = k > 0$, model (1) becomes the following additive form of risk-value models:

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[3] For the lotteries with zero-expected values, our standard measure of risk can be a suitable measure for perceived risk. However, for general lotteries, the standard measure of risk may not be appropriate. This is because the standard measure of risk is only a measure of "pure" risk that is independent of any payoffs with certainty. Nevertheless, empirical studies have shown that subjects' perceived risk decreases as a positive constant amount is added to all outcomes of a lottery (Coombs and Lehner, 1981; Keller, Sarin and Weber, 1986). This is also one of the major axioms in Pollatsek and Tversky’s (1970) risk theory. However, the joint factor $f(\bar{X})[R(X') - R(0)]$ with a decreasing $f(\bar{X})$ can appropriately capture this phenomenon. This measure also has many other appealing properties for perceived risk and can unify a large body of empirical evidence (Jia, Dyer and Butler, 1996).

$$E[U(\bar{X},\ X')] = V(\bar{X}) - k[R(X') - R(0)]. \tag{2}$$

When $\varphi(\bar{X}) = -V(\bar{X}) > 0$, then model (1) reduces to the following multiplicative form:

$$E[U(\bar{X},\ X')] = V(\bar{X})R(X') \tag{3}$$

where $R(0) = 1$ and $V(0) = 1$. In model (3), $R(X')$ serves as a value discount factor due to risk.

In this paper, we treat an individual's wealth level in a generic way, so that X could include one's initial level of wealth and a lottery. If we state the level of one's wealth explicitly, say $X = w + Y$, then $\bar{X} = w + \bar{Y}$ and $X' = Y'$, where w is one's initial wealth level and Y is a "true" lottery. Notice that the standard measure of risk is independent of wealth level and any payoffs with certainty. Thus, the risk-value model (1) can also be written as

$$E[U(w+\bar{Y},\ Y')] = V(w+\bar{Y}) - \varphi(w+\bar{Y})[R(Y') - R(0)]. \tag{4}$$

The advantage of model (4) is that it can consider the effect of wealth level on risky decision making in an obvious way. The wealth level may influence preference through the value measure and the tradeoff factor. For example, we may choose a decreasing function for the tradeoff factor such that perceived riskiness has a decreasing impact on preference when an individual's wealth level increases.

3. Dominance and Risk Propensities in Risk-Value Models

In this section, we discuss dominance conditions and risk propensities for the risk-value model (1). These conditions can help a decision maker specify appropriate functional forms for the risk measure, the value measure and the tradeoff factor. The results derived in this section can also be applied to the simplified risk-value models (2) and (3).

3.1. Dominance Conditions

When two lotteries have the same standard risk to an individual, then the individual should prefer the lottery with a higher expected value. This defines a dominance condition.

DEFINITION 3. The mean-dominance condition is satisfied if $(\bar{X}+\Delta,\ X') >_{\tilde{p}} (\bar{X},\ X')$ for any $\Delta > 0$.

THEOREM 2. For the risk-value model (1), the mean-dominance condition holds if and only if $V'(\bar{X}) - \varphi'(\bar{X})[R(X') - R(0)] > 0$ for all lotteries, where V' and φ' are the first derivatives of the value measure and the tradeoff factor respectively.

The mean-dominance condition is required for all risk-value models. Based on the conditions in Theorem 2, we can specify some constraints for the choices of the value function and the tradeoff factor. For example, for an increasing function $V(\bar{X})$ and $R(X') > R(0)$, a nonincreasing $\varphi(\bar{X})$ will always satisfy the mean-dominance condition.

The mean-dominance condition is a special case of first order stochastic dominance. A lottery Y is said to be first order stochastically dominated by another lottery X if $F_Y(t) \geq F_X(t)$ for all $t \in (-\infty,+\infty)$, where $F_Y(t)$ and $F_X(t)$ are the cumulative probability distribution functions of the lotteries Y and X respectively. Machina (1982) argued that the first order stochastic dominance condition should be satisfied by individuals. However, Tversky and Kahneman (1986) found that the dominance condition is usually obeyed in transparent problems, but frequently violated in nontransparent ones.

If the first order stochastic dominance condition is required in risk-value tradeoffs, we can employ Machina's (1982) local utility theory to establish a condition for a continuously differentiable risk-value model (i.e., the value measure, the tradeoff factor and the risk measure in (1) are all continuously differentiable).

THEOREM 3. For the risk-value model (1), $V'(\bar{X}) - \varphi'(\bar{X})[R(X') - R(0)] + \varphi(\bar{X})\{u'(x - \bar{X}) - \mathrm{E}[u'(X')]\} \geq 0$ for all lotteries and all x if and only if the first order stochastic dominance condition is satisfied, where V' and φ' are the first derivatives of the value measure and the tradeoff factor respectively, u' is the first derivative of the utility function for the standard measure of risk, and x is a real variable.

Since $u'(x-\bar{X})-\mathrm{E}[u'(X')]$ (assuming that u is increasing) can be both positive and negative for different x and all lotteries, the mean-dominance condition in Theorem 2 $V'(\bar{X})-\mathrm{f}\,'(\bar{X})[R(X')-R(0)]>0$ is necessary for the first order stochastic dominance condition in Theorem 3. This shows a relationship between these two dominance conditions.

3.2. Mean-Risk Aversion

The following definition specifies a necessary and sufficient condition for a nonincreasing (or decreasing) function of the tradeoff factor $\mathrm{f}\,(\bar{X})$.

DEFINITION 4. An individual is mean-risk averse if for some lotteries $(\bar{X},\ X')$, $(\bar{Y},\ Y')\in \mathbf{P}$ where $\bar{Y}>\bar{X}$ and $X' >_p Y'$, then

$$\{0.5,\ (\bar{X}+\Delta,\ X');\ 0.5,\ (\bar{Y}+\Delta,\ Y')\}\ \geq_{\tilde{\mathbf{p}}}\ \{0.5,\ (\bar{X}+\Delta,\ Y');\ 0.5,\ (\bar{Y}+\Delta,\ X')\}$$

for any constant Δ. If a strict preference relation holds for the above condition, then the individual is strictly mean-risk averse.

The condition specified in Definition 4 is analogous to the concept of multivariate risk aversion proposed by Richard (1975). If the condition holds for lotteries $(\bar{X},\ X')$, $(\bar{Y},\ Y')$ with $\bar{Y}>\bar{X}$ and $X' >_p Y'$ (i.e., Y' is riskier than X'), then you would not prefer the even-chance gamble on the pairings of the lower mean $\bar{X}$ with the bigger risk Y' and the higher mean $\bar{Y}$ with the smaller risk X' rather than the even-chance gamble on the pairings of $\bar{X}$ with X' and $\bar{Y}$ with Y'. Furthermore, this preference condition needs to hold for any common constant adding to the lotteries.

THEOREM 4. An individual is mean-risk averse (or strictly mean-risk averse) if and only if the tradeoff factor $\mathrm{f}\,(\bar{X})$ in (1) is a nonincreasing (or strictly decreasing) function.

Therefore, by this theorem, the tradeoff factor $f(\bar{X})$ can not be increasing if a decision maker is mean-risk averse. Furthermore, an additive risk-value model (2) will not be appropriate if a decision maker is strictly mean-risk averse. As we mentioned earlier, a decreasing function for $f(\bar{X})$ is also required for an appropriate measure of perceived risk in the risk-value model (1).

3.3. Lottery-Risk Aversion and Global Risk Aversion.

Arrow (1965) and Pratt (1964) proposed $r(x) = -u''(x)/u'(x)$ as a local measure of one's risk aversion for utility models by considering the risk premium for a small risk lottery. Let $p(\bar{X}, X')$ be the risk premium of a lottery $(\bar{X}, X')$ for an individual in our risk-value structure. We define the certainty equivalent for a lottery as follows.

DEFINITION 5. The certainty equivalent (CE) of a lottery $(\bar{X}, X') \in \mathbf{P}$ is determined by $(CE, 0) \sim_{\tilde{\mathbf{p}}} (\bar{X}, X')$.[4]

Thus, the risk premium $p(\bar{X}, X')$ can be determined by $[(\bar{X} - p(\bar{X}, X')), 0] \sim_{\tilde{\mathbf{p}}} (\bar{X}, X')$, where $\bar{X} - p(\bar{X}, X')$ is the certainty equivalent. Based on the traditional concept, risk aversion implies a positive risk premium for a lottery. Similarly, we define risk aversion for our risk-value models as follows.

DEFINITION 6. For a nondegenerate lottery $(\bar{X}, X') \in \mathbf{P}$, if the risk premium $p(\bar{X}, X') > 0$ for an individual, then the individual is lottery-risk averse for this lottery; if $p(\bar{X}, X') < 0$, then lottery-risk seeking; and if $p(\bar{X}, X') = 0$, then lottery-risk neutral. If these conditions hold for all nondegenerate lotteries $(\bar{X}, X') \in \mathbf{P}$, then we say that the individual is globally risk averse, globally risk seeking and globally risk neutral for all lotteries, respectively.

THEOREM 5. The risk-value model (1) with an increasing value function $V(\bar{X})$ is:

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[4] According to this definition, if the value measure is linear, the risk-value model (1) will represent the certainty equivalent for a lottery; that is, $CE = \bar{X} - f(\bar{X})[R(X') - R(0)]$. This can be used as a pricing model for risky alternatives

i) lottery-risk averse if and only if $R(X') > R(0)$; lottery-risk seeking if and only if $R(X') < R(0)$; and lottery-risk neutral if and only if $R(X') = R(0)$.

ii) globally risk averse if and only if the utility function u for $R(X')$ is strictly concave; globally risk seeking if and only if u is strictly convex; and globally risk neutral if and only if u is linear.

For the normalized utility function $u(0) = 0$, $R(X') > R(0)$ simply means $R(X') > 0$. Even though an individual may have a standard measure of risk based on some utility function that is both locally risk averse and locally risk seeking with respect to different ranges of outcomes or different wealth levels, the individual will be lottery-risk averse if $R(X') > R(0)$, and lottery-risk seeking if $R(X') < R(0)$. Note that the concept of lottery-risk aversion is different from that of global risk aversion unless the utility function for the standard measure of risk is concave.

Rothschild and Stiglitz (1970) proposed a theory for increasing risk by mean preserving spreads. They established an equivalence theorem for the expected utility model, second degree stochastic dominance, and mean preserving spreads when lotteries have the same expected values. Their central idea is that if we add some uncorrelated noise to a lottery, the new lottery should be riskier than the original, and therefore less preferable for risk averse individuals.

In nonexpected utility theory, Machina (1982) also used the mean preserving spread as a basic condition for his local utility function. For continuously differentiable risk-value models, the second order derivative of the local utility function (see (b1) in the appendix) is $u_l''(x; X) = \varphi(\bar{X})u''(x - \bar{X})$. This implies that the local utility function is strictly concave for all lotteries if and only if the utility function u for the standard measure of risk is strictly concave. Thus, based on Machina (1982, Theorems 2 and 3), an individual with the risk-value model (1) will always be averse to mean preserving spreads if his u is strictly concave (i.e., the individual is globally risk averse). This shows a consistency of behavior between our risk-value model and the traditional expected utility model for mean preserving spreads.

3.4. Constant Risk Aversion and Decreasing Risk Aversion

Now we consider the effect on the risk premium when an individual's wealth level changes (recall that the wealth level is implied in $\bar{X}$) or when the lottery changes by adding a certain amount of money, say $\Delta > 0$; then $[(\bar{X}+\Delta-\pi(\bar{X}+\Delta, X')), 0] \sim_{\tilde{p}} (\bar{X}+\Delta, X')$. The relationships between $\pi(\bar{X}, X')$ and $\pi(\bar{X}+\Delta, X')$ can reflect three different risk aversion attitudes, defined as follows.

DEFINITION 7. For a risk averse individual (either globally risk averse or lottery-risk averse for the lotteries considered) and a constant $\Delta > 0$, the individual is constant risk averse if $\pi(\bar{X}+\Delta, X') = \pi(\bar{X}, X')$, decreasing risk averse if $\pi(\bar{X}+\Delta, X') < \pi(\bar{X}, X')$, and increasing risk averse if $\pi(\bar{X}+\Delta, X') > \pi(\bar{X}, X')$.

The following result characterizes the three different risk aversion attitudes for model (1), when $V(\bar{X})$ is twice continuously differentiable and $\varphi(\bar{X})$ is once continuously differentiable.

THEOREM 6. If $V(\bar{X})$ is increasing and $R(X') > R(0)$, then the risk-value model (1) is

i) decreasing risk averse if and only if $m(\bar{X}) < -\varphi'(\bar{X})/\varphi(\bar{X})$ for nonincreasing $\varphi'(\bar{X})/\varphi(\bar{X})$;

ii) constant risk averse if and only if $m(\bar{X}) = -\varphi'(\bar{X})/\varphi(\bar{X})$ = constant; and

iii) increasing risk averse if and only if $m(\bar{X}) > -\varphi'(\bar{X})/\varphi(\bar{X})$, for nondecreasing $\varphi'(\bar{X})/\varphi(\bar{X})$,

where $m(\bar{X}) = -V''(\bar{X})/V'(\bar{X}) \geq 0$.

We may consider $m(\bar{X}) = -V''(\bar{X})/V'(\bar{X})$ to be a relative measure of the strength of value (Dyer and Sarin, 1982).

Note that the concepts of risk aversion attitudes in our risk-value structure are quite different from the traditional concepts of risk aversion attitudes in the classic utility model (Arrow, 1965; Pratt, 1964). In a utility model, for example, the concept of decreasing risk aversion must be based on a concave utility function. But in our risk-value model, the concept of decreasing risk aversion can be based on lottery-risk aversion which does not necessarily imply a concave utility function (or global risk aversion) for the standard measure of risk. According to Theorem 6, risk aversion attitudes in the risk-value model are not even relevant to the choice of the standard measure of risk (e.g., we may use a piecewise utility model). The only requirement for $R(X')$ is that $R(X') > R(0)$ for those lotteries considered.

However, for continuously differentiable and concave utility models, our results in Theorem 6 are consistent with Arrow's (1965) and Pratt's (1964) definitions of risk aversion attitudes. To show this, we can use the Taylor expansion for an expected utility model around the mean of a lottery (with small risk),

$$\mathrm{E}[u(X)] \approx u(\bar{X}) + \frac{1}{2} u''(\bar{X}) \mathrm{E}[(X - \bar{X})^2]. \tag{5}$$

It can be seen that (5) is a special case of our risk-value model (1) when $V(\bar{X}) = u(\bar{X})$, $R(X') = \mathrm{E}[(X - \bar{X})^2]$ and $\phi(\bar{X}) = -u''(\bar{X})/2$ (for risk aversion, $u''(\bar{X}) < 0$). Thus, a decreasing $\phi(\bar{X})$ corresponds to $u'''(\bar{X}) > 0$. Applying the decreasing risk averse result of Theorem 6 for the expected utility model (5), we obtain $u''(\bar{X})/u'(\bar{X}) > u'''(\bar{X})/u''(\bar{X})$, which implies decreasing risk aversion in the Arrow-Pratt sense. Thus, the concept of decreasing risk aversion in our risk-value models includes that of decreasing risk aversion in traditional utility models as a special case.

The usefulness of Theorem 6 for risk-value models is obvious. They can help us identify appropriate functional forms for the value measure and the tradeoff factor. For instance, if a decision maker is constant risk averse, then by $m(\bar{X}) = -\phi'(\bar{X})/\phi(\bar{X})$, we must have $V(\bar{X}) = a\int \phi(\bar{X}) d\bar{X} + b$, where $a > 0$ and b are constant. Therefore, if the decision maker has a constant tradeoff factor, i.e., the additive model (2), then his value function must be linear. For another example, if a decision maker is decreasing risk averse and has a linear value function, then we must choose a decreasing function for the tradeoff factor based on the condition $m(\bar{X}) < -\phi'(\bar{X})/\phi(\bar{X})$.

4. Examples of Risk-Value Models and Decision Paradoxes

In this section, we provide some examples of our risk-value models and investigate their descriptive power for some observed risky choice behavior and the decision paradoxes that violate the independence axiom of expected utility theory.

According to the risk-value model (1), the standard measure of risk, the value function, and the tradeoff factor can be treated independently. Thus, we can properly choose functional forms for each of them according to different theoretical and empirical considerations. Some examples of the standard measure of risk $R(X')$ can be found in Jia and Dyer (1996). The value measure $V(\bar{X})$ should be chosen as an increasing function and may have the same functional form as a utility model. For appropriate risk averse behavior, as discussed in the last section, the tradeoff factor $\varphi(\bar{X})$ should be either a decreasing function or a positive constant. In Theorem 6, $\varphi'(\bar{X})/\varphi(\bar{X})$ equal to a constant is the only choice that can lead to all the three types of risk aversion for the risk-value model. This suggests a simple choice for the tradeoff factor: $\varphi(\bar{X}) = ke^{-b\bar{X}}$, where $k > 0$ and $b \geq 0$.

Table 1 provides some examples of the risk-value model based on simple combinations of several standard measures of risk and value functions. For different individuals and different decision problems, we may need to choose different functional forms of risk-value models. Readers can also select their own favorite ones to make some other risk-value models. Here we are particularly interested in three types of risk-value models, namely exponential risk-value models, generalized disappointment models, and moments risk-value models. In addition to illustrating the flexibility of the risk-value model, these examples exhibit interesting characteristics, which will be discussed separately as follows.

4.1. Exponential Risk-Value Models

If the standard measure of risk is based on exponential or linear plus exponential utility models, then $R(X') = \mathrm{E}[e^{-c(X-\bar{X})} - 1]$. A risk-value model with this measure of risk is globally risk averse; that is, it always averse to mean preserving spreads. To be compatible with the form of the standard measure of risk, we can also choose the same form of exponential functions, but with different parameters, for the value measure $V(\bar{X})$ and the tradeoff factor f $(\bar{X})$, which leads to model (3B) in Table 1. When $a = b = c$ and $h = k$, this model reduces to an exponential utility model. Otherwise, these two models are different. When $b > a$, model (3B) is decreasing risk averse by Theorem 6 even though the traditional exponential utility model is constant risk averse.

Choosing a linear function or a linear plus exponential function for $V(\bar{X})$ leads to models (3A) or (3C) in Table 1 respectively. Model (3A) is decreasing risk averse based on Theorem 6. Model (3C) includes a linear plus exponential utility model as a special case when $a = b = c$ and $h = k$. It is decreasing risk averse if $b \geq a$.

In order to see the descriptive power of an exponential risk-value model, we consider a special case of model (3B) in Table 1. When $a = b$ and $h = k$, this model reduces to the following simple multiplicative form:

$$\mathrm{E}[U(\bar{X},\ X')] = -\ ke^{-a\bar{X}}\mathrm{E}[e^{-c(X\ -\ \bar{X})}]. \tag{6}$$

This model is constant risk averse according to Theorem 6, and has the same risk attitude as an exponential utility model. According to Theorem 3, model (6) satisfies the first order stochastic dominance condition if and only if $ke^{-(a-c)\bar{X}}[(a-c)\mathrm{E}(e^{-cX}) + ce^{-cx}]$ ^{3} 0 for all lotteries and all x. Thus, $a \geq c$ is sufficient to maintain first order stochastic dominance for model (6).

The exponential risk-value model (6) can also be written as $\mathrm{E}[U(\bar{X}, X')]$ = $\mathrm{E}[-ke^{-cX\ -\ (a\ -\ c)\bar{X}}]$. When $X \neq 0$, we can further transform the model to $\mathrm{E}[-ke^{-c_f X}]$, where $c_f = c + (a - c)\bar{X} / X$. This model has some similarity to Becker and Sarin's (1987) lottery dependent exponential model,[5] and it provides a natural interpretation, based on risk and value tradeoffs, for lottery dependent decision models.

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[5] In Becker and Sarin's model, the parameter c_f is an expectation of some function over a lottery, i.e., $c_f = \mathrm{E}[h(X)]$, and it is predetermined before calculating the lottery-dependent expected utility. However, the h function is difficult to assess.

The simple risk-value model (6) can be used to explain some well known decision paradoxes. First, let us consider Allais Paradox (Allais, 1953, 1979) which is based on the following pairs of lotteries:

X_1: win \$1 million for sure versus X_2: 0.10 chance of \$5 million
0.89 chance of \$1 million
0.01 chance of \$0
X_3: 0.11 chance of \$1 million versus X_4: 0.10 chance of \$5 million
0.89 chance of \$0 0.90 chance of \$0

Many experiments have shown that a majority of subjects have the preference pattern of X_1 over X_2 but X_4 over X_3, which violates the independence axiom of expected utility theory. To capture this preference pattern by (6), a necessary and sufficient condition can be found as follows:

$$c-\frac{1}{390,000}log[g_2(c)]<a<c-\frac{1}{390,000}log[g_1(c)],$$

where $g_1(c)=e^{-c\,1000000}/(0.1e^{-c\,5000000}+0.89e^{-c1000000}+0.01)$ and $g_2(c)=(0.11e^{-c1000000}+0.89)$ $/(0.1e^{-c5000000}+0.90)$. For a numeric example, when c = 0.0005, then 0.00051 < a < 0.00177 will be sufficient to predict the subjects' preference pattern. The Allais Paradox is now known to be a special case of a general empirical phenomenon called the common consequence effect (e.g., see Machina, 1987).

Another type of paradox is the common ratio effect, which involves pairs of lotteries of the following form:

X_1: win \$3000 for sure versus X_2: 0.80 chance of \$4000
0.20 chance of \$0
X_3: 0.25 chance of 3000 versus X_4: 0.20 chance of \$4000
0.75 chance of \$0 0.80 chance of \$0

It has been observed that a majority of the subjects prefer X_1 to X_2 and X_4 to X_3 (see Kahneman and Tversky, 1979), which is inconsistent with any expected utility model. Based on the exponential risk-value model (6), we can find the following necessary and sufficient condition for the observed preference pattern:

$$c+\frac{1}{50}\,log[g_4(c)]<a<c+\frac{1}{200}\,log[g_3(c)],$$

where $g_3(c)=(0.8e^{-c4000}+0.2)/e^{-c3000}$ and $g_4(c)=(0.25e^{-c3000}+0.75)/(0.2e^{-c4000}+0.8)$. For c = 0.0005, 0.00051 < a < 0.00216 is sufficient to predict the phenomenon of the common ratio effect. Further, c = 0.0005, 0.00051 < a < 0.00177 will be sufficient for the model (6) to predict both the preference patterns of the Allais Paradox and the common ratio effect simultaneously. Note that the first stochastic dominance condition is also satisfied for this choice of parameters a and c (i.e., a > c).

Even with the simple form of risk-value model, the model (6) shows a great deal of descriptive power for risky choice behavior. To gain more flexibility in modeling risky preferences such as decreasing risk aversion, we can select some other forms of exponential or linear plus exponential risk-value models provided in Table 1.

4.2. Generalized Disappointment Models

Bell (1985) proposed a disappointment model for decision making under uncertainty. According to Bell, disappointment is a psychological reaction to an outcome that does not meet a decision maker's expectation. Bell used the mean of a lottery as a decision maker's psychological expectation. If an outcome that is smaller than the expected value occurs, the decision maker would be disappointed. Otherwise, the decision maker would be elated. Although Bell's development of the disappointment model has an intuitive appeal, his model is only applicable to the lotteries with two outcomes.

Following Bell's basic ideas, we can develop a more general disappointment model based on our risk-value framework. Consider the following piece-wise linear utility model:

$$u(x) = \begin{cases} ex & \text{when } x \geq 0 \\ dx & \text{when } x < 0 \end{cases} \tag{7}$$

where $d,\ e > 0$ are constant. Decision makers who are averse to downside risk or losses should have $d > e$, as illustrated in Figure 1. The standard measure of risk for this utility model can be obtained as follows:

$$R(X') = d\mathrm{E}^{-}[|X - \bar{X}|] - e\mathrm{E}^{+}[|X - \bar{X}|] \tag{8}$$

where $\mathrm{E}^{-}[|X - \bar{X}|] = \sum_{x_i < \bar{X}} p_i |x_i - \bar{X}|$ and $\mathrm{E}^{+}[|X - \bar{X}|] = \sum_{x_i > \bar{X}} p_i (x_i - \bar{X})$. According to Bell's (1985) basic idea, $d\mathrm{E}^{-}[|X - \bar{X}|]$ should be a general measure of expected disappointment and $e\mathrm{E}^{+}[|X - \bar{X}|]$ a general measure of expected elation, and then overall psychological satisfaction is measured by $-R(X')$, which is the converse of the standard measure of risk (8).

If we assume a linear value measure and a constant tradeoff factor, then we can have the following risk-value model based on the standard measure of risk (8):

$$\mathrm{E}[U(\bar{X},\ X')] = \bar{X} - \{d\mathrm{E}^{-}[|X - \bar{X}|] - e\mathrm{E}^{+}[|X - \bar{X}|]\}. \tag{9}$$

For a two-outcome lottery, model (9) reduces to Bell's disappointment model. Thus, we call the risk-value model (9) a "generalized disappointment model." Note that the risk-value model (9) will be consistent with the piece-wise linear utility model (7) when the lotteries considered have the same means. It is also globally risk averse when $d > e$.[6]

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[6] This generalized disappointment model combined with a measure of regret has been used in a marketing study regarding consumer satisfaction and valuation (Inman, Dyer, and Jia, 1997).

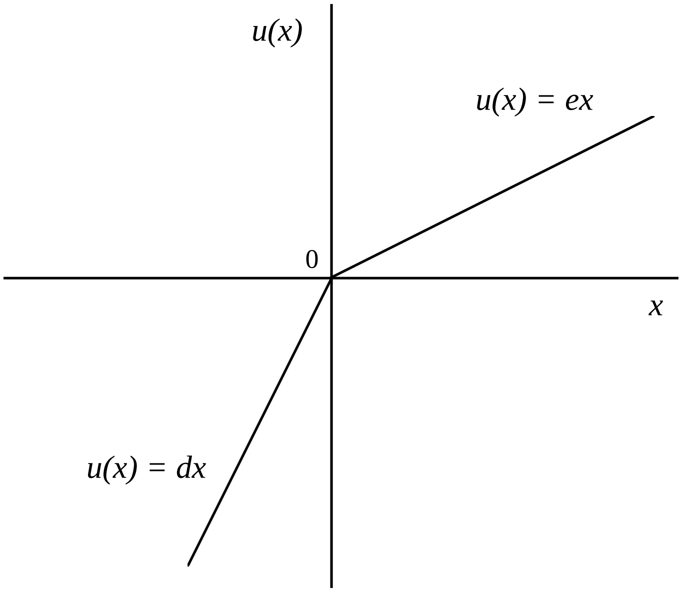

Figure 1. A piece-wise linear utility function.

Using his two-outcome disappointment model, Bell (1985) gave an explanation for the common ratio effect (see Section 4.1). Now we can use the generalized disappointment model (9) to explain the Allais Paradox, which involves an alternative with three outcomes (see Section 4.1). Based on model (9), we can have $\mathrm{E}[U(\overline{X}_1, X'_1)] = 1$, $\mathrm{E}[U(\overline{X}_2, X'_2)] = 1.39 - 0.361(d-e)$, $\mathrm{E}[U(\overline{X}_3, X'_3)] = 0.11 - 0.0979(d-e)$, and $\mathrm{E}[U(\overline{X}_4, X'_4)] = 0.5 - 0.45(d-e)$. For $X_1 >_p X_2$, we need $(d-e) > 1.080$; and for $X_4 >_p X_3$, we need $(d-e) < 1.108$. Thus, $1.108 > (d-e) > 1.080$ is sufficient to predict the subjects' preference pattern for the Allais Paradox.

For Bell's disappointment model and our model (9), we may question the assumption that disappointment and elation are proportional to the difference between the expected value and an outcome. Then we should use some nonlinear functions for disappointment and elation. Another concern for Bell's model and our model (9) is that they imply constant risk aversion. Thus, they are not appropriate for decreasing risk averse behavior. These considerations are captured by

Model (4A) provided in Table 1. When $q_1 = q_2 = 1$ and $b = 0$, this model reduces to model (9). When $e = 0$ and $q_2 = 2$, model (4A) becomes a mean-semivariance model.[7]

Finally, our generalized disappointment models are also different from Loomes and Sugden's (1986) development. In their basic model, disappointment (or elation) is measured by some function of the difference between the utility of outcomes and the expected utility of a lottery. In their analysis, they further assume a linear "utility" measure of wealth and the same sensation intensity for both disappointment and elation, so that their model has the form, $\bar{X} + \mathrm{E}[D(X - \bar{X})]$, where $D(x - \bar{X}) = -D(\bar{X} - x)$, and D is continuously differentiable and convex for $x > \bar{X}$ (thus concave for $x < \bar{X}$). Even though this model is different from our generalized disappointment models (9) and (4A), it is a special case of our risk-value model with a linear measure of value, a constant tradeoff factor, and a specific form of the standard measure of risk (i.e., $R(X') = -\mathrm{E}[D(X - \bar{X})]$, where $D(x - \bar{X}) = -D(\bar{X} - x)$). Loomes and Sugden (1986) used this model to provide an explanation for the choice behavior that violates Savage's (1954) sure-thing principle.

4.3. Moments Risk-Value Models

In some decision situations, it may be difficult to know the distributions of lotteries. But moments may be estimated easily by historic data. Especially in the area of finance, people often use mean and variance to make tradeoffs for financial decision making because of their operational advantages and reasonable approximation for modeling decision problems (see Markowitz, 1959, 1987, 1991; Sharpe, 1970, 1991). In the past, expected utility theory has been used as a foundation for moments models. However, the theory itself has been repeatedly challenged as a principle for reasonable decision making.

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[7] Since model (9) and some other models in Table 1 are not continuously differentiable, Machina's local utility function for them does not exist. The necessary and sufficient condition of first order stochastic dominance condition for these models needs to be developed.

Now we can provide a better foundation, i.e., the risk-value theory, for developing moments models. For example, the widely used mean-variance model, $\bar{X}$ - $k\,\mathrm{E}[(X-\bar{X})^2]$ (where $k > 0$), is a simple risk-value model with variance as the standard measure of risk and a constant tradeoff factor. Sharpe (1970, 1991) basically assumed this type of mean-variance model in his analysis for portfolio selection and the Capital Asset Pricing Model. However, under the expected utility framework, this mean-variance model is based on the assumptions that the investor has an exponential utility function and returns are jointly normally distributed. Our risk-value theory provides an alternative foundation for the mean-variance model, even though variance may not be a satisfactory measure of risk. According to our Theorem 6, this mean-variance model is constant risk averse. To obtain a decreasing risk averse mean-variance model, we can simply use a decreasing tradeoff factor, i.e., $\bar{X} - ke^{-b\bar{X}}\mathrm{E}[(X-\bar{X})^2]$, where b, $k > 0$. Model (1A) in Table 1 offers a more general form which includes the mean-variance model and a mean-absolute standard deviation model as special cases when $q = 2$ and $q = 1$ respectively.

For many decision problems, mean-variance models are over simplified. Based on our risk-value framework, we can develop some richer moments models for risky decision making. First, let us consider the following simple three moments model:

$$\mathrm{E}[u\,(\bar{X},\, X')] = \bar{X} \text{ - } k\,\{\mathrm{E}[(X-\bar{X})^2] \text{ - } c\,\mathrm{E}[(X-\bar{X})^3]\} \qquad (10)$$

where c, $k > 0$. The standard measure of risk in (10) is defined by variance and skewness, which is based on a three-order polynomial utility function.[8] According to Theorem 3, $1-3kc\mathrm{E}[(X-\bar{X})^2]-2k(x-\bar{X})+3kc(x-\bar{X})^2$ must be nonnegative for all x and all lotteries considered in order to maintain first order stochastic dominance for model (10).

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[8] Note that incorporating other higher moments such as peakedness into the standard measure of risk is straight forward, which can be based on a higher order polynomial utility function. In fact, the standard measure of risk for any continuously differentiable utility function can be approximated by moments through the Tayler expansion formula (see Jia and Dyer, 1996).

The three moments model (10) can be either lottery-risk averse or lottery-risk seeking, depending on the distribution of a lottery. For symmetric bets or lotteries not highly skewed (e.g., an insurance policy) such that $\mathrm{E}[(X-\bar{X})^2] > c\mathrm{E}[(X-\bar{X})^3]$, model (10) will be lottery-risk averse. But for highly positive skewed lotteries (e.g., lottery tickets) such that the skewness overwhelms the variance, i.e., $\mathrm{E}[(X-\bar{X})^2] < c\mathrm{E}[(X-\bar{X})^3]$, then model (10) will exhibit lottery-risk seeking behavior. Therefore, an individual with preferences described by the moments model (10) would purchase both insurance and lottery tickets simultaneously.

Markowitz (1952) noticed that individuals of all wealth levels have the same tendency to purchase insurance and lottery tickets whether they are poor or rich. This observed behavior contradicts a common assumption of expected utility theory that preference ranking is defined over ultimate levels of wealth. But whether our risk-value model is risk averse or risk seeking is determined only by the standard measure of risk, which is independent of an individual's wealth level (refer to the form of risk-value model (4)). In particular, for the three moments model (10), the change of wealth level just causes a parallel shift for the model, which will not affect the risk attitude and the choice behavior of this model. This is consistent with Markowitz's observation.

To further see some implications of the three moments model (10) for some nonexpected utility models, we consider a two-outcome lottery $(p, x;\ 1-p, y)$. Substituting these values into the model (10), we can have

$$\begin{aligned}
\mathrm{E}[u(\bar{X}, X')] &= px + (1-p)y - kp(1-p)(x-y)^2 + kcp(1-p)(1-2p)(x-y)^3 \\
&= y + [p - kp(1-p)(x-y) + kcp(1-p)(1-2p)(x-y)^2](x-y) \\
&= y + \mathrm{p}(x,p,y)(x-y) \\
&= \mathrm{p}(x,p,y)x + [1-\mathrm{p}(x,p,y)]y
\end{aligned}$$

where

$$\mathrm{p}(x,p,y) = p - kp(1-p)(x-y) + kcp(1-p)(1-2p)(x-y)^2 . \tag{11}$$

A major feature of Kahneman and Tversky's (1979) Prospect theory is the use of a decision weight (i.e., a nonlinear function of probability) instead of the probability (also see Chew and

MacCrimmon, 1979; Quiggin, 1982; Fishburn, 1983; Yaari, 1987; and Tversky and Kahneman, 1992). For a linear value measure, $\pi(x,p,y)$ could be interpreted as a decision weight. This decision weight model (11) can have a shape similar to that of Prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), as illustrated in Figure 2. That is, $\pi(x,p,y) < p$ when $p > p^*$; and $\pi(x,p,y) > p$ when $p < p^*$, where $p^* = \frac{1}{2}[1 - \frac{1}{c(x-y)}]$ is the cross-over point of the decision weight and probability. It can be seen that this twisted shape of the decision weight is mainly caused by skewness. The decision weight $\pi(x,p,y)$ also depends on the difference between outcomes. For $x > y$, the p^* value is below 1/2; and for $x < y$, the p^* value is above 1/2. Figure 2 clearly shows this effect.

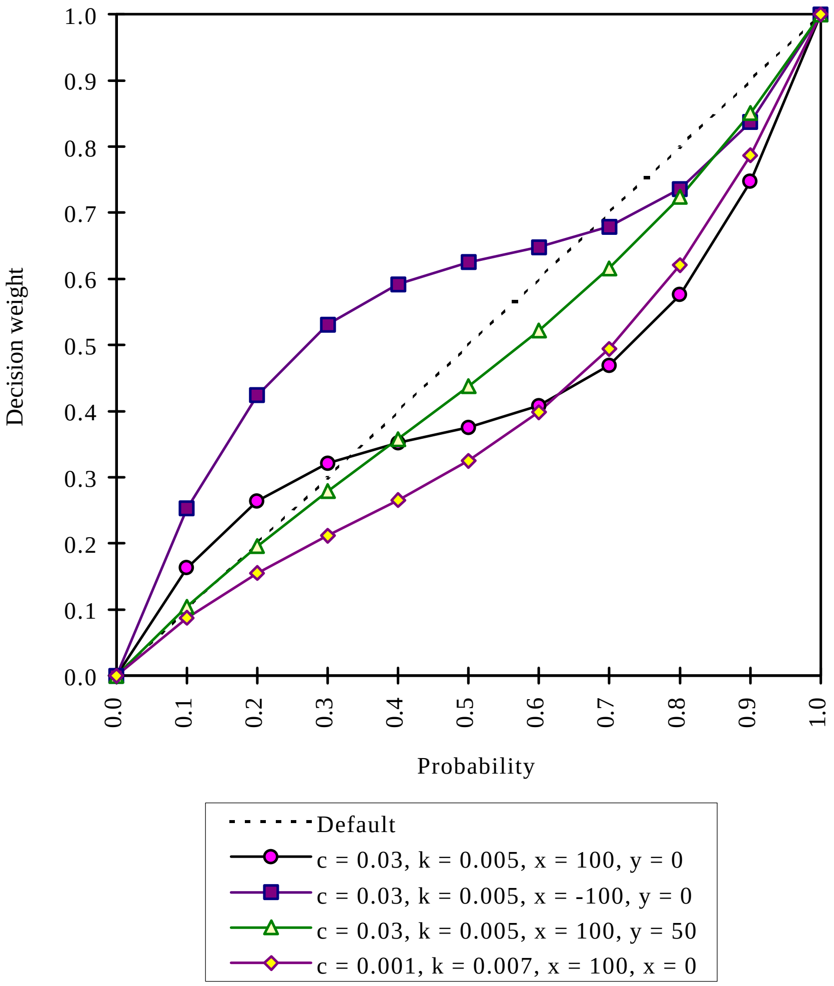

Figure 2. Decision weights based on the three moments model with two-outcome lotteries.

Several empirical studies suggest that the decision weight is scenario dependent (e.g., Currim and Sarin, 1989; Lattimore, Baker and Witte, 1992). In a new version of Prospect theory, Tversky and Kahneman (1992) considered different weighting functions for positive outcomes and negative outcomes. Hogarth and Einhorn (1990) found that outcome sizes also have an effect on the shape of decision weight. In fact, even the choice of value measure may affect the shape of decision weight (e.g., see Hershey and Schoemaker, 1980). Thus, a decision weight that is a function of probability only may not be able to provide a robust model of risky choice behavior. However, our interpretation for the "decision weight" is different from those non-expected utility models, and is based on the intuitive idea of risk-value tradeoffs.

Finally, some other three moments models are provided in Table 1. Model (2A) can be used for decreasing risk averse behavior (when it is also lottery-risk averse, i.e., $\mathrm{E}[(X-\bar{X})^2] > c\,\mathrm{E}[(X-\bar{X})^3]$ for lotteries considered). Models (2B) and (2C) provide nonlinear measures for the value function, and can be decreasing risk averse when $b > a$.

5. Conclusion

In this paper, we have incorporated the intuitively appealing idea of risk-value tradeoffs into decision making under risk, and developed a new decision methodology, the risk-value theory. This theory provides a significant extension of Coombs' Portfolio theory and other mean-risk studies. With the new theoretical foundation, the risk-value framework ties together two streams of research: one in developing preference models and the other in modeling risk judgments, and unifies a wide range of decision phenomena including both normative and descriptive aspects.

In this development, we have refined and generalized a substantial number of previously proposed decision theories and models, ranging from the mean-variance model in finance to disappointment models (Bell, 1985; Loomes and Sugden, 1986) in decision science. We have also created many new risk-value models. Specifically, we have developed three useful classes of decision models based on our theory: exponential risk-value models, generalized disappointment models, and moments risk-value models. These models are very flexible in modeling preferences. They also provide new resolutions for observed risky choice behavior and the decision paradoxes that violate the independence axiom of the expected utility theory.

Even though some other non-expected utility theories that have been proposed (e.g., Prospect theory and other weighted utility theories) may produce the same predictions for the decision paradoxes as our theory, we have offered a new justification for them based on a more appealing and realistic notion of risk-value tradeoffs. This approach provides a unified explanation for empirical facts with regard to both preferences and risk perceptions. This has not been accomplished by other non-expected utility theories and models. In particular, since the role of risk is merely considered implicitly in these decision theories and models,[9] they are not compatible with the choice behavior that is based on risk and mean return tradeoffs as often faced in financial management and other applied fields. Therefore, these theories and models offer little guidance in practice for this type of decision making. From an operational point of view, our risk-value models are not limited by the size of problems (e.g., lotteries with continuous distributions). They are mathematically tractable and realistically usable for large scale decision problems.

The risk-value theory can retain many appealing properties of the traditional expected utility theory, such as transitivity, first order stochastic dominance, decreasing risk aversion and mean preserving spreads. In particular, our risk-value models reduce to expected utility models for lotteries that have the same expected values. Fishburn (1989) pointed out, "in view of the accumulated evidence for persistent and predictable violation of expected utility, new theories

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[9] There are some non-expected utility models that do attempt to treat risk (or “additional" risk) as an explicit factor. Allais' (1953, 1979) mean-variance (in terms of cardinal utility) model and Hagen's (1979) first three moments (also in terms of cardinal utility) model are examples of these models, which can be represented as a "value" measure plus a "risk" term. Since the "measures of risk" in these models are defined at the level of utility, they are different from our framework. It is not clear if these "measures of risk" can capture the empirical evidence of perceived risk.

have been proposed to accommodate such violations without abandoning too much of the mathematical elegance of the traditional theories." Our risk-value theory is a further contribution toward achieving this goal.

The most important assumption in this study is risk independence, which leads to a separable form of risk-value models. Although some other weaker condition could be used to derive a risk-value model that has more descriptive power, this reduces the elegance of our basic risk-value form, and increases operational difficulty. Of course, if the increased complexity of risk-value tradeoffs is necessary for some descriptive purposes, then a more complicated risk-value model can be employed (see Jia, 1995).

The major purpose of this study is to establish a descriptive theory of decision making under risk. However, since our risk-value models retain many desirable normative properties, they may also be used for prescriptive purposes when decision makers are willing to deviate from the normative utility preference and make their decisions based on risk-value tradeoffs, as in financial decision making. For instance, our risk-value models provide extensions with a new theoretical foundation for the traditional mean-variance analysis (e.g., Markowitz, 1959, 1987, 1991; Sharpe, 1964, 1970, 1991) so that some more appealing decision models can be developed for financial modeling, such as a mean-semivariance model, a disappointment model, and a three moments model. We believe that the potential for contributions of our risk-value models in finance is very exciting. And also applications of our risk-value models in some other fields such as economics, insurance, and risk management should be promising.

6. Appendix

PROOF OF THEOREM 1

Define a conditional preference order $>_{\tilde{\mathbf{p}}}^{\overline{w}}$ as $X' >_{\tilde{\mathbf{p}}}^{\overline{w}} Y'$ iff $(\overline{w}, X') >_{\mathbf{p}} (\overline{w}, Y')$, where X', Y' P^{o}, and a conditional utility function $u_{\overline{w}}: P^{o} \rightarrow \mathrm{Re}$ for each $\overline{w} \in \mathbf{X}_1$ by $\mathrm{E}[u_{\overline{w}}(X')] = \mathrm{E}[U(\overline{w}, X')]$, with $\mathrm{E}[u_{\overline{w}}(X')] > \mathrm{E}[u_{\overline{w}}(Y')]$ iff $X' >_{\tilde{\mathbf{p}}}^{\overline{w}} Y'$. It can be seen that each

$u_{\overline{w}}$ is linear on P^{o}. Because the risk independence condition holds, then for some $\overline{w}_o \in \mathbf{X}_1$, $X' >^{\overline{w}_0}_{\tilde{\mathbf{p}}} Y'$ iff $X' >^{\overline{w}}_{\tilde{\mathbf{p}}} Y'$ for all $\overline{w} \in \mathbf{X}_1$. This implies that all conditional preference orders on P^{o} are identical and the conditional utility functions $u_{\overline{w}_o}$ and $u_{\overline{w}}$ preserve the same order. Thus, $u_{\overline{w}_o}$ and $u_{\overline{w}}$ must be a positive linear transformation of each other; that is, for each $\overline{w} \in \mathbf{X}_1$ there exist some real numbers $\mathrm{f}(\overline{w}) > 0$ and $g(\overline{w})$ such that

$$\mathrm{E}[u_{\overline{w}}(X')] = g(\overline{w}) + \mathrm{f}(\overline{w})\mathrm{E}[u_{\overline{w}_o}(X')]$$

or

$$\mathrm{E}[\,U(\overline{w}, X')\,] = g(\overline{w}) + \mathrm{f}(\overline{w})\mathrm{E}[U(\overline{w}_o, X')]. \qquad \text{(a1)}$$

By setting $\overline{w} = \overline{X}$ and $\overline{w}_o = 0$, we obtain

$$\mathrm{E}[\,U(\overline{X}, X')\,] = g(\overline{X}) + \mathrm{f}(\overline{X})\mathrm{E}[U(0, X')]. \qquad \text{(a2)}$$

Since the consistence condition is satisfied, we have $\mathrm{E}[U(0, X')] = \mathrm{a}\mathrm{E}[u(X')] + \mathrm{b}$, where u is the single-attribute von Neumann-Morgenstern utility function, and a > 0 and b are constants. Thus, $\mathrm{E}[U(0, X')] = -R(X')$ (simply set a = 1 and b = 0). Substituting this into (a2), we have

$$\mathrm{E}[\,U(\overline{X}, X')\,] = g(\overline{X}) - \mathrm{f}(\overline{X})R(X'). \qquad \text{(a3)}$$

When $X = \overline{X}$, it becomes

$$U(\overline{X}, 0) = g(\overline{X}) - \mathrm{f}(\overline{X})\,R(0).$$

Letting $U(\overline{X}, 0) = V(\overline{X})$, then this gives

$$g(\overline{X}) = V(\overline{X}) + \mathrm{f}(\overline{X})\,R(0). \qquad \text{(a4)}$$

Substituting (a4) into (a3), we obtain the desired result:

$$\mathrm{E}[\,U(\overline{X}, X')\,] = V(\overline{X}) - \mathrm{f}(\overline{X})[R(X') - R(0)]. \qquad \text{(a5)}$$

Conversely, we can easily verify that the risk independence condition and the consistence condition are satisfied by the risk-value form (a5).

If other three functions F, ψ and G also satisfy (a5), then by uniqueness, we must have

$$F(\bar{X}) - \psi(\bar{X})[G(X') - G(0)] = a\{V(\bar{X}) - \phi(\bar{X})[R(X') - R(0)]\} + b \qquad \text{(a6)}$$

where $a > 0$. When $X = \bar{X}$, we directly have $F(\bar{X}) = a\,V(\bar{X}) + b$. Thus, (a6) can be reduced to the following:

$$\psi(\bar{X})[G(X') - G(0)] = a\,\phi(\bar{X})[R(X') - R(0)]. \qquad \text{(a7)}$$

Since both R and G are based on a single-attribute utility function, one must be a positive linear transformation of the other, i.e., $G(X') = c\,R(X') + d$, where $c > 0$ and d are constants. Substituting this into (a7), we can obtain $\psi(\bar{X}) = a / c\, \phi(\bar{X})$.

Conversely, if $F(\bar{X}) = aV(\bar{X}) + b$, $G(X') = cR(X') + d$, and $\psi(\bar{X}) = a/c\,\phi(\bar{X})$, where a, $c > 0$ and b, d are constants, then we can have $F(\bar{X}) - \psi(\bar{X})[G(X') - G(0)] = a\{V(\bar{X}) - \phi(\bar{X})[R(X') - R(0)]\} + b$ Thus, $F(\bar{X}) - \psi(\bar{X})[G(X') - G(0)]$ is a positive linear transformation of $V(\bar{X}) - \phi(\bar{X})[R(X') - R(0)]$, and F, ψ and G satisfy (a5) when V, ϕ and R satisfy (a5).

PROOF OF THEOREM 2

If the mean-dominance condition is assumed, then $(\bar{X} + \Delta,\ X') >_{\tilde{p}} (\bar{X},\ X')$ for $\Delta > 0$. Thus, we have

$$V(\bar{X} + \Delta) - \phi(\bar{X} + \Delta)[R(X') - R(0)] > V(\bar{X}) - \phi(\bar{X})[R(X') - R(0)]$$

or

$$V(\bar{X} + \Delta) - V(\bar{X}) > [\phi(\bar{X} + \Delta) - \phi(\bar{X})][R(X') - R(0)].$$

Dividing the both sides of this inequation by Δ and letting $\Delta \to 0$, we obtain the desired result in Theorem 2. The opposite procedure of the above proof shows the converse of Theorem 2.

PROOF OF THEOREM 3

Based on Machina's (1982) local utility theory, we can find the local utility function for our risk-value model (1) as follows:

$$u_l(x;\ X) = \{V'(\bar{X}) - \phi'(\bar{X})[R(X') - R(0)]\}x + \phi(\bar{X})\{u(x - \bar{X}) - \mathrm{E}[u'(X')]x\} \qquad \text{(b1)}$$

where V' and ϕ' are the first derivatives of value measure and tradeoff factor respectively, u' is the first derivative of the utility function for the standard measure of risk, and x is a real variable. According to Machina (1982, Theorem 1), the first order stochastic dominance condition is satisfied if and only if $u_l(x;\ X)$ is nondecreasing in x for all lotteries. That is, the first derivative of the local utility function $u'_l(x;\ X)$ with respect to x is nonnegative. Thus, we have the following condition:

$$V'(\bar{X}) - \phi'(\bar{X})[R(X') - R(0)] + \phi(\bar{X})\{u'(x - \bar{X}) - \mathrm{E}[u'(X')]\} \geq 0$$

for all lotteries and all x.

PROOF OF THEOREM 4

According to the definition of mean-risk aversion and the risk-value model (1), we have

$$\frac{1}{2}\{V(\bar{X}+\Delta) - \phi(\bar{X}+\Delta)[R(X') - R(0)]\} + \frac{1}{2}\{V(\bar{Y}+\Delta) - \phi(\bar{Y}+\Delta)[R(Y') - R(0)]\} \geq$$

$$\frac{1}{2}\{V(\bar{X}+\Delta) - \phi(\bar{X}+\Delta)[R(Y') - R(0)]\} + \frac{1}{2}\{V(\bar{Y}+\Delta) - \phi(\bar{Y}+\Delta)[R(X') - R(0)]\}$$

or

$$\phi(\bar{X}+\Delta)[R(Y') - R(X')] \geq \phi(\bar{Y}+\Delta)[R(Y') - R(X')].$$

Since $X' >_p Y'$ implies $R(Y') > R(X')$, then $\phi(\bar{X}+\Delta) \geq \phi(\bar{Y}+\Delta)$. By the assumption that $\bar{X} < \bar{Y}$ or $\bar{X} + \Delta < \bar{Y} + \Delta$ for any Δ, ϕ must be nonincreasing. Similarly, we can prove that if the strict mean-risk aversion condition holds, ϕ must be decreasing. To show the converse, we only need follow the opposite procedure of the above proof.

PROOF OF THEOREM 5

We only prove the risk averse case here. The proofs for the other cases are similar.

Based on Definition 5 for the certainty equivalent, we have

$$V(CE) = V(\bar{X} - \pi(\bar{X}, X')) = V(\bar{X}) - \phi(\bar{X})[R(X') - R(0)].$$

For lottery-risk aversion, we can easily observe that if V is an increasing, then $\pi(\bar{X}, X') > 0$ iff $R(X') > R(0)$ (recall that $\phi(\bar{X}) > 0$). For global risk aversion, we have $\pi(\bar{X}, X') > 0$ iff $R(X') > R(0)$ for all nondegenerate lotteries $(\bar{X}, X') \in \mathbf{P}$. This requires that the utility function for the standard measure of risk be strictly concave. To see this, notice $\mathrm{E}(X') = 0$ for all standard risk $X' \in \mathrm{P}^{\mathrm{o}}$. Thus, $R(X') > R(0)$ or $\mathrm{E}[u(X')] < u(0)$ is equivalent to $\mathrm{E}[u(X')] < u[\mathrm{E}(X')]$. By Jessen's inequality, u must be strictly concave.

PROOF OF THEOREM 6

Here we only provide a proof for the case of decreasing risk aversion. Proofs for the other cases are similar. According to the risk-value model (1), the risk premium $\pi(\bar{X}, X')$ of a lottery $(\bar{X}, X')$ is determined by

$$V(\bar{X} - \pi(\bar{X}, X')) = V(\bar{X}) - \phi(\bar{X})[R(X') - R(0)] \tag{c1}$$

and also the risk premium $\pi(\bar{X} + \Delta, X')$ of a lottery $(\bar{X} + \Delta, X')$ is

$$V(\bar{X} + \Delta - \pi(\bar{X} + \Delta, X')) = V(\bar{X} + \Delta) - \phi(\bar{X} + \Delta)[R(X') - R(0)] \tag{c2}$$

where $\Delta > 0$ is a change of an individual's wealth level or a certain payoff. Because $R(X') > R(0)$ and V is increasing, both $\pi(\bar{X}, X')$ and $\pi(\bar{X} + \Delta, X')$ are positive. For decreasing risk aversion, $\pi(\bar{X} + \Delta, X') < \pi(\bar{X}, X')$. Thus, by (c2), we must have

$$V(\bar{X} + \Delta - \pi(\bar{X}, X')) < V(\bar{X} + \Delta) - \phi(\bar{X} + \Delta)[R(X') - R(0)]. \tag{c3}$$

By (c3) - (c1) (simply let $\pi = \pi(\bar{X}, X')$), we find

$$V(\bar{X} + \Delta - \pi) - V(\bar{X} - \pi) < V(\bar{X} + \Delta) - V(\bar{X}) - [\phi(\bar{X} + \Delta) - \phi(\bar{X})][R(X') - R(0)]$$

or

$$\frac{V(\bar{X}+\Delta-\pi)-V(\bar{X}-\pi)}{\Delta} < [\frac{V(\bar{X}+\Delta)-V(\bar{X})}{\Delta}]-[\frac{\phi(\bar{X}+\Delta)-\phi(\bar{X})}{\Delta}][R(X')-R(0)].$$

Let $\Delta \to 0$, we obtain

$$V'(\bar{X}-\pi) < V'(\bar{X})-\phi'(\bar{X})[R(X')-R(0)]$$

or

$$V'(\bar{X})-V'(\bar{X}-\pi) > \phi'(\bar{X})[R(X')-R(0)]. \tag{c4}$$

From (c1), we also have $V(\bar{X})-V(\bar{X}-\pi)=\phi(\bar{X})[R(X')-R(0)]$. Dividing both sides of (c4) by this, we have

$$\frac{V'(\bar{X})-V'(\bar{X}-\pi)}{V(\bar{X})-V(\bar{X}-\pi)} > \frac{\phi'(\bar{X})}{\phi(\bar{X})}\ . \tag{c5}$$

For a small risk, $\pi(X) \to 0$, (c5) becomes

$$\frac{V''(\bar{X})}{V'(\bar{X})} > \frac{\phi'(\bar{X})}{\phi(\bar{X})}. \tag{c6}$$

Letting $m(\bar{X})=-V''(\bar{X})/V'(\bar{X})$, we obtain the desired result.

Conversely, if (c6) holds for any $\bar{X} = x$, then we can rewrite (c6) as

$$\frac{d\log[V'(x)]}{dx} > \frac{d\log[\phi(x)]}{dx}$$

and integrate it from x_0 to $x_0 + \Delta$ (where $\Delta > 0$) to yield

$$\log\frac{V'(x_0+\Delta)}{V'(x_0)} > \log\frac{\phi(x_0+\Delta)}{\phi(x_0)}$$

or

$$\frac{V'(x_0+\Delta)}{V'(x_0)} > \frac{\phi(x_0+\Delta)}{\phi(x_0)}. \tag{c7}$$

According to the Mean Value Theorem of differential calculus, for the "two" functions, $V(x)$ and $f(x)=V(x+\Delta)$, there at least exists a $c \in (\bar{X}-\pi,\ \bar{X})$ such that

$$\frac{V(\bar{X}+\Delta)-V(\bar{X}+\Delta-\mathrm{p})}{V(\bar{X})-V(\bar{X}-\mathrm{p})}=\frac{V'(c+\Delta)}{V'(c)} \qquad \text{(c8)}$$

Letting $x_0 = c$ in (c7) and comparing it with (c8), we find

$$\frac{V(\bar{X}+\Delta)-V(\bar{X}+\Delta-\mathrm{p})}{V(\bar{X})-V(\bar{X}-\mathrm{p})}>\frac{\mathrm{f}(c+\Delta)}{\mathrm{f}(c)}. \qquad \text{(c9)}$$

Since $\mathrm{f}'(x)/\mathrm{f}(x)$ is nonincreasing, then for $\bar{X} > c$ and $\Delta > 0$, we must have

$$\int_c^{c+\Delta}\frac{\mathrm{f}'(x)}{\mathrm{f}(x)}dx \geq \int_{\bar{X}}^{\bar{X}+\Delta}\frac{\mathrm{f}'(x)}{\mathrm{f}(x)}dx$$

or

$$\frac{\mathrm{f}(c+\Delta)}{\mathrm{f}(c)} \geq \frac{\mathrm{f}(\bar{X}+\Delta)}{\mathrm{f}(\bar{X})}. \qquad \text{(c10)}$$

Combining (c9) and (c10), we obtain

$$\frac{V(\bar{X}+\Delta)-V(\bar{X}+\Delta-\mathrm{p})}{V(\bar{X})-V(\bar{X}-\mathrm{p})}>\frac{\mathrm{f}(\bar{X}+\Delta)}{\mathrm{f}(\bar{X})}. \qquad \text{(c11)}$$

By (c1), we can have $\mathrm{f}(\bar{X})=\dfrac{V(\bar{X})-V(\bar{X}-\mathrm{p})}{R(X')-R(0)}$. Substituting this into (c11), we obtain

$$V(\bar{X}+\Delta)-V(\bar{X}+\Delta-\mathrm{p})>\mathrm{f}(\bar{X}+\Delta)[R(X')-R(0)]$$

or

$$V(\bar{X}+\Delta-\mathrm{p})<V(\bar{X}+\Delta)-\mathrm{f}(\bar{X}+\Delta)[R(X')-R(0)]. \qquad \text{(c12)}$$

Comparing (c12) with (c2) (recall $\mathrm{p}=\mathrm{p}(\bar{X}, X')$), we must have $\mathrm{p}(\bar{X}, X') < \mathrm{p}(\bar{X}+\Delta, X')$, which implies decreasing risk aversion.

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Table 1
Some examples of risk-value models

		A	B	C
$V(\bar{X})$ $R(X')$		$\bar{X}$	$-he^{-a\bar{X}}$	$\bar{X}-he^{-a\bar{X}}$
1	$\mathrm{E}[\lvert X-\bar{X}\rvert^{q}]$	$\bar{X}-ke^{-b\bar{X}}\mathrm{E}[\lvert X-\bar{X}\rvert^{q}]$	$-he^{-a\bar{X}}-ke^{-b\bar{X}}\mathrm{E}[\lvert X-\bar{X}\rvert^{q}]$	$\bar{X}-he^{-a\bar{X}}-ke^{-b\bar{X}}\mathrm{E}[\lvert X-\bar{X}\rvert^{q}]$
2	$\mathrm{E}[(X-\bar{X})^{2}]$ $-c\,\mathrm{E}[(X-\bar{X})^{3}]$	$\bar{X}-ke^{-b\bar{X}}\{\mathrm{E}[(X-\bar{X})^{2}]$ $-c\,\mathrm{E}[(X-\bar{X})^{3}]\}$	$-he^{-a\bar{X}}-ke^{-b\bar{X}}\{\mathrm{E}[(X-\bar{X})^{2}]$ $-c\,\mathrm{E}[(X-\bar{X})^{3}]\}$	$\bar{X}-he^{-a\bar{X}}-ke^{-b\bar{X}}\{\mathrm{E}[(X-\bar{X})^{2}]$ $-c\,\mathrm{E}[(X-\bar{X})^{3}]\}$
3	$\mathrm{E}[e^{-c(X-\bar{X})}-1]$	$\bar{X}-ke^{-b\bar{X}}\mathrm{E}[e^{-c(X-\bar{X})}-1]$	$-he^{-a\bar{X}}-ke^{-b\bar{X}}\mathrm{E}[e^{-c(X-\bar{X})}-1]$	$\bar{X}-he^{-a\bar{X}}-ke^{-b\bar{X}}\mathrm{E}[e^{-c(X-\bar{X})}-1]$
4	$d\,\mathrm{E}^{-}[\lvert X-\bar{X}\rvert^{q_2}]$ $-e\,\mathrm{E}^{+}[\lvert X-\bar{X}\rvert^{q_1}]$	$\bar{X}-e^{-b\bar{X}}\{d\,\mathrm{E}^{-}[\lvert X-\bar{X}\rvert^{q_2}]$ $-e\,\mathrm{E}^{+}[\lvert X-\bar{X}\rvert^{q_1}]\}$	$-he^{-a\bar{X}}-e^{-b\bar{X}}\{d\,\mathrm{E}^{-}[\lvert X-\bar{X}\rvert^{q_2}]$ $-e\,\mathrm{E}^{+}[\lvert X-\bar{X}\rvert^{q_1}]\}$	$\bar{X}-he^{-a\bar{X}}-e^{-b\bar{X}}\{d\,\mathrm{E}^{-}[\lvert X-\bar{X}\rvert^{q_2}]$ $-e\,\mathrm{E}^{+}[\lvert X-\bar{X}\rvert^{q_1}]\}$

Note: 1) All parameters used in this table are positive or nonnegative.

2) In this table, we use $f(\bar{X})=ke^{-b\bar{X}}$ (or $f(\bar{X})=e^{-b\bar{X}}$), for risk-value models. As a special case, $f(\bar{X})$ may be chosen as a positive constant (i.e., $b=0$) for some specific decision problems. Readers may also choose some other nonincreasing functions for $f(\bar{X})$.

3) In the last row, the standard measure of risk $R(X')=d\,\mathrm{E}^{-}[\lvert X-\bar{X}\rvert^{q_2}]-e\,\mathrm{E}^{+}[\lvert X-\bar{X}\rvert^{q_1}]$, where $\mathrm{E}^{-}[\lvert X-\bar{X}\rvert^{q_2}]=\sum_{x_i<\bar{X}}p_i\lvert x_i-\bar{X}\rvert^{q_2}$ and $\mathrm{E}^{+}[\lvert X-\bar{X}\rvert]^{q_1}=\sum_{x_i>\bar{X}}p_i(x_i-\bar{X})^{q_1}$, is based on a piecewise linear plus power utility model (Jia and Dyer, 1996).