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MEASURES OF PERCEIVED RISK

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Abstract

Based on our previous work on the standard measure of risk, this paper presents two classes of measures for perceived risk by considering a two-dimensional structure, the mean of a lottery and its standard risk. One of the classes of our risk measures presumes that there is no risk if and only if there is no uncertainty involved and the other allows that different degenerate lotteries may be evaluated with different values of "risk". The former has more prescriptive appeal in risky decision making and the later has more descriptive power for subjective risk judgments. Our risk measures can also take into account the asymmetric effects of losses and gains on perceived risk. It is demonstrated that our perceived risk models unify a large body of empirical evidence regarding risk judgments and can be incorporated into preference models (i.e., based on risk-value tradeoffs) in a clear fashion.

(Perceived risk; risk measurement; risk-value tradeoffs)

1. Introduction

In our previous papers, we proposed a standard measure of risk, which is based on the converse of the expected utility of normalized distributions or lotteries with zero-expected values, and used it to derive risk-value models for risky choices within both an expected utility framework and a non-expected utility framework (Jia and Dyer, 1995, 1996; Dyer and Jia 1997). Though the standard measure of risk has some descriptive power for risk judgment, it is more normative in nature. In particular, since the standard measure of risk eliminates the effect of the mean of a lottery, it only measures the "pure" risk of the lottery, and may not be appropriate for modeling perceptions of risk. The purpose of this study is to treat the concept of risk as a primitive and to develop measurement models for perceived risk based on the standard measure of risk.

Over the last 25 years, researchers have expended much effort toward developing and testing models of the perceived risk of lotteries. Pollatsek and Tversky (1970) wrote:

> The various approaches to study of risk share three basic assumptions. 1. Risk is regarded as a property of options (e.g., gambles, courses of action) that affects choices among them. 2. Options can be meaningfully ordered with respect to their riskiness. 3. The risk of an option is related in some way to the dispersion, or the variance, of its outcomes.

These ideas expressed in an early study on risk are still meaningful to today's research. As stated in assumption 1, risk is a characteristic of a lottery that affects decisions. This should be a major reason why we study the nature of perceived risk. A measure of perceived risk may be used as a variable in preference models, such as Coombs' Portfolio theory (Coombs, 1969; Coombs and Meyer, 1969; Coombs and Huang, 1970b; Coombs, 1975) in which a choice among lotteries is a compromise between maximizing expected value and optimizing the level of perceived risk. However, the "risk" measure in Coombs' Portfolio theory is left essentially undefined, and is considered to be an independent theory. This has stimulated a long stream research on the measure of perceived risk.

Although perceived risk might be regarded as a vague and subjective notion, empirical studies have demonstrated that subjects are able to consistently order lotteries with respect to their riskiness, and that risk judgments satisfy some basic axioms (e.g., see Keller, Sarin and Weber, 1986; Weber and Bottom, 1990). Thus, as stated in assumption 2, the term "riskiness" should be meaningful to subjects.

Following assumption 3, many additional assumptions have been made about the change in perceived riskiness resulting from changes in lotteries, such as:

- Perceived risk increases when there is an increase in range, variance, or expected loss (e.g., see Coombs and Huang, 1970a).
- Perceived risk decreases if a constant positive amount is added to all outcomes of a lottery (Coombs and Lehner, 1981; Keller, Sarin and Weber, 1986).
- Perceived risk increases if all outcomes of a lottery with zero-mean are multiplied by a positive constant greater than one (Coombs and Meyer, 1969).
- Perceived risk increases if a lottery with zero-mean is repeated many times (Coombs and Meyer, 1969).

These empirically verified properties provide basic guidelines for developing and evaluating measures of perceived risk.

In the following section, we give a technical review of previously proposed models of perceived risk. As we shall see, these developments are not completely satisfactory according to empirical studies. In Section 3, we present our measures of perceived risk based on a two dimensional structure of the standard risk of a lottery and its mean. Theoretical and empirical evaluations of the proposed measures of perceived risk are provided in section 4. It is shown that our perceived risk models unify a large body of empirical evidence. Finally, in section 5, we summarize this study and discuss its implications in decision making under risk. Proofs of theorems are included in the appendix.

2. Technical review of some perceived risk models

The literature contains various attempts to define risk based on different assumptions about how perceived risk is determined and changed. In this section, we review some previously proposed models for perceived risk and their key assumptions. We focus on those that are closely related to the present study.

2.1. Studies by Coombs and his Associates

In early studies, risk judgments were represented by using the moments of a distribution and their transformations. Expected value, variance, skewness, range, and the number of repeated plays have been investigated as possible determinates of risk judgments (Coombs and Pruitt, 1960; Coombs and Meyer, 1969; Coombs and Huang, 1970a). Coombs and Huang (1970a) considered several composition functions of three indices corresponding to transformations on perceived risk, and their study supported a distributive model. However, evidence to the contrary of the distributive model was also found (Barron, 1976). A study by Coombs and Lehner (1981) concluded that a simple polynomial distributive model with variables measuring such general characteristics of a distribution as expectation, dispersion and asymmetry cannot capture perceived risk because of some complex interactions among these variables.

Coombs and Lehner (1984) further considered perceived risk as a direct function of outcomes and probabilities, with no intervening distribution parameters. They assumed a bilinear model similar to that of prospect theory (Kahneman and Tversky, 1979). Their experiment supported the proposition that the notion of perceived risk can be decomposed into contributions from good and bad components, and the bad components play a larger role than the good ones.

2.2. Pollatsek and Tversky's Risk Theory

An important milestone in the study of perceived risk is the axiomatization of risk theory developed by Pollatsek and Tversky (1970). Let P denote a set of simple probability distributions or lotteries $\{X, Y, Z, \ldots\}$[1] and $\geq_R$ be a binary risk relation (meaning at least as risky as). Pollatsek and Tversky assumed seven axioms for a risk system, which leads to a real-valued risk measure R on P as follows:

$$R(X) = -\theta\bar{X} + (1-\theta)\mathrm{E}[(X-\bar{X})^2], \tag{1}$$

where $0 < \theta \leq 1$, $\bar{X}$ is the mean of a lottery X and E represents the operation of expectation.

However, the empirical validity of model (1) was criticized by Coombs and Bowen (1971b) who showed that factors other than mean and variance, such as skewness, affect perceived risk. In the system of Pollatsek and Tversky's axioms, the continuity condition based on the central limit theorem is directly responsible for the mean-variance model (1).

Another empirically questionable assumption of Pollatsek and Tversky's axioms is the additive independence condition, i.e., for X, Y and Z in P, $X \geq_R Y$ if and only if $X \circ Z \geq_R Y \circ Z$, where "$\circ$" denotes the binary operation of adding independent random variables (i.e., the convolution of their density functions). This independence condition implies that risk is additive, a phenomenon that was not supported by empirical studies (Coombs and Bowen, 1971a; Nygren, 1977).

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[1] For convenience, we will use X, Y and Z to refer to random variables, probability distributions or lotteries interchangeably.

Nevertheless, some other assumptions in the system of Pollatsek and Tversky's axioms are very appealing, such as positivity and scalar monotonicity. Because they are important to the present study, we briefly introduce them here. According to the positivity axiom, if K is a degenerate lottery with an outcome $k > 0$, then $X \geq_R X \circ K$ for all X in P. In other words, the addition of a positive and sure amount to a lottery would decrease its perceived risk. This property is considered a universal property of perceived risk, and has been confirmed by several empirical studies (e.g., see Coombs and Lehner, 1981; Keller, Sarin and Weber, 1986).

Another appealing axiom in Pollatsek and Tversky's risk theory is scalar monotonicity, which means, for all X, Y in P with E(X)=E(Y)=0, (1) $bX \geq_R X$ for $b > 1$; and (2) $X \geq_R Y$ if and only if $bX \geq_R bY$ for $b > 0$. This axiom asserts that: (1) for lotteries with zero expectation, risk increases when the lottery is multiplied by a real number $b > 1$ (also see Coombs and Meyer, 1969); and (2) the risk ordering is preserved under a scale change of the lotteries (e.g., from dollars to pennies). Pollatsek and Tversky regarded the positivity axiom and part (1) of the monotonicity axiom as necessary assumptions for any theory of risk.

In a recent study, Rotar and Sholomitsky (1994) weakened the part (2) of the scalar monotonicity axiom (coupled with some other additional conditions) to arrive at a more flexible risk model that is a finite linear combination of cumulants of higher order moments. However, because Rotar and Sholomitsky's risk model still retains the additive independence condition as a basic assumption, their model would be subject to the same criticisms regarding the additivity of risk (e.g., see Coombs and Bowen, 1971a; Fishburn, 1988).

2.3. Luce's Risk Models and Others

Subsequent to the criticisms of Pollatsek and Tversky's risk measure, Luce (1980, correction 1981) approached the problem of risk measurement in a different way. He began with a multiplicative structure of risk. Based on two decomposition rules and two aggregation rules, he proposed four measures of risk including an expected logarithm model and an expected power model. However, Luce's risk models did not receive positive support from an experimental test by Keller, Sarin and Weber (1986).

Influenced by empirical investigations, Luce and Weber (1986) proposed the Conjoint Expected Risk (CER) model, a revision of Luce's original power model. The CER model has the following form:

$$R(X) = A_0 \Pr(X = 0) + A_+ \Pr(X > 0) + A_- \Pr(X < 0)$$
$$+ B_+ \mathrm{E}[X^{K+} \mid X > 0] \Pr(X > 0) + B_- \mathrm{E}[|X|^{K-} \mid X < 0] \Pr(X < 0), \qquad (2)$$

where A_0, A_+ and A_- are probability weights, and B_+ and B_- are weights of the conditional expectations raised to some positive powers, K_+ and K_-. The major advantage of the CER model is that it allows for asymmetric effects of transformations on positive and negative outcomes. Weber (1988) showed that the CER model describes risk judgments reasonably well. But one might argue that the CER model contains so many parameters (seven in total) that the model is flexible enough to fit subject's data.

Weber and Bottom (1990) recently tested the adequacy of the axioms underlying the CER model and found that the conjoint structure assumptions about the effect of change of scale transformations on risk hold for negative-outcome lotteries, but not for positive-outcome lotteries. Thus, Luce and Weber’s (1986) multiplicative structure of risk, including the multiplicative independence assumption (i.e., for positive (or negative)-outcome-only lotteries X and Y, $X \geq_R Y$ if and only if $bX \geq_R bY$ for $b > 0$) may not be valid. Note that this assumption is related to Pollatsek and Tversky’s (1970) scalar monotonicity axiom. However, Pollatsek and Tversky only assumed this condition for the lotteries with zero-expected values.

Sarin (1987) also extended Luce's (1980) work by considering the effect on perceived risk of adding a constant to a lottery, and proposed an expected exponential risk model. Luce's models as well as Sarin's model employ the expectation principle (Huang, 1971) implied by the independence axiom of expected utility theory. But empirical studies have shown that this assumption may not be valid for risk judgments (Coombs and Lehner, 1984; Keller, Sarin and

Weber, 1986).[2] This cast a doubt on any perceived risk models that are directly based on the expectation principle.

Fishburn (1982, 1984) explored risk measurement from a rigorous axiomatic perspective. In his two-part study on axiomatizations of perceived risk, he considered lotteries separated into gains and losses relative to the target zero. He assumed that there is no risk if and only if there is no chance of getting a loss. This rules out additive forms of risk measures but allows forms that are multiplicative in losses and gains. Because Fishburn's models contain free functions, it is difficult to test them empirically.

In summary, since the pioneering work of Coombs and his associates' on perceived risk, several formal theories and models have been proposed. But none of them is fully satisfactory. This encourages us to search further for a better measure of perceived risk.

3. Two-Attribute Models for Perceived Risk

In this section, we propose two-attribute models for perceived risk based on the mean of a lottery and the standard measure of risk that we developed previously. We also suggest several explicit functional forms based on our measures of perceived risk.

3.1. A Two-Attribute Structure for Perceived Risk

A common approach in previous studies of perceived risk is to look for different factors underlying a lottery that are responsible for risk perceptions, and then consider some separation or aggregation rules to obtain a risk measurement model. As reviewed in the last section, different kinds of multiple dimensional structures of risk have been considered, which can be summarized as follows:[3]

- {mean, variance} (Pollatsek and Tversky, 1970);

[2] Weber and Bottom (1990) showed, however, that the independence axiom is not violated for risk judgments, but that the culprit is the so-called probability accounting principle (see Luce and Weber, 1986).

[3] See Payne (1973) for a review of some other risk studies based on multidimensional stimuli.

• {mean, range, skewness} (Coombs and Huang, 1970a; Coombs and Lehner, 1981);

• {positive outcome and probability, negative outcome and probability} (Coombs and Lehner, 1984);

• {loss probability, distribution of loss, gain probability, distribution of gain} (Fishburn, 1982, 1984);

• {probabilities for loss, gain, and zero, positive outcomes, negative outcomes} (Luce and Weber, 1986).

In an earlier study (Jia and Dyer, 1996), we considered decomposing a lottery X into its mean $\overline{X}$ and its standard risk, $X' = X - \overline{X}$, and proposed a standard measure of risk as follows:

$$R(X') = -\mathrm{E}[u(X')] = -\mathrm{E}[u(X - \overline{X})] \tag{3}$$

where $u(\cdot)$ is a von Neumann-Morgenstern utility function. The mean of a lottery serves as a status quo for measuring the standard risk.

For lotteries with zero-expected values, the standard measure of risk can provide a suitable measure for perceived risk. However, the standard measure of risk would not be appropriate for modeling subjects' perceived risk for general lotteries since the standard measure of risk is independent of expected value or any certain payoffs.[4] As we discussed earlier, empirical studies have shown that subjects' perceived risk decreases as a positive constant amount is added to all outcomes of a lottery (Coombs and Lehner, 1981; Keller, Sarin and Weber, 1986).

In order to incorporate the effect of the mean of a lottery on perceived risk, we consider a two-attribute structure for evaluating perceived risk; that is, $(\overline{X}, X')$. In fact, a lottery X can be represented by X' and $\overline{X}$ exclusively, e.g., $X = \overline{X} + X'$. Thus, $(\overline{X}, X')$ is simply a natural extension of the representation of the lottery X. This two-attribute structure has an intuitive implication in risk judgment. When people make a risk judgment for lotteries, they may first

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[4] If $Y = X + k$, where k is a constant, then $Y' = Y - \overline{Y} = X - \overline{X} = X'$.

consider the variations or uncertainty of the lotteries, measured by X', and then take into account the effect of the expected values of the lotteries on the uncertainty perceived initially, or vice versa.

Let **P** be the set of all simple probability distributions, including degenerate distributions, on a nonempty product set, $\mathbf{X}_1 \times \mathbf{X}_2$, of outcomes, where $\mathbf{X}_i \subseteq$ Re, i = 1, 2, and Re is the set of real numbers. For our special case, the outcome of a lottery X on $\mathbf{X}_1$ is fixed at its mean $\bar{X}$; thus, the marginal distribution on $\mathbf{X}_1$ is degenerate with a singleton outcome $\bar{X}$. We assume $|\bar{X}| < \infty$; that is, the mean of a lottery is bounded. For the second attribute, the marginal distribution on $\mathbf{X}_2$ is $X' \in P^{\circ}$, where P° is the set of normalized probability distributions with zero expected values. Therefore, $(\bar{X}, X')$ denotes the distribution in **P** that yields $\bar{X} \in \mathbf{X}_1$ with probability 1 coupled with $x' \in \mathbf{X}_2$ with probability X', where x' is a realization on the risk attribute. Because $\bar{X}$ is a constant, the two "marginal distributions" $(\bar{X}, X')$ are sufficient to determine a unique distribution in **P**.

Let $>_{\tilde{\mathbf{R}}}$ be a strict risk relation, $\sim_{\tilde{\mathbf{R}}}$ an indifference risk relation, and $\geq_{\tilde{\mathbf{R}}}$ a weak risk relation on **P**. We assume a two-attribute case of the expectation principle and other necessary conditions[5] for the risk ordering $>_{\tilde{\mathbf{R}}}$ on **P** such that for all $(\bar{X}, X'), (\bar{Y}, Y') \in \mathbf{P}$, $(\bar{X}, X') >_{\tilde{\mathbf{R}}} (\bar{Y}, Y')$ if and only if $R_p(\bar{X}, X') > R_p(\bar{Y}, Y')$, where R_p is defined as follows:

$$R_p(\bar{X}, X') = \mathrm{E}[U_R(\bar{X}, X')] \qquad (4)$$

and U_R is a real-valued function unique up to similar positive linear transformations. Note that because the marginal distribution for the first attribute is degenerate, the expectation, in fact, only needs to be taken over the marginal distribution for the second attribute, which in turn is the original distribution of a lottery X over the standard risk $X' = X - \bar{X}$.

3.2. Basic Forms of Perceived Risk Models

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[5] These conditions are analogous to those of multiattribute utility theory (e.g., see Fishburn, 1970).

Model (4) provides a general measure of perceived risk based on two attributes, the mean and standard risk of a lottery. In order to obtain separable forms of perceived risk models, we make the following assumptions about risk judgment.

ASSUMPTION 1. For $X', Y' \in P^o$, if there exists a $w^o \in$ Re for which $(w^o, X') >_{\tilde{R}} (w^o, Y')$, then $(w, X') >_{\tilde{R}} (w, Y')$ for all $w \in$ Re.

ASSUMPTION 2. For $X', Y' \in P^o$, $(0, X') >_{\tilde{R}} (0, Y')$ if and only if $X' >_R Y'$.

ASSUMPTION 3. For $(\bar{X}, X') \in \mathbf{P}$, then $(\bar{X}, X') >_{\tilde{R}} (\bar{X}+\Delta, X')$ for any constant $\Delta > 0$.

Assumption 1 is an independence condition, which says that the risk ordering for two lotteries with the same mean will not switch when the common mean changes to any other value. Compared with Pollatsek and Tversky's (1970) additive independence condition, Assumption 1 is weaker since it uses a pair of lotteries with the same mean and a common constant as a variable instead of a common lottery. Coombs (1975) considered a similar assumption for a riskiness ordering, i.e., $X \geq_R Y$ if and only if $X + k \geq_R Y + k$, where $E(X) = E(Y)$ and k is a constant. However, our formulation is based on a two-attribute structure, which leads to a separable risk function for $\bar{X}$ and X', as we shall discuss.

Assumption 2 postulates a relationship between the two risky binary relations, $>_{\tilde{R}}$ and $>_R$ (where $>_R$ is a strict risk relation on P^o), so that for any zero-expected lotteries, the risk judgments made by $R_p(0, X')$ and by the standard measure of risk $R(X')$ are consistent. The last assumption implies that if two lotteries have the same "pure" risk, X', then the lottery with a larger mean will be perceived less risky than the one with a lower mean, which has been suggested by previous studies (Pollatsek and Tversky, 1970; Coombs and Lehner, 1981; Keller, Sarin and Weber, 1986).

THEOREM 1. The two-attribute perceived risk model (4) can be decomposed into the following form:

$$R_p(\bar{X}, X') = g(\bar{X}) + \psi(\bar{X}) R(X') \quad (5)$$

if and only if Assumptions 1-3 are satisfied, where $\psi(\bar{X}) > 0$, $g'(\bar{X}) < -\psi'(\bar{X})R(X')$, and $R(X')$ is the standard measure of risk.

According to this theorem, the measure of perceived risk can be constructed by a combination of the standard measure of risk and the effect of the mean. Theorem 1 postulates a constraint on the choice of functions $g(\bar{X})$ and $\psi(\bar{X})$ in model (5). If $\psi(\bar{X})$ is a constant, then the condition, $g'(\bar{X}) < -\psi'(\bar{X})R(X')$, becomes $g'(\bar{X}) < 0$; i.e., $g(\bar{X})$ is a decreasing function of $\bar{X}$. Otherwise, a nonincreasing function $g(\bar{X})$ and a decreasing function $\psi(\bar{X})$ should be sufficient to satisfy the condition $g'(\bar{X}) < -\psi'(\bar{X})R(X')$ (assuming $R(X') > 0$).

For risk judgments, we may require that any degenerate lotteries should have no risk (e.g., Bell, 1988, 1995). The concept of risk would not be evoked under conditions of certainty; no matter how bad a certain loss may be, it is a sure thing and, therefore, riskless. This point of view can be represented by the following assumption.

ASSUMPTION 4. For any $w \in \mathrm{Re}$, $(w, 0) \sim_{\tilde{R}} (0, 0)$.

THEOREM 2. The two-attribute perceived risk model (4) can be represented as follows:

$$R_p(\bar{X}, X') = \psi(\bar{X})[R(X') - R(0)] \quad (6)$$

if and only if assumptions 1-4 are satisfied, where $\psi(\bar{X}) > 0$ is a decreasing function.

When $g(\bar{X}) = -R(0)\psi(\bar{X})$ as required by Assumption 4, the general risk model (5) reduces to the multiplicative risk model (6). This multiplicative risk model captures the effect of the mean on perceived riskiness in an appealing way; increasing (decreasing) the mean reduces (increases) perceived riskiness in a proportional manner.

Finally, note that our two-attribute perceived risk models (5) and (6) are not simple expected forms; we decompose a lottery into a two-attribute structure and only apply the expectation principle to normalized lotteries with zero-expected values. Thus, our perceived risk models can avoid the drawbacks of a simple expected risk model (Coombs and Lehner, 1984; Keller, Sarin and Weber, 1986). Further, we believe that the expectation principle should be appropriate for the lotteries with zero-expected values.

3.3. Some Examples

As a special case of our perceived risk model (5), when $g(\bar{X})$ is linear, $\psi(\bar{X})$ is constant and $R(X')$ is variance, the risk model (5) reduces to Pollatsek and Tversky's (1970) mean-variance model (3). But Pollatsek and Tversky's risk model may be considered over simplified. To obtain Rotar and Sholomitsky's (1994) generalized moments model, we just need to choose the standard measure of risk based on a polynomial utility model.

Based on the general structure of the perceived risk model (5), we can select some appropriate functional forms for $g(\bar{X})$, $\psi(\bar{X})$ and $R(X')$ to construct many different models for perceived risk. In Jia and Dyer (1996), we have proposed some explicit models for the standard measure of risk $R(X')$. Those models can be used directly in constructing functional forms of perceived risk models (5) and (6). An example for $\psi(\bar{X})$ is $\psi(\bar{X}) = ke^{-b\bar{X}}$, where $k > 0$ and $b \geq 0$ (when $b = 0$, $\psi(\bar{X})$ becomes a constant k), and a simple choice for $g(\bar{X})$ is $g(\bar{X}) = -a\bar{X}$, where $a > 0$ is a constant. Some functional forms of the perceived risk model (5) are provided as follows:

$$R_p(\bar{X}, X') = -a\bar{X} + ke^{-b\bar{X}}\,\mathrm{E}[e^{-c(X - \bar{X})}], \tag{7}$$

$$R_p(\bar{X}, X') = -a\bar{X} + ke^{-b\bar{X}}\,\{\mathrm{E}[(X - \bar{X})^2] - c\,\mathrm{E}[(X - \bar{X})^3]\}, \tag{8}$$

$$R_p(\bar{X}, X') = -a\bar{X} + e^{-b\bar{X}}\,\{d\mathrm{E}^-[|X - \bar{X}|^{\theta_2}] - e\mathrm{E}^+[|X - \bar{X}|^{\theta_1}]\}, \tag{9}$$

where a, b, c, d, e, k, q_1 and q_2 are nonnegative constants. When $b = 0$, these perceived risk models become additive forms. For different individuals and different situations, we may need to choose different models for describing perceived risk.

Similarly, some examples of the multiplicative risk model (6) are given as follows:

$$R_p(\bar{X}, X') = ke^{-b\bar{X}} \mathrm{E}[e^{-c(X - \bar{X})} - 1], \tag{10}$$

$$R_p(\bar{X}, X') = ke^{-b\bar{X}} \{\mathrm{E}[(X - \bar{X})^2] - c\,\mathrm{E}[(X - \bar{X})^3]\}, \tag{11}$$

$$R_p(\bar{X}, X') = e^{-b\bar{X}} \{d\mathrm{E}^-[|X - \bar{X}|^{q_2}] - e\mathrm{E}^+[|X - \bar{X}|^{q_1}]\}. \tag{12}$$

Note that perceived risk models (9) and (12) contain the standard measure of risk that includes many financial risk measures as special cases (Jia and Dyer, 1996). These perceived risk models show how financial measures of risk and psychological measures of risk (i.e., measures of perceived risk) can be related and unified. In particular, for a given level of mean, minimizing the perceived risk defined by (5) or (6) will be equivalent to minimizing the standard risk. Our measures of perceived risk provide a clear way to simplify the decision criterion of minimizing perceived risk (or maximizing perceived risk for a risk seeker), as suggested in Coombs' Portfolio theory (Coombs, 1969; Coombs and Meyer, 1969; Coombs and Huang, 1970b; Coombs, 1975).

4. Theoretical and Empirical Evaluations

In this section, we further investigate some properties and the empirical validity of our perceived risk models based on previous experimental studies.

4.1. Properties of the Measures of Perceived Risk

Models (5) and (6) can have many appealing properties for measuring perceived risk, which are summarized as follows:

i) For an independent zero-mean lottery Z', $R_p(\overline{X}, X'+Z') > R_p(\overline{X}, X')$.

ii) For a zero-mean lottery X' and a constant b > 1, $R_p(0, \text{b}X') > R_p(0, X')$.

iii) Perceived risk increases if a lottery X' with zero-mean is repeated many times.

iv) For any lottery and any constant $c > 0$, $R_p(\overline{X} + c, X') < R_p(\overline{X}, X')$.

v) For two lotteries with the same mean $\overline{X} = \overline{Y}$, if $R_p(\overline{X}, X') > R_p(\overline{Y}, Y')$, then $R_p(\overline{X} + k, X') > R_p(\overline{Y} + k, Y')$ for any constant k.

vi) For lotteries with negative means only, $R_p(\text{b}\overline{X}, \text{b}X') > R_p(\overline{X}, X')$ for any constant b > 1.

The first property (i) concerns mean-preserving spreads (Rothschild and Stiglitz, 1970) and states that the perceived risk for a lottery increases when we add some random noise to it.[6] Property (ii) is another type of mean-preserving spreads, which was reported by Coombs and Meyer (1969) in their experiment using lotteries with zero-expected values.[7] Properties (i) and (ii) also imply that risk increases when there is an increase in variance or range, holding means unchanged (see Coombs and Huang, 1970b).

When a lottery with a zero mean is played many times independently (i.e., the property (iii)), it is equivalent to using property (i) over and over again, which results in an increasing risk according to the mean-preserving spreads property. This empirical evidence was observed by Coombs and Meyer (1969) in an experimental study.

Property (iv) is our presumption that perceived risk decreases as the mean of a lottery increases, which is considered a universal property of perceived risk and has been confirmed by several empirical studies (Coombs and Lehner, 1981; Keller, Sarin and Weber, 1986). Properties (ii) and (iv) are also basic assumptions in Pollatsek and Tversky's (1970) risk theory. Although

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[6] In order to satisfy the property of mean-preserving spreads (Rothschild and Stiglitz, 1970), we should use, in general, a concave utility function for the standard measure of risk (see Jia and Dyer, 1996).

[7] The implication of property (ii) will be further discussed in the following subsection.

their risk system is simplified by some other assumptions, Pollatsek and Tversky regarded these two properties as necessary conditions for any theory of risk.

Property (v) is the most important presumption (Assumption 1) for our perceived risk models. It means that the risk order is invariant under the additive translation; that is, if one lottery is perceived as riskier than another at the same expected value level, then translating the lotteries by adding the same constant to all outcomes will not reverse the risk order. Coombs (1975) provided some empirical evidence of this. The last property (vi) will be discussed in the next section.

4.2. Comparison with Luce and Weber's CER Model

Property (vi) is particularly interesting. Several authors assumed that for a lottery X and a constant $b > 1$, $R(bX) > R(X)$ (Luce, 1980; Luce and Weber, 1986; Sarin and Weber, 1993). Our model states that the condition is only true for lotteries that have negative means or zero-means (i.e., property (ii)).

Luce and Weber (1986) used multiplicative independence and archimedean conditions in their Conjoint Expected Risk (CER) model (2). The multiplicative independence condition implies that for any positive (or negative)-outcome-only lottery X, $b\,X >_R a\,X$ if and only if $b > a > 0$,[8] and the multiplicative archimedean condition requires that for two lotteries X and Y, if $X >_R Y$, then there exists a constant $b > 0$ such that $b\,Y \geq_R X$. However, an empirical study by Weber and Bottom (1990) showed that the multiplicative independence and archimedean conditions only hold for negative-outcome lotteries but not for positive-outcome lotteries. This raises a question regarding the assumptions of Luce and Weber's CER model.

The empirical findings of Weber and Bottom (1990) can be explained by our perceived risk models. As an example, let us consider model (12), which has some similarity to the CER

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[8] As a special case when $a = 1$, this condition becomes $bX \geq_R X$ or $R(bX) > R(X)$ for $b > 1$.

model (2). For simplicity, when the distribution is symmetric and $q_1 = q_2 = q$, this model can be rewritten as:

$$R_p(\bar{X}, X') = ke^{-b\bar{X}}\mathrm{E}[|X - \bar{X}|^{q}] \quad (13)$$

where $k = (d - e)/2$. Multiplying the lottery X by a positive constant $b > 0$, we have

$$R_p(b\bar{X}, bX') = kb^{q}e^{-bb\bar{X}}\mathrm{E}[|X - \bar{X}|^{q}]. \quad (14)$$

If the lottery X has negative outcomes only, then its mean must also be negative. Thus $e^{-bb\bar{X}}$ will increase as b increases. The function b^{q} is also an increasing function of b . Therefore, $R_p(b\bar{X}, bX')$ must increase as b increases, which is consistent with the multiplicative independence condition as well as the multiplicative archimedean condition. However, for a lottery with positive outcomes, the mean must be positive. Then $e^{-bb\bar{X}}$ will decrease as b increases. Even though b^{q} is an increasing function of b , the overall effects of the two factors may cancel out, leaving $R_p(b\bar{X}, bX')$ essentially unchanged, or changed without a systematic direction. Figure 1 provides a numerical example of this phenomenon.

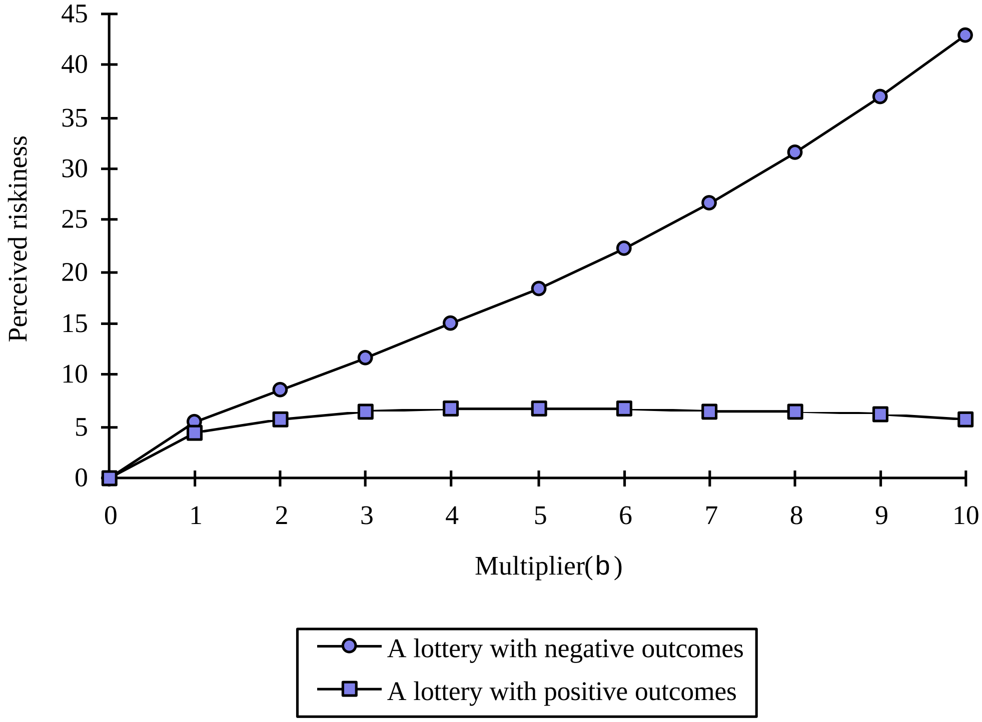

Figure 1. The effects of positive mean and negative mean on perceived risk.

(This figure is based on model (14) using k = 1, b = 0.01, q = 0.5, $\bar{X}$ = - 10 for the negative outcome lottery, $\bar{X}$ = 10 for the positive outcome lottery, and $E[|X-\bar{X}|^{q}]$ = 5 for both lotteries.)

A major strength of Luce and Weber's CER model is that it allows for asymmetric impacts of positive and negative outcomes on perceived risk. Empirical studies of preference under risk also show that the effect of loss on preference is larger than that of gain (e.g., Fishburn and Kochenberger, 1979; Kahneman and Tversky, 1979). This coincides with similar findings in risk judgments (Coombs and Lehner, 1983; Luce and Weber, 1986; Weber, 1988). To consider the asymmetric effects, we can use our perceived risk models (9) and (12), which are based on the standard measure of risk with a piecewise power utility function.

In order to make a further comparison with Luce and Weber's CER model, we consider the following form of a piecewise utility model:

$$u(x) = \begin{cases} k_1 + e x^{\theta_1} & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -k_2 - d\,|x|^{\theta_2} & \text{for } x < 0, \end{cases} \qquad (15)$$

where d, e, k_1, k_2, θ_1, and θ_2 are nonnegative constants. As illustrated in Figure 2, this utility function has a "jump" between preferences for a loss and a gain, which is measured by k_1 and k_2 (unless $k_1 = k_2 = 0$). If an individual's preference for a loss has a larger influence on his or her utility than that of a gain, then $k_2 > k_1$.

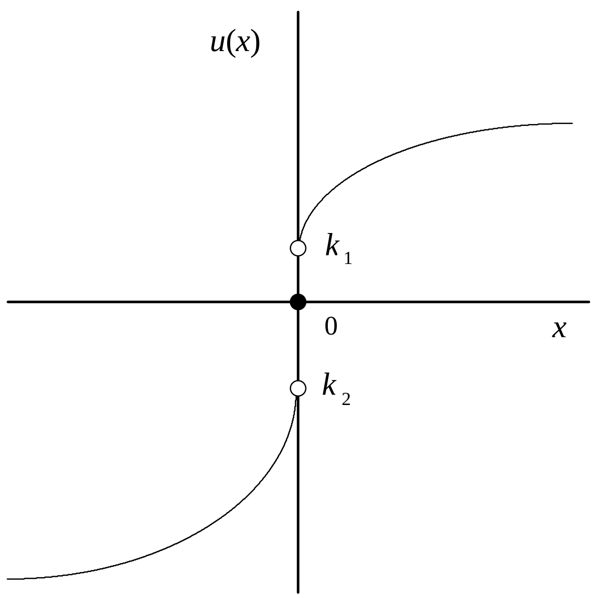

Figure 2. A piecewise utility model

If we take the converse form of model (15) as a risk function, then Luce and Weber's CER model (2) can be obtained by taking the expectation of the converse model (letting $k_1 = -A_+$,

$k_2 = A_-$, $e = - B_+$, $d = B_-$, $q_1 = K_+$, $q_2 = K_-$ and $A_0 = 0$). This provides an interpretation for the CER model.[9]

Based on the utility function (15), we have the following standard measure of risk:

$$R(X') = k_1 \Pr(X < \bar{X}) - k_2 \Pr(X > \bar{X}) + d\mathrm{E}^-[|X - \bar{X}|^{q_2}] - e\mathrm{E}^+[|X - \bar{X}|^{q_1}]. \tag{16}$$

In this case, if the distribution of a lottery is asymmetric, i.e., $\Pr(X < \bar{X}) \neq \Pr(X > \bar{X})$, the probability of relative loss (with respect to the mean) $\Pr(X < \bar{X})$ and the probability of relative gain $\Pr(X > \bar{X})$ become the factors that would influence one's perceived risk. Based on our perceived risk models (5) and (6) and the standard measure of risk (16), we propose the following additional measures of perceived risk:

$$R_p(\bar{X}, X') = -a\bar{X} + \{k_1 \Pr(X < \bar{X}) - k_2 \Pr(X > \bar{X}) + d\mathrm{E}^-[|X - \bar{X}|^{q_2}] - e\mathrm{E}^+[|X - \bar{X}|^{q_1}]\}, \tag{17}$$

$$R_p(\bar{X}, X') = e^{-b\bar{X}} \{k_1 \Pr(X < \bar{X}) - k_2 \Pr(X > \bar{X}) + d\mathrm{E}^-[|X - \bar{X}|^{q_2}] - e\mathrm{E}^+[|X - \bar{X}|^{q_1}]\}. \tag{18}$$

These models may be viewed as revisions of Luce and Weber's CER model. When $k_1 = k_2 = 0$, models (17) and (18) reduce to models (9) and (12) respectively. Thus, the risk models (17) and (18) should be more flexible for risk judgments.

5. Conclusions

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[9] According to this interpretation for the CER model, the constants A_+ and A_- in the model (15) should be negative and positive respectively if we assume the risk function is monotonic (setting $A_0 = 0$). This is consistent with Luce and Weber's (1986) experimental study, but not with Weber's (1988) experimental results.

In this paper, we have reviewed previous studies about perceived risk and proposed new measures of perceived risk. Our measures of perceived risk are based on a two-dimensional structure, the mean of a lottery and its standard risk, for evaluating the riskiness perceived by subjects. Some of our risk measures also take into account the asymmetric effects of losses and gains on perceived risk. We have demonstrated that our measures of perceived risk can unify a large body of empirical evidence about risk judgments and capture people's perceptions of risk better than many previously proposed measures of perceived risk.

In particular, our measures of perceived risk show a clear relationship between financial measures of risk and psychological measures of risk. They can also be incorporated into preference models in a clear fashion based on a tradeoff between perceived risk and expected value. In our other papers (Jia and Dyer, 1995, 1996), we proposed risk-value models that have the intuitive form of the subjective value of mean less the measure of perceived risk (6). Thus, our risk-value framework provides a unified approach to both risk judgment and preference modeling.

In this development, we have offered two classes of perceived risk measures. One of the classes presumes that there is no risk if and only if there is no uncertainty involved, and the other allows different degenerate lotteries to be evaluated with different values of "risk". The former has more prescriptive appeal in risky decision making, and the latter has more descriptive power for subjective risk judgments. In fact, most of perceived risk measures that have been proposed previously allow degenerate lotteries to have different evaluation values of risk (e.g., Pollatsek and Tversky, 1970; Coombs and Huang, 1970a; Coombs and Lehner, 1984; Luce, 1980; Luce and Weber, 1986; Sarin, 1987; Rotar and Sholomitsky, 1994).

As we argued earlier, from a prescriptive point of view, it is questionable to associate a measure of risk with a sure thing. However, some experiments did show that subjects may or may not give different ratings for different degenerate lotteries. For example, in a study by Nygren (1977), 13 subjects rated the (degenerate) lottery (10, 0.5; 10) (i.e., the outcome is 10 for sure) less risky than (0, 0.5; 0), and 22 subjects rated them equally risky. For the subjects who rated both lotteries as having the same riskiness, the form of perceived risk measure (5) must be ruled out. In Nygren's study, no attempt was made to define risk; subjects were encouraged to make judgments on the basis of what risk meant to them. It could be argued that the 13 subjects who rated the riskiness of these degenerate lotteries different may have confounded risk with preference. Anyhow, there were still more then half of the subjects who did think sure things have no risk (or rated them as having the same value of riskiness).

In Fishburn's (1982, 1984) systems of risk, he presumed that there is no risk unless a loss probability is greater than zero. In the multiplicative form of our risk measure (6), it is assumed that there is no risk if and only if an alternative is a sure thing. Many people would agree that "risk" should only be associated with the uncertainty of outcomes (e.g., Bell, 1988, 1995; Sarin and Weber, 1993). We believe that the notion of risk can be made much clearer to people if we state an explicit reference point for "zero risk," such as any sure thing has no risk. Given the explicit anchor, risk judgments would be more consistent. We leave this point for further empirical studies.

6. Appendix

<u>PROOF OF THEOREM 1</u>

Let x' be a realization of $X' \in P^{o}$. For $(w,x') \in \mathbf{X_1} \times \mathbf{X_2}$, the two-attribute risk function $U_R(w,x')$ can be decomposed as follows:

$$U_r(w,x') = g(w) + \psi(w) u_R(x') \tag{a1}$$

if and only if the independence condition (Assumption 1) holds, where $\psi(w) > 0$. (Note that the independence condition is analogous to utility independence in multiattribute utility theory, e.g., see Keeney and Raiffa, 1976, pp. 224-229.) Taking expectation over (a1), we get

$$R_p(\bar{X}, X') = \mathrm{E}[U_R(\bar{X}, X')] = g(\bar{X}) + \psi(\bar{X}) \mathrm{E}[u_R(X')]. \tag{a2}$$

By Assumption 2, we have the following two consistent orderings:

$$g(0)+\psi(0)\mathrm{E}[u_R(X')] > g(0)+\psi(0)\mathrm{E}[u_R(Y')]$$

or

$$\mathrm{E}[u_R(X')] > \mathrm{E}[u_R(Y')]$$

if and only if

$$R(X') > R(Y') \text{ or } -\mathrm{E}[u(X')] > -\mathrm{E}[u(Y')].$$

Thus, u_R must be a positive linear transformation of $-u$ and (a2) can be written as

$$R_p(\bar{X}, X') = g(\bar{X}) + \psi(\bar{X})R(X'), \tag{a3}$$

which is the perceived risk model (5).

Based on Assumption 3, $(\bar{X}, X') >_{\mathbf{R}} (\bar{X}+\Delta, X')$ for any constant $\Delta > 0$, we should have

$$g(\bar{X})+\psi(\bar{X})R(X') > g(\bar{X}+\Delta)+\psi(\bar{X}+\Delta)R(X')$$

or

$$-[\psi(\bar{X}+\Delta)-\psi(\bar{X})]R(X') > g(\bar{X}+\Delta)-g(\bar{X}). \tag{a4}$$

Dividing both sides of (a4) by Δ and taking the limit as $\Delta \to 0$, we obtain the following condition for the measure of perceived risk:

$$g'(\bar{X}) < -\psi'(\bar{X})R(X').$$

PROOF OF THEOREM 2

Following the proof above, if Assumption 4 is further required, we then have $R_p(\bar{X},0) = R_p(0,0)$ for any $\bar{X} \in$ Re. It is convenient to set $R_p(0,0) = 0$. Thus, based on (a3), we have

$$g(\bar{X})+\psi(\bar{X})R(0) = 0$$

or

$$g(\overline{X}) = -\psi(\overline{X})R(0). \qquad \text{(a5)}$$

Substituting (a5) into (a3), we obtain the desired form:

$$R_p(\overline{X}, X') = \psi(\overline{X})[R(X') - R(0)].$$

By Assumption 3, $\psi(\overline{X})$ must be a decreasing function of $\overline{X}$.

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