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Generalized Disappointment Models

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"Blessed is he who expects nothing, for he shall never be disappointed"
-- Benjamin Franklin

Abstract

Based on our risk-value framework, this paper presents generalizations and extensions for the disappointment models that were proposed by Bell (1985) and Loomes and Sugden (1986). We provide explicit functional forms for modeling the effect of disappointment on risky choice behavior. Our generalized disappointment models can explain a number of decision paradoxes, and offer additional insights into nonexpected utility preferences based on the intuitive notions of disappointment and risk-value tradeoffs.

1. Introduction

In our previous studies (Jia and Dyer, 1995, 1996; Dyer and Jia, 1997), we proposed risk-value models such that decisions can be made based on the intuitively appealing idea of risk-value tradeoffs. We demonstrated that these risk-value models are very flexible in modeling preferences, and provide new resolutions for observed risky choice behavior and some decision paradoxes.

Let X be a random variable representing a lottery, and $\bar{X}$ the mean of that lottery; then $X' = X - \bar{X}$ is a standard risk variable with zero expected value. The basic form of the risk-value model can be written as follows:

$$f\left(\bar{X}, X'\right) = V\left(\bar{X}\right) - \mathrm{f}\left(\bar{X}\right)\left[R(X') - R(0)\right] \tag{1}$$

where $V\left(\bar{X}\right)$ is an increasing function of value measure for the subjective value of the mean $\bar{X}$, $\mathrm{f}\left(\bar{X}\right) > 0$ is a tradeoff factor that may depend on the mean $\bar{X}$, $R\left(X'\right) = -\mathrm{E}\left[u\left(X'\right)\right] = -\mathrm{E}\left[u\left(X - \bar{X}\right)\right]$ is a standard measure of risk (Jia and Dyer, 1995, 1996), and $R(0) = -u(0)$ is a constant, where $u(\cdot)$ is a utility function (von Neumann and Morgenstern, 1947), and the symbol E represents the expectation operator.

Model (1) can be either risk averse or risk prone depending on whether $R\left(X'\right) > R(0)$ or $R\left(X'\right) < R(0)$ respectively. To see this, we consider the certainty equivalent (CE) of a lottery X such that $f(CE,\ 0) = V(CE)$ $= V\left(\bar{X}\right) - \mathrm{f}\left(\bar{X}\right)\left[R\left(X'\right) - R(0)\right]$ or equivalently $\mathrm{f}\left(\bar{X}\right)\left[R(X') - R(0)\right] = V\left(\bar{X}\right) - V(CE)$. Since $V\left(\bar{X}\right)$ is increasing, $R\left(X'\right) > R(0)$ implies $\bar{X} > CE$, which we call lottery-risk aversion for the lottery considered. Conversely, $R\left(X'\right) < R(0)$ implies $\bar{X} < CE$, which we call lottery-risk seeking. When $u(\cdot)$ is concave, $R(X') > R(0)$ will be always true for all lotteries. Then we call the risk-value model (1) globally risk averse.

Note that the concept of lottery-risk aversion is different from that of global risk aversion unless the utility function for the standard measure of risk is concave. Lottery-risk aversion and lottery risk seeking are determined based on both the nature of lottery considered and the nature of the standard measure of risk. Even though an individual may have the standard measure of risk based on some utility function that is both locally concave and locally convex with respect to different ranges of outcomes, the individual will be lottery-risk averse to the lottery considered if $R(X') > R(0)$.

The term $\varphi(\bar{X})[R(X') - R(0)]$ in the risk-value model (1) plays a role in measuring the perceived risk of a lottery if $\varphi(\bar{X})$ is a decreasing function of the mean $\bar{X}$ (Jia, Dyer and Butler, 1996).In this case, preference can be represented by the difference between value and perceived risk.

In this risk-value model (1), the utility function for the standard measure of risk, the value function, and the tradeoff factor can be treated independently. Thus, we can choose appropriate functional forms for each of them according to different theoretical and empirical considerations. This makes the risk-value model very flexible in modeling risky choice behavior.

In this paper, we investigate a specific type of the standard measure of risk, called disappointment risk, based on previous studies (Bell, 1985; Loomes and Sugden, 1986). Though we proposed generalized disappointment models in our early paper (Jia and Dyer, 1995), these models deserve more detailed studies. In particular, we provide further empirical investigations and insights for these generalized disappointment models.

2. Bell's disappointment model

Bell (1985) first proposed a disappointment model for decision making under uncertainty. According to Bell, disappointment is a psychological reaction to an outcome that does not meet a decision maker's expectation. For a lottery (x, p, y) (i.e., the lottery has a probability p of yielding \$$x$ and a probability $(1-p)$ of yielding \$$y$), where $x > y$, Bell used the mean $\overline{X} = px + (1-p)y$ as the decision maker's psychological expectation. If y occurs, the decision maker would be disappointed because what he/she receives is less than expected, a priori. The measure of disappointment is assumed to be proportional to the difference between the expectation and the outcome y; i.e., disappointment $= d\left(\overline{X} - y\right) = d\left[px + (1-p)y - y\right] = dp(x-y)$, where $d \geq 0$ is a constant. On the other hand, if the decision maker receives x, $\left(x > \overline{X}\right)$, he/she will be “elated”. This elation is also assumed to be proportional to the difference between what he received and what he expected; i.e., elation $= e\left(x - \overline{X}\right) = e\left[x - px - (1-p)y\right] = e(1-p)(x-y)$, where $e \geq 0$ is a constant.

Bell took the expectation of disappointment (negative effect) and elation, and combined them into the decision maker's expected overall psychological satisfaction: $p(\text{elation}) + (1-p)(\text{disappointment}) = (e-d)\,p(1-p)(x-y)$. Bell also interpreted the opposite of psychological satisfaction, i.e. $(d-e)\,p(1-p)(x-y)$, as a measure of the psychological risk involved in the simple lottery for the decision maker. Finally, he assumed that the decision maker's preference is based on expected economic payoff and the psychological satisfaction, and that the preference relation is additive. Then Bell's disappointment model can be written as the following equivalent forms:

$$\left[px-(1-p)y\right]+(e-d)\,p(1-p)(x-y) \qquad (2a)$$

or

$$\left[px-(1-p)y\right]-(d-e)\,p(1-p)(x-y) \qquad (2b)$$

or

$$y+(x-y)\mathrm{p}(p) \qquad (2c)$$

where $\mathrm{p}(p)=p-(d-e)\,p(1-p)$ is a decision weight and $(d-e)$ measures the relative influence of disappointment and elation on preference. The form of (2b) reflects the tradeoff between mean value and the psychological risk for constructing preference. Bell showed that this simple model of disappointment can explain the common ratio effect, Ellsberg's paradox, and other observed risky choice behavior.

Bell's development of the disappointment model (2) has intuitive appeal. However, since his model only applies to lotteries with two outcomes, its application is limited. Following Bell's basic ideas, we offer more general disappointment models based on our risk-value framework.

3. A simple extension of Bell's disappointment model

First, we discuss a simple extension of Bell's disappointment model. Consider a standard measure of risk that is based on the following piece-wise linear utility model:

$$u(x)=\begin{cases} ex & \text{when } \; x\geq 0 \\ dx & \text{when } \; x<0 \end{cases} \qquad (3)$$

where d, $e > 0$ are constant. For decision makers who are averse to downside risk or losses, $d > e$ is required to capture this preference. This condition makes the piece-wise linear utility function concave, as illustrated in Figure 1. Thus, it implies risk aversion for lotteries with both gain and loss outcomes.[1]

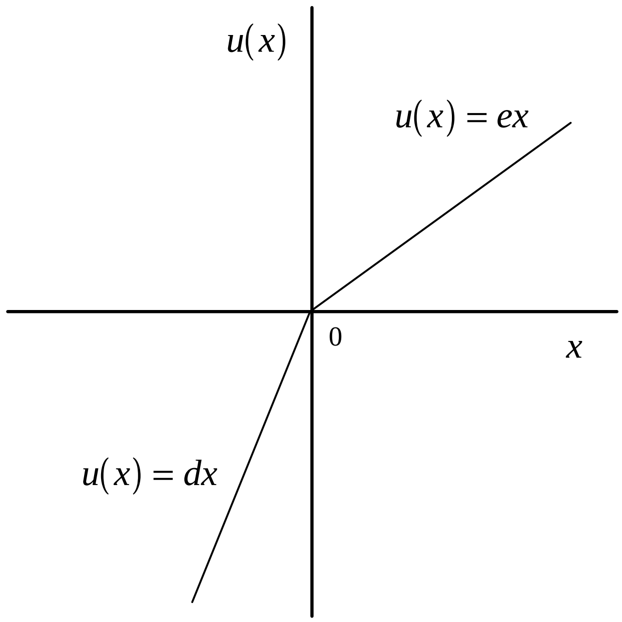

Figure 1. A piece-wise linear utility function.

The associated standard measure of risk for this piece-wise linear utility model can be obtained as follows:

$$R(X') = d\mathrm{E}^{-}\left[\left|X - \overline{X}\right|\right] - e\mathrm{E}^{+}\left[\left|X - \overline{X}\right|\right] \tag{4}$$

where $\mathrm{E}^{-}\left[\left|X - \overline{X}\right|\right] = \sum_{x_i < \overline{X}} p_i \left|x_i - \overline{X}\right|$, $\mathrm{E}^{+}\left[\left|X - \overline{X}\right|\right] = \sum_{x_i \geq \overline{X}} p_i \left|x_i - \overline{X}\right|$, and p_i is the probability associated with the outcome x_i. The standard measure of risk makes

[1] The discussion here ignores the effect of wealth level on preference over a lottery. However, when we use the piece-wise linear function for the standard measure of risk, it is independent of wealth level and any certain payoffs because the reference point is moved from zero to the expected value of a lottery.

appropriate use of the piece-wise linear utility function by normalizing lotteries at their means. Thus any non-degenerate lottery will have an expected loss $\mathrm{E}^{-}\left[\left|X-\bar{X}\right|\right]$ and an expected gain $\mathrm{E}^{+}\left[\left|X-\bar{X}\right|\right]$ relative to the mean. Their impacts on preference are measured by $d\mathrm{E}^{-}\left[\left|X-\bar{X}\right|\right]$ (negative effect) and $e\mathrm{E}^{+}\left[\left|X-\bar{X}\right|\right]$ respectively.

Following Bell's (1985) basic idea, $d\mathrm{E}^{-}\left[\left|X-\bar{X}\right|\right]$ is the measure of expected disappointment and $e\mathrm{E}^{+}\left[\left|X-\bar{X}\right|\right]$ is the measure of expected elation, and overall psychological satisfaction is measured by $-R(X')$, which is the converse of the standard measure of risk.

If the value measure is linear and the tradeoff factor is constant in (1),[2] then we can have the following risk-value model based on the standard measure of risk (4):

$$f\left(\bar{X}, X'\right) = \bar{X} - \left\{d\mathrm{E}^{-}\left[\left|X-\bar{X}\right|\right] - e\mathrm{E}^{+}\left[\left|X-\bar{X}\right|\right]\right\}. \qquad (5a)$$

We call (5a) a “generalized disappointment model.” This model is consistent with Bell's two key assumptions. That is, (1) disappointment and elation are proportional to the difference between outcome and expectation; and (2) one would be risk neutral if it were not for the effects of disappointment and elation. It is easy to verify that for a two-outcome lottery (x, p, y), our model (5a) reduces to Bell's model (2). That is, Bell's disappointment model (2) is a special case of the risk-value model (5a). Our treatment

[2] See Jia (1995) regarding preference conditions for a linear value measure and a constant tradeoff factor for the risk-value model (1).

here also offers a clear interpretation for the parameters *d* and *e* (see Figure 1) in Bell's disappointment model (2). When *d* > *e*, model (5a) is globally risk averse for all lotteries. In fact, this two parameter model can be further simplified by noticing the fact $E^{-}\left[\left|X-\bar{X}\right|\right]=E^{+}\left[\left|X-\bar{X}\right|\right]$.

PROPOSITION 1. The disappointment model (5a) has the following two equivalent forms:

$$f\left(\bar{X}, X'\right)=\bar{X}-\lambda\, E^{-}\left[\left|X-\bar{X}\right|\right] \tag{5b}$$

or

$$f\left(\bar{X}, X'\right)=\bar{X}-(\lambda/2) E\left[\left|X-\bar{X}\right|\right]. \tag{5c}$$

where $\lambda = d - e$ and $E\left[\left|X-\bar{X}\right|\right]$ is the absolute standard deviation. (The proof is provided in the appendix.)

The implication of this simplification is that the influence of disappointment and elation on preference is simply determined by their difference. This leads to an extremely simple form of nonlinear expected utility model containing one parameter only. For risk aversion in models (5b) and (5c), we need $\lambda = d - e > 0$. Thus, the notion of risk aversion is consistent with the assumption of the asymmetric effect of disappointment and elation in these simple models.

Since the value measure is the mean, the generalized disappointment model (5a), or equivalently (5b) and (5c), also provides a direct measure of the certainty equivalent for a lottery. We can assess the disappointment model by asking the decision maker to provide the certainty equivalent for a risky prospect. Then by (5c),

$\lambda = 2(\overline{X} - CE) / \mathrm{E}[|X - \overline{X}|]$, where CE is the assessed certainty equivalent for a risky prospect.

The generalized disappointment model (5a), or equivalently (5b) and (5c), will be consistent with the piece-wise linear utility model (3) when the lotteries considered have zero-expected values. But for general lotteries, this model will deviate from the expected utility preference. In fact, the disappointment model can be transformed into an alternative form of weighted utility models (e.g., Karmarkar, 1978; Kahneman and Tversky, 1979; Chew and MacCrimmon, 1979).

Without loss of generality, we assume that outcomes of a lottery have the rank order: $x_{m+n} > \ldots x_k \ldots > x_{m+1} > \overline{X} > x_m > \ldots x_j \ldots > x_1$. So we use x_j to represent a outcome of the lottery that is associated with disappointment, and x_k to represent a outcome associated with elation.

PROPOSITION 2. Model (5a), or equivalently (5b) and (5c), has the following alternative representation:

$$f(\overline{X},\, X') = \sum_{j=1}^{m} \pi^{+}(p_j)x_j + \sum_{k=m+1}^{m+n} \pi^{-}(p_k)x_k \tag{5d}$$

where $\pi^{+}(p_j) = \left(1 + \lambda \sum_{k=m+1}^{m+n} p_k\right)p_j$ and $\pi^{-}(p_k) = \left(1 - \lambda \sum_{j=1}^{m} p_j\right)p_k$.

Model (5d) shows how nonlinear decision weights can also be interpreted in terms of the effect of disappointment and elation. For the case of risk aversion (i.e., $\lambda > 0$), the probabilities associated with disappointment outcomes will be overweighted (i.e., $\pi^{+}(p_j) > p_j$) and the probabilities associated with elation outcomes will be underweighted (i.e., $\pi^{-}(p_k) < p_k$). Model (5d) also provides a generalization of Bell's probability weighting form (2c) for two-outcome lotteries.

Finally in this section, we provide a condition of first order stochastic dominance for the disappointment models (5a) - (5d).

PROPOSITION 3. Models (5a) - (5d) satisfy the first order stochastic dominance condition if $|\lambda| = |d - e| < 1$.

This condition guarantees that stochastically dominating lotteries are preferred. The first order stochastic dominance condition should be required for prescriptive decision making. However, it may not be satisfied in actual choice problems. As found by Tversky and Kahneman (1986), this dominance condition is usually obeyed in transparent problems and frequently violated in less clear cases.

4. Choice Implications of the Generalized Disappointment Model

In this section, we use the Allais paradox (Allais, 1953, 1979) to explore the choice behavior implied by our disappointment model.

4.1. Allais paradox

Using the two-outcome disappointment model (2), Bell (1985) gave an explanation for the common ratio effect (Allais, 1953; Kahneman and Tversky, 1979). Now we can use the generalized disappointment model (5) to explain the Allais paradox (Allais, 1953, 1979), which involves an alternative with three outcomes in the following pairs of lotteries:

X_1:	win \$1 million for sure	versus	X_2:	0.10 chance of \$5 million
				0.89 chance of \$1 million
				0.01 chance of \$0
X_3:	0.11 chance of \$1 million	versus	X_4:	0.10 chance of \$5 million
	0.89 chance of \$0			0.90 chance of \$0

Many experiments have shown that a majority of subjects have the preference pattern of X_1 over X_2 but X_3 over X_4, which violates the independence axiom of expected utility theory.

Using our generalized disappointment model (5), we can have $f\left(\overline{X}_1, X_1'\right)$ = 1, $f\left(\overline{X}_2, X_2'\right)$ = 1.39 - 0.361λ, $f\left(\overline{X}_3, X_3'\right)$ = 0.11 - 0.0979λ, and $f\left(\overline{X}_4, X_4'\right)$ = 0.5 - 0.45λ. For the preference pattern of X_1 over X_2, we need λ > 1.080; and for X_4 over X_3, we need λ < 1.108. Thus, 1.108 > λ > 1.080 will lead to the subjects' preference pattern for the Allais paradox.[3]

4.2. Probability triangle diagram

To further study the choice behavior implied by the generalized disappointment model, the probability triangle diagram will be useful. The Allais paradox involves the three payoff levels: 0, 1, and 5 million dollars. Various lotteries can be constructed by

[3] Since the required l values for Allais paradox are larger than 1, they may lead to violation of the first order stochastic dominance condition for some choice problems. In particular, for some lotteries based on the three outcomes of Allais paradox, the violation may happen when the middle outcome has a large probability, as can be seen in Figure 2 (i.e., the southwestern corner of the triangle).

assigning different probabilities to the three payoffs. Since the probabilities sum to 1, only two of the probabilities are needed to define a lottery. If we use the probability of the best payoff as the vertical axis and the probability of the worst payoff as the horizontal axis, we can obtain the probability triangle, as illustrated in Figure 2. A point in this triangle represents a lottery.

Based on this diagram, implications of choice models can be observed from indifference curves over lotteries. These indifference curves should be upward-sloping lines with northwest as the direction of increasing preference. In particular, the expected utility theory makes a specific requirement that indifference curves be parallel straight lines. However, empirical studies found this requirement is systematically violated by actual responses to some choice problems (for a recent review, see Camerer, 1995).

Using the disappointment model (5) for the payoffs of the Allais paradox, we can generate the indifference curves in the triangle as in Figures 2 and 3. The dotted line $\left(p_3 = 1/4\, p_1\right)$ separates the triangle into two areas. This is because the middle payoff, \$1 million, can impact either disappointment or elation, depending on the expectation of the lottery being evaluated.

When $p_3 > 1/4\, p_1$, this payoff will be treated as a disappointment and the indifference curves are determined by $f_1 = (1 - p_1 + 4p_3) - \lambda(4 + p_1 - 4p_3)p_3$. As shown in Figure 2, increasing disappointment makes the indifference curves more concave and steeper, and leads to more risk aversion, especially when the chance of getting \$1 million is relatively high.

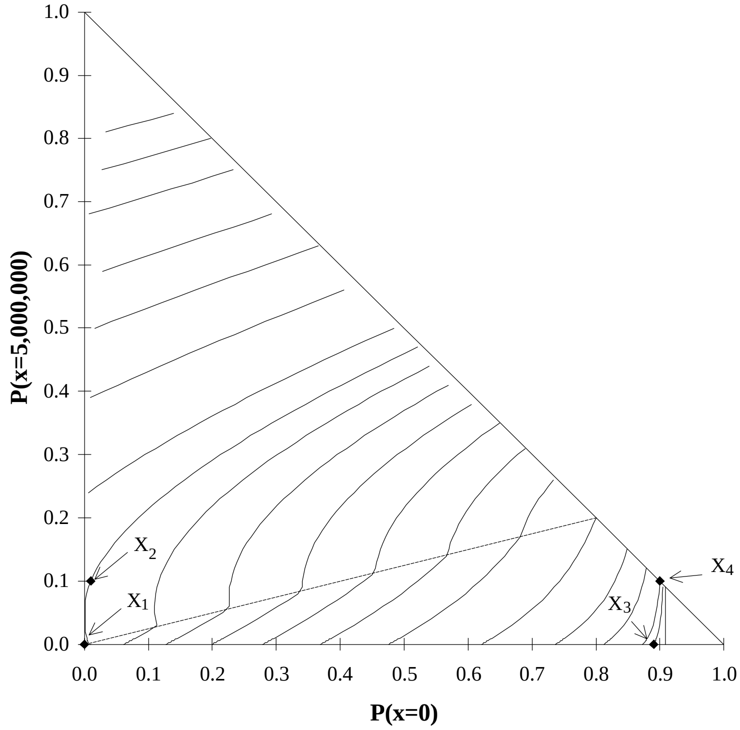

Figure 2. Indifference curves (l = 1.1) and the Allais paradox.

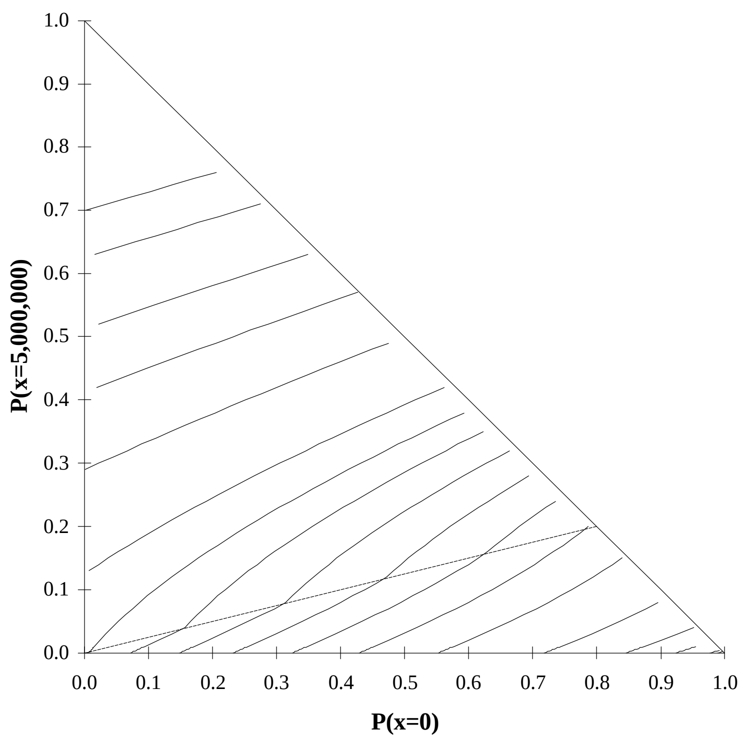

Figure 3. Indifference curves with l = 0.8.

When $p_3 < 1/4\, p_1$, the middle payoff will be regarded as an elation and the indifference curves are based on a different function $f_2 = (1 - p_1 + 4p_3) - \lambda\,(4 + p_1 - 4p_3)p$. When p_3 is large (close to 1), the disappointment term will have less effect and the linear expectation term will play a more important role in determining the indifference curves. Thus those indifference curves in the northwest corner should be approximately parallel straight lines. Similarly, indifference curves in the southeast corner should be approximately parallel straight lines. However, the implied behavior between the northwest corner and the southeast corner will not be symmetric since these two nonlinear terms in f_1 and f_2 have a different degree of impact on preference.

The behaviors of indifference curves also depend on the value of the parameter λ. The larger the value of λ, the more nonlinear the indifference curves. This impact is illustrated by comparing Figures 2 and 3.

We plot the four lotteries X_1, X_2, X_3, and X_4 associated with the Allais paradox in the triangle in Figure 2. The choice of the parameter $\lambda = 1.1$ leads to the indifference curves that capture the Allais paradox. In the area above the dotted line, the indifference curves are concave and fanning in, especially in the lower part of this area. In the area below the dotted line, the indifference curves are convex and fanning out for the most part (there is some fanning in in the southeast corner for this case).

Accumulated evidence from recent empirical studies indicates a mixed fanning picture of the indifference curves of subjects, and a strict fanning out hypothesis (Machina, 1982) can be rejected (Neilson, 1992; Camerer, 1995). For example, in an experimental study using the Allais payoffs, Conlisk (1989) found fanning in in the upper-left area. Prelec (1990) found strong fanning in close to the lower edge and southeast corner.

These empirical findings are largely consistent with the indifference curves in Figure 2, implied by the simple one-parameter disappointment model (5).

4.3. Risk-value diagram

One drawback for the probability triangle is the restriction to lotteries with three possible outcomes only. Many practical problems, however, require larger probability spaces. Our risk-value framework can offer an alternative graphic way of representing risky choice behavior. Similar to the mean-variance structure, we can consider risk (measured here by the absolute standard deviation) and value (measured by the mean) as two dimensions, as in Figure 4. Thus, a point in the risk-value plane represents a lottery. An individual's indifference curves in terms of mean and standard risk are given by the solutions to the equations, $\bar{X} - (\lambda/2)\mathrm{E}\left[\left|X - \bar{X}\right|\right]$ = constant. These indifference curves are simply parallel straight lines with the slope equal to the tradeoff factor. Clearly, all northwest movements lead to increasing preference if the individual is risk averse (i.e., $\lambda > 0$).

To explain the Allais paradox, we use $\lambda = 1.1$ and plot the four lotteries X_1, X_2, X_3, and X_4 associated with the paradox in Figure 4. Because the point representing X_2 is below the indifference line passing through the point X_1, the lottery X_1 is preferred to the lottery X_2. Since the point X_4 is above the indifference line passing through the point X_3, X_4 is preferred to X_3. Indeed, various preference patterns are possible in this two dimensional space. Thus, the risk-value framework provides a flexible way of modeling preferences. In particular, the diagram of risk-value indifference curves can handle lotteries with more than three outcomes (e.g., continuous distributions).

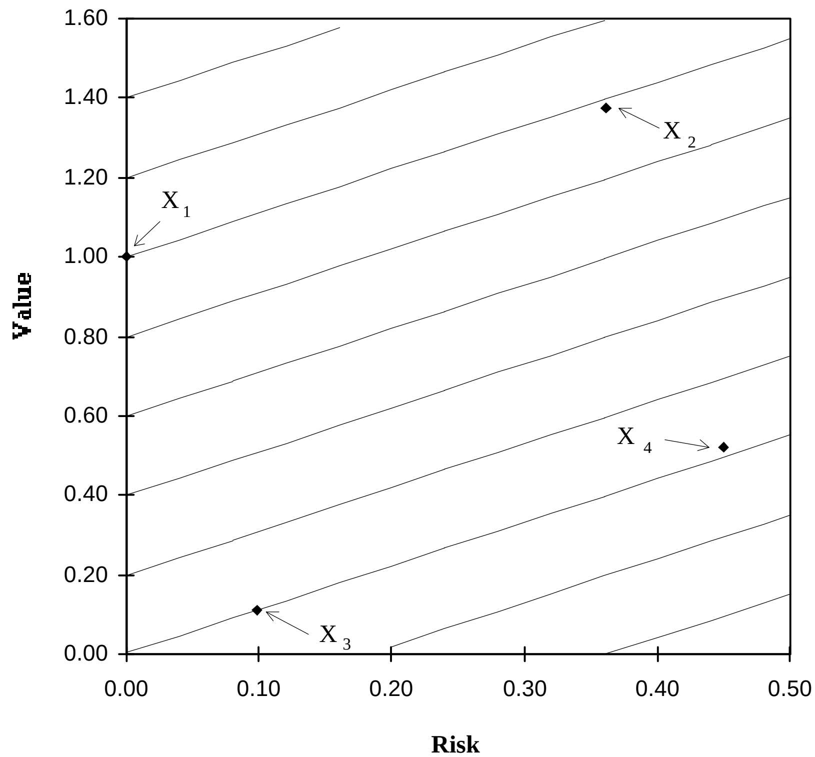

Figure 4. Risk-value indifference curves (λ = 1.1) and the Allais paradox.

There are some intuitive arguments regarding the preference pattern of the Allais paradox; e.g., it seems people are less risk averse in the choice for the second pair of the lotteries X_3 and X_4 compared with the choice for the first pair X_1 and X_2. To accommodate this observation based on risk-value tradeoffs, we define a coefficient of risk similar to the coefficient of variation in statistics, based on the ratio of the standard measure of risk to the mean, i.e., $r = \left(\mathrm{E}\left[\left|X-\overline{X}\right|\right]\right)/\overline{X}$ (see Dyer and Jia, 1996). Then the disappointment model (5c) can be written as $f\left(\overline{X}, X'\right) = \overline{X}\left[1-\lambda/2\left(\mathrm{E}\left[\left|X-\overline{X}\right|\right]\right)/\overline{X}\right]$ $= \overline{X}\left(1-\lambda r/2\right)$. In the first lottery pair of the Allais paradox, r = 0 for X_1, and r = 0.519

for X_2; and in the second pair, r = 1.78 for X_3, and r = 1.80 for X_4. Thus, the difference of the coefficient of risk between X_3 and X_4 is tiny so that the choice using the risk-value tradeoff is mainly determined by the expected value ($\bar{X}_3$ = 0.11 versus $\bar{X}_4$ = 0.5). This shows a kind of "risk neutral" behavior in choice. However, in determining the certainty equivalents for these lotteries (which are 0.00231 and 0.005 for X_3 and X_4 respectively), the judgment is truly risk averse. Thus, our risk-value framework offers additional insight into the observed choice behavior.

5. A General Form of Disappointment Model

For disappointment models (2) and (5), we may question the assumption that disappointment and elation are linearly proportional to the difference between the expected value and an outcome. In this case, we should consider some nonlinear measures for disappointment and elation, which will be discussed in this section.

If an individual is locally risk averse for gains but locally risk seeking for losses (Kahneman and Tversky, 1979), then we can use a piecewise power utility model (e.g., Fishburn and Kochenberger, 1979; Tversky and Kahneman, 1992). Luce and Weber (1986) also used a piecewise power utility model as a major component of their perceived risk measure to model the possibility that losses have a larger effect on perceived risk than gains. Consider the following piecewise power utility model for the standard measure of risk:

$$u(x) = \begin{cases} ex^{\theta_1}, & \text{when } x \geq 0 \\ -d|x|^{\theta_2}, & \text{when } x < 0 \end{cases} \qquad (6)$$

where q_1, q_2, e and d are constants. When $q_1 = q_2 =$ 1, model (6) reduces to the piecewise linear model (3). Figure 5 illustrates some utility curves that are based on model (6).

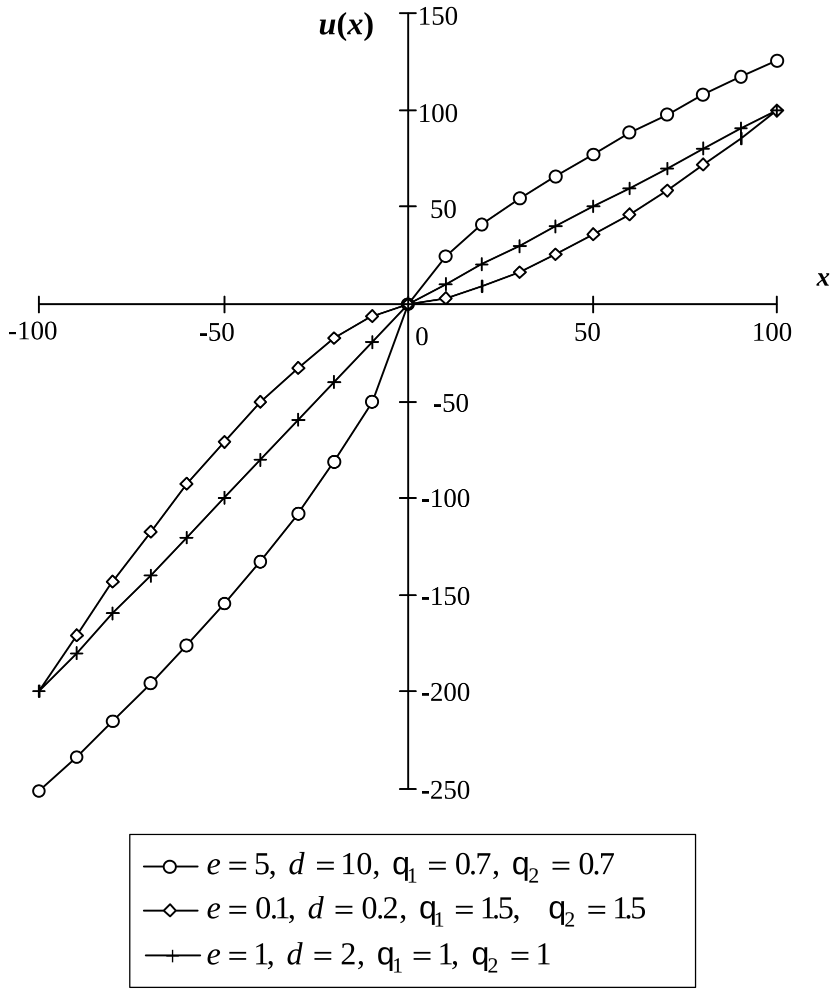

Figure 5. Utility curves based on the piecewise linear plus power model (6).

The corresponding standard measure of risk for the utility model (6) can be obtained as follows:

$$R(X') = d\mathrm{E}^{-}\left[\left|X - \bar{X}\right|^{\theta_2}\right] - e\mathrm{E}^{+}\left[\left|X - \bar{X}\right|^{\theta_1}\right], \qquad (7)$$

where $\mathrm{E}^{-}\left[\left|X - \bar{X}\right|^{\theta_2}\right] = \sum_{x_i < \bar{X}} p_i \left|x_i - \bar{X}\right|^{\theta_2}$, $\mathrm{E}^{+}\left[\left|X - \bar{X}\right|^{\theta_1}\right] = \sum_{x_i \geq \bar{X}} p_i \left|x_i - \bar{X}\right|^{\theta_1}$, and p_i is the probability associated with the outcome x_i. The standard measure of risk (7) includes several commonly used measures of risk in the financial literature as special cases, such as semi-variance (when θ_1 = 0 and θ_2 = 2) and semi-moment (when θ_1 = 0). If $\theta_1 = \theta_2$ = 1, the standard measure of risk (7) reduces to (4), or the absolute standard deviation. Thus, (7) offers a more general measure of disappointment risk.

Another characteristic of Bell's disappointment model (2) and our model (5) is that they imply constant risk aversion (Jia and Dyer, 1995). Thus, these models are not appropriate for decreasing risk averse behavior. To obtain a disappointment model that is decreasing risk averse with the standard measure of risk (7), we propose the following form of risk-value model:

$$f\left(\bar{X}, X'\right) = \bar{X} - \phi\left(\bar{X}\right)\left\{d\mathrm{E}^{-}\left[\left|X - \bar{X}\right|^{\theta_2}\right] - e\mathrm{E}^{+}\left[\left|X - \bar{X}\right|^{\theta_1}\right]\right\} \qquad (8)$$

where $\phi\left(\bar{X}\right) > 0$ is a decreasing function, e.g., $\phi\left(\bar{X}\right) = e^{-a\bar{X}}$, where $a \geq 0$. This model also implies that psychological satisfaction (Bell, 1985), here measured by $-R(X')$, may have a decreasing effect on preference as the expectation increases. When $\theta_1 = \theta_2 = 1$ and $\phi\left(\bar{X}\right) = 1$, model (8) simply reduces to model (5). When e = 0 and θ_2 = 2, this model becomes a mean-semivariance model. Because of the linear function of value, model (8) also measures the certainty equivalent of a lottery for decision makers.

5.1. Decision weights

To see some implications of model (8) for nonlinear expected utility preference, we discuss the decision weights implied by this model. In prospect theory, Kahneman and Tversky (1979) suggest that people replace probabilities with decision weights when evaluating risky outcomes. For a simple lottery (x, p, y), the model of prospect theory has the form, $v(y) + \pi(p)[v(x) - v(y)]$, where $v(\cdot)$ is a value function and $\pi(p)$ is the decision weight associated with probability *p*.

Using our risk-value model (8) for a two-outcome lottery, we have

$$
\begin{aligned}
f &= px + (1-p)y - \left[d(1-p)\big(px + (1-p)y - y\big)^{\theta_2} - ep\big(x - px - (1-p)y\big)^{\theta_1}\right] \\
&= px + (1-p)y - d(1-p)p^{\theta_2}(x-y)^{\theta_2} + ep(1-p)^{\theta_1}(x-y)^{\theta_1} \\
&= y + \pi(x, p, y)(x - y)
\end{aligned}
$$

where

$$
\pi(x, p, y) = p - d(1-p)p^{\theta_2}(x-y)^{\theta_2 - 1} + ep(1-p)^{\theta_1}(x-y)^{\theta_1 - 1} \qquad (9)
$$

For a linear value measure (Yaari, 1987), $\pi(x, p, y)$ could be interpreted as a decision weight. This decision weight is well behaved at the endpoints, i.e., $\pi(x, 0, y) = 0$ and $\pi(x, 1, y) = 1$, consistent with a recent development of prospect theory (Tversky and Kahneman, 1992).

Model (9) shows that outcomes generally have an effect on the decision weight; only when $\theta_1 = \theta_2 = 1$, does the decision weight not depend on the outcome (see the form of model (5d)). In this case, $\pi(x, p, y) = \pi(p) = p - (d - e)p(1 - p)$, which is the decision weight implied in Bell's model (2c) or our model (5d). Tversky and Kahneman (1992) proposed different decision weight functions for positive outcomes and negative outcomes. More generally, Hogarth and Einhorn (1990) considered the effect of outcome sizes on decision weights. Weighted utility theories (Chew and MacCrimmon, 1979; Chew, 1983; Fishburn, 1983) also use a function of outcomes to weight probabilities.

In order to study the effect of disappointment and elation on the decision weight, we let $\theta_1 = \theta_2 = \theta$, and then model (9) reduces to the following form:

$$\pi(x, p, y) = p - \left[dp^{\theta-1} - e(1-p)^{\theta-1}\right]p(1-p)(x-y)^{\theta-1}. \qquad (10)$$

In this model, the asymmetric effects of disappointment and elation should be solely reflected by the relative magnitude of the parameters *d* and *e*. This decision weight can be either convex or concave, or even mixed, depending on the choice of parameters. For the case of a mixed shape, the decision weight has the cross-over point at $p^* = 1 \big/ \left(1 + \sqrt[\theta-1]{d/e}\right)$. By cross-over we mean the point where the decision weight changes from underweighting to overweighting of the probabilities, or vice versa. Note that the cross-over point is determined independent of outcomes. When *d* = *e*, we get $p^* = 0.5$, which corresponds to Karmarkar's (1978) model. When $d > e$ and $\theta > 1$, we have $p^* < 0.5$. Empirical studies suggest that the decision weight has a cross-over point below 0.5, usually between 0.1 and 0.3 (Camerer, 1995). According to our model, this is due to the asymmetric effect of disappointment and elation on preference.[4]

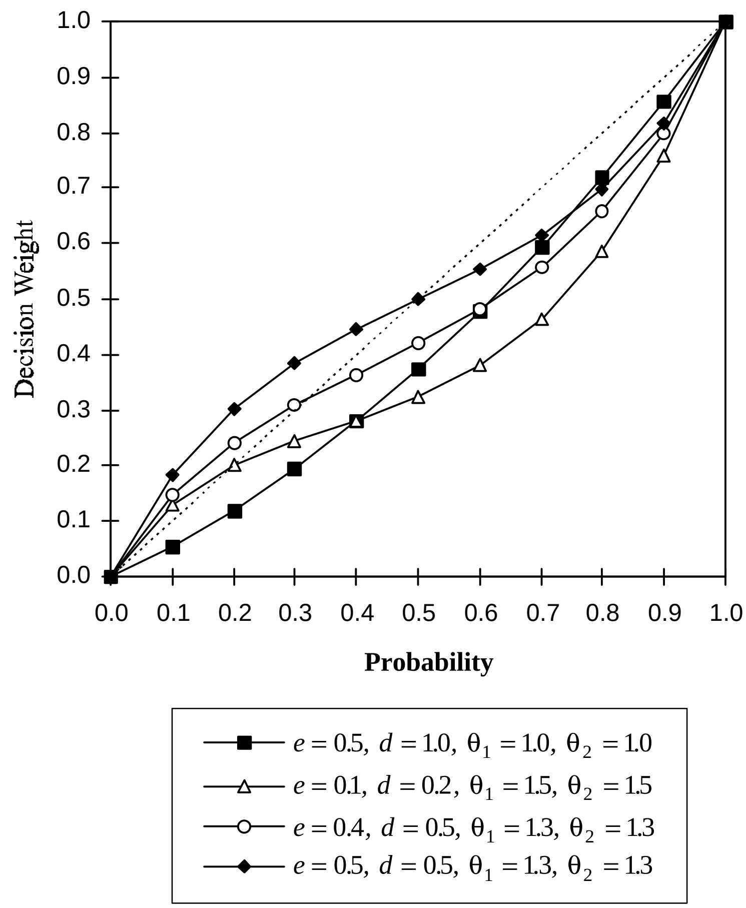

Figure 6. Decision weights based on model (10).

(The outcomes used are x - y = 10.)

Figure 6 illustrates some different shapes of the decision weight determined by using different parameters in model (10). It can be seen that the decision weight implied in our disappointment model is very flexible. Although model (10) is similar to some decision weight models that have been proposed, our framework is based on the more intuitively appealing notion of disappointment.

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[4] The choice $e > d$ and $q < 1$ also leads to $p^* < 0.5$. However, this contradicts the intuition that

5.2. Reflection effect

Another important property of the risk-value model (8) is that it can be either lottery-risk averse or lottery-risk seeking. To see this, let us consider the following pairs of lotteries provided by Kahneman and Tversky (1979):

X_1:	*win* \$3000 for sure	versus	X_2:	0.80 chance of *winning* \$4000
				0.20 chance of \$0
X_3:	*lose* \$3000 for sure	versus	X_4:	0.80 chance of *losing* \$4000
				0.20 chance of \$0

Note that the second pair of lotteries are obtained by reversing the signs of outcomes of the first pair. Kahneman and Tversky found that a majority of the subjects prefer X_1 to X_2 and X_4 to X_3, which implies different risk attitudes when considering gains and losses, i.e., risk aversion in the positive domain and risk seeking in the negative domain. They conjectured that the reflection of prospects around 0 reverses the preference order, and labeled this pattern the reflection effect.

Although this phenomenon can be easily explained by the piecewise utility model (6), we show that a corresponding risk-value model (8), which uses the mean as a reference point, is also able to model the reflection effect. Consider a simplified version of model (8) with $q_1 = q_2 = q$ and $f\left(\bar{X}\right) = 1$. For a simple example, we choose q = 1.3 and $d = e = 0.5$ (note that this choice is consistent with one set of the parameters used for the

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disappointment has a larger impact on preference than elation.

decision weight in Figure 6). Then we have $R(X_2')$ = 1226 and $R(X_4')$ = -1226, which means that the model (8) is lottery-risk averse for the lottery X_2 and lottery-risk seeking for the lottery X_4. Thus, $f\left(\overline{X}_1, X_1'\right)$ = 3,000 > $f\left(\overline{X}_2, X_2'\right)$ = 1,974 and $f\left(\overline{X}_3, X_3'\right)$ = -3,000 < $f\left(\overline{X}_4, X_4'\right)$ = -1,974, which predicts the reflection effect. In fact, for q = 1.3, *e* > 0.082 and *d* > 0.082 will be sufficient for the model to predict this behavior. Since model (5) or Bell's model (2) can only be either globally risk averse (i.e., λ = *d* - *e* > 0) or globally risk seeking (i.e., λ = *d* - *e* < 0), they can not explain the reflection effect.

5.3. Comparison with Loomes and Sugden's model

Finally, note that our generalized disappointment models are different from Loomes and Sugden's (1986) development. In their basic model, disappointment (or elation) is measured by some function of the difference between the utility of outcomes and the expected utility of a lottery. In their analysis, they further assume a linear "utility" (or in our terms, value) measure of wealth and the same intensity of sensation for both disappointment and elation, so that their model has the form, $\overline{X} + \mathrm{E}\left[D\left(X - \overline{X}\right)\right]$, where $D\left(x - \overline{X}\right) = -D\left(\overline{X} - x\right)$, $D(\cdot)$ is continuously differentiable, convex for $x > \overline{X}$ and concave for $x < \overline{X}$.[5] Loomes and Sugden used this model to provide an explanation for the choice behavior that violates Savage's (1954) sure-thing principle in the form of the common consequence effect.

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[5] This is consistent with our choice of parameters for the decision weights in Figure 6, and explaining the reflection effect.

Even though Loomes and Sugden's model is different from our generalized disappointment models (5) and (8), it is a special case of our risk-value model (1) with a linear measure of value, a constant tradeoff factor, and a special form of our standard measure of risk. Our development also provides an explicit functional form for Loomes and Sugden's model. In addition, our model does not have the restriction $D\left(x-\overline{X}\right)=-D\left(\overline{X}-x\right)$ and allows an asymmetric treatment between disappointment and elation.

6. Conclusion

Following Bell's (1985) work, we have generalized the disappointment model for lotteries with more than two outcomes. This generalization is based on our risk-value framework and an appropriate measure of disappointment risk. With only one parameter, the generalized disappointment model is powerful in describing risky choice behavior. It is extremely easy to assess the parameter based on a certainty equivalent argument. In particular, this model is associated with the absolute standard deviation, which has been suggested as a robust measure of risk in statistics and other applied fields. Our development provides a further understanding about this measure and formally incorporates it into a preference model that captures the asymmetric effect of disappointment and elation. The parsimony of this generalized disappointment model will make it useful in practice.[6]

To consider the possible nonlinear impact of disappointment and elation on preference, we have proposed a more general form of disappointment model. This model is more flexible in modeling nonlinear expected utility preferences. In particular, the generalized disappointment model can be either lottery-risk averse or lottery-risk seeking, which allows an individual to shift his or her risk attitudes when the nature of lotteries changes. The model also implies decision weights that have been proposed by some other risky choice theories. Our analysis suggests that the phenomenon of a decision weight with a cross-over point below 0.5 may be caused by the asymmetric and nonlinear effect of disappointment and elation on preference. Compared with Loomes and Sugden's (1986) development, our generalized disappointment models have greater flexibility and explicit functional forms that can be used easily.

Based on the notions of disappointment and risk-value tradeoffs, we have offered additional insights into some observed risky choice behavior and the decision paradoxes that violate the independence axiom of expected utility theory. Since our development is compatible with Bell's (1985) and Loomes and Sugden's (1986), we share the desirable properties of their models. For example, the disappointment models proposed in this paper allow the violation of the compound probability axiom but retain the principle of dynamic consistency. Thus, they can predict both the common ratio effect and the isolation effect (Loomes and Sugden, 1986).

This development uses the expected value of a lottery as the reference point regarding the measures of disappointment and elation, which is the same as Bell (1985) and Loomes and Sugden (1986). The expected value is a convenient and probabilistically appealing reference point. There are other possible reference points that might be considered, such as the mode of a distribution, the certainty equivalent of a lottery (Gul,

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[6] This generalized disappointment model combined with a measure of regret has been used in a marketing study regarding consumer satisfaction and valuation (Inman, Dyer, and Jia, 1997).

1991), the expected utility of a lottery (Loomes and Sugden, 1986), a reference lottery (Bell, 1982; Loomes and Sugden, 1982; Inman, Dyer and Jia, 1997), and the aspiration level of a decision maker. It would be interesting to consider these alternative reference points in our models in the future.

7. Appendix

<u>Proof of Proposition 1</u>

Without loss of generality, we assume that outcomes of a lottery have the rank order, $x_{m+n} > \ldots x_{m+1} > \overline{X} > x_m \ldots > x_1$. Then model (5a) can be written in the following form:

$$\begin{aligned} f &= \overline{X} - \left\{ d\mathrm{E}^{-}\left[\left|X - \overline{X}\right|\right] - e\mathrm{E}^{+}\left[\left|X - \overline{X}\right|\right]\right\} \\ &= \sum_{i=1}^{m+n} p_i x_i - d\sum_{j=1}^{m} p_j\left(\sum_{i=1}^{m+n} p_i x_i - x_j\right) + e\sum_{k=m+1}^{m+n} p_k\left(x_k - \sum_{i=1}^{m+n} p_i x_i\right) \end{aligned} \quad \text{(a1)}$$

We need to prove that $\sum_{k=m+1}^{m+n} p_k\left(x_k - \sum_{i=1}^{m+n} p_i x_i\right) = \sum_{j=1}^{m} p_j\left(\sum_{i=1}^{m+n} p_i x_i - x_j\right)$, which is presented as follows:

$$\begin{aligned}\sum_{k=m+1}^{m+n} p_k\left(x_k-\sum_{i=1}^{m+n} p_i x_i\right) &= \sum_{k=m+1}^{m+n} p_k x_k - \sum_{k=m+1}^{m+n} p_k\left(\sum_{i=1}^{m+n} p_i x_i\right)\\ &= \sum_{k=m+1}^{m+n} p_k x_k - \left(1-\sum_{j=1}^{m} p_j\right)\left(\sum_{i=1}^{m+n} p_i x_i\right)\\ &= \sum_{k=m+1}^{m+n} p_k x_k - \sum_{i=1}^{m+n} p_i x_i + \left(\sum_{j=1}^{m} p_j\right)\left(\sum_{i=1}^{m+n} p_i x_i\right)\\ &= -\sum_{j=1}^{m} p_j x_j + \left(\sum_{j=1}^{m} p_j\right)\left(\sum_{i=1}^{m+n} p_i x_i\right)\\ &= \sum_{j=1}^{m} p_j\left(\sum_{i=1}^{m+n} p_i x_i - x_j\right).\end{aligned}$$

Thus, model (a1) can be reduced to either of the following two forms:

$$f = \sum_{i=1}^{m+n} p_i x_i - (d-e)\sum_{j=1}^{m} p_j\left(\sum_{i=1}^{m+n} p_i x_i - x_j\right) = \overline{X} - \lambda\, \mathrm{E}^{-}\left[\left|X-\overline{X}\right|\right] \qquad \text{(a2)}$$

or

$$\begin{aligned} f &= \sum_{i=1}^{m+n} p_i x_i - \frac{(d-e)}{2}\left(\sum_{j=1}^{m} p_j\left(\sum_{i=1}^{m+n} p_i x_i - x_j\right) + \sum_{k=m+1}^{m+n} p_k\left(x_k - \sum_{i=1}^{m+n} p_i x_i\right)\right)\\ &= \overline{X} - \frac{\lambda}{2}\mathrm{E}\left[\left|X-\overline{X}\right|\right] \qquad \text{(a3)}\end{aligned}$$

where $\lambda = d-e$ and $\mathrm{E}\left[\left|X-\overline{X}\right|\right]$ is the absolute standard deviation.

Proof of Proposition 2

Based on model (a2), we can obtain the following alternative form of the disappointment model:

$$
\begin{aligned}
f &= \sum_{i=1}^{m+n} p_i x_i - \lambda \sum_{j=1}^{m} p_j \left(\sum_{i=1}^{m+n} p_i x_i - x_j \right) \\
&= \sum_{i=1}^{m+n} p_i x_i - \lambda \sum_{j=1}^{m} p_j \left(\sum_{i=1}^{m+n} p_i x_i \right) + \lambda \sum_{j=1}^{m} p_j x_j \\
&= \left(1 - \lambda \sum_{j=1}^{m} p_j \right) \left(\sum_{i=1}^{m+n} p_i x_i \right) + \lambda \sum_{j=1}^{m} p_j x_j \\
&= \left(1 - \lambda \sum_{j=1}^{m} p_j \right) \left(\sum_{j=1}^{m} p_j x_j + \sum_{k=m+1}^{m+n} p_k x_k \right) + \lambda \sum_{j=1}^{m} p_j x_j \\
&= \left(1 - \lambda \sum_{j=1}^{m} p_j + \lambda \right) \left(\sum_{j=1}^{m} p_j x_j \right) + \left(1 - \lambda \sum_{j=1}^{m} p_j \right) \left(\sum_{k=m+1}^{m+n} p_k x_k \right) \\
&= \left(1 + \lambda \sum_{k=m+1}^{m+n} p_k \right) \left(\sum_{j=1}^{m} p_j x_j \right) + \left(1 - \lambda \sum_{j=1}^{m} p_j \right) \left(\sum_{k=m+1}^{m+n} p_k x_k \right) \\
&= \sum_{j=1}^{m} \pi^{+}(p_j) x_j + \sum_{k=m+1}^{m+n} \pi^{-}(p_k) x_k \qquad \text{(a4)}
\end{aligned}
$$

where $\pi^{+}(p_j) = \left(1 + \lambda \sum_{k=m+1}^{m+n} p_k \right) p_j$ and $\pi^{-}(p_k) = \left(1 - \lambda \sum_{j=1}^{m} p_j \right) p_k$.

Proof of Proposition 3

Based on Machina (1982), the local utility functions of model (a4) can be derived as follows:

$$
U(x_j; P) = \frac{\partial f}{\partial p_j} = \left(1 + \lambda \sum_{k=m+1}^{m+n} p_k \right) x_j - \lambda \left(\sum_{k=m+1}^{m+n} p_k x_k \right) \quad \text{for all } x_j < \overline{X} \qquad \text{(a5)}
$$

and

$$
U(x_k; P) = \frac{\partial f}{\partial p_k} = \left(1 - \lambda \sum_{j=1}^{m} p_j \right) x_k + \lambda \left(\sum_{j=1}^{m} p_j x_j \right) \quad \text{for all } x_k \geq \overline{X} . \qquad \text{(a6)}
$$

To satisfy the first order stochastic dominance condition, the local utility functions (a5) and (a6) must be increasing in x_j and x_k in each distribution, respectively. This requires

$\left(1+\lambda\sum_{k=m+1}^{m+n}p_k\right)$ > 0 and $\left(1-\lambda\sum_{j=1}^{m}p_j\right)$ > 0 for any distribution. Thus, $1>\lambda>-1$ will be sufficient to satisfy these two conditions.

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