Group Decisions with Multiple Criteria*

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February 18, 2003

Abstract

We consider a decision problem where a group of individuals evaluates multi-attribute alternatives. We explore the minimal required agreements that are sufficient to specify the group utility function. A surprising result is that, under some conditions, a bilateral agreement among pairs of individuals on a single attribute is sufficient to derive the multi-attribute group utility. The bilateral agreement between a pair of individuals could be on the weight of an attribute, on an attribute evaluation function, or on willingness to pay.

We focus on the case in which each individual's utility function is additive. We show that the group utility can be represented as the weighted sum of group attribute weights and, more remarkably, of attribute evaluation functions. These group attribute evaluation functions are in turn weighted sums of individual attribute evaluation functions.

1 Introduction

In this paper, we focus on group decisions where a group of individuals or a committee collectively shares the responsibility for choosing among alternative proposals for action. Individual or committee members may have different views on the relative merit of each proposal. Therefore, the problem boils down to how one should

*The authors are thankful to the seminar participants at Duke University, The Wharton School, INSEAD (Singapore), and Informs Hawaii for several helpful suggestions.

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aggregate the views/preferences of the committee members to arrive at a preferred decision.

Arrow (1963) and Sen (1970) have shown that, in general, there is no procedure for combining individual rankings into a group ranking without violating some rather reasonable assumptions. Interpersonal comparison of preferences is the Achilles Heel in escaping from the trap that Arrow's Impossibility Theorem has so eloquently laid out. In the spirit of Keeney and Raiffa (1976), we will assume that individuals are willing to perform interpersonal comparisons of utility or welfare.

Our contribution is to provide explicit ways to elicit the utility comparisons to derive the group utility. We show that it is possible to restrict these utility comparisons at a pair level. Our strategy is to seek compromise and agreement on a chosen parameter from the two individuals comprising the pair. Thus, for example, two individuals who attach different importance weights to an attribute of interest are asked to compromise and to come up with an agreement on their common weight for the attribute in question.

This paper represents an important application of Baucells and Shapley (2000) theory to multiattribute decision problems. Further, we provide decision analytic procedures to elicit utility comparisons among individuals. Our results therefore could be directly adapted to practical applications of group decisions with multiple criteria.

We will assume that individual preferences satisfy the von Neumann-Morgenstern utility axioms. For organizational decisions, an individual may represent a department such as Marketing or R&D. The individual would presumably reflect the preferences of the department. Marketing, for example, may believe that an early entry to market (attribute: time to market entry) is more important, but R&D may believe that the features of the introduced product (attribute: product features) is a more important attribute. The group utility (or company utility in this case) is derived by forming bilateral agreements that seek a compromise on the attribute weight. Thus, depending on the context of the decision, individual utility may reflect an impact on oneself, or an impact on the department or constituency that the individual represents. In Section 2, we summarize some key results of Baucells and Shapley (2000) that show how bilateral agreements among $(n-1)$ pairs of individuals are enough to derive the group utility function. The operationalization of interpersonal weights through decision analytic techniques is discussed.

In the remainder of this paper, we consider the multiattribute decision problem where the consequences or the outcomes of a decision are measured using multiple attributes. Examples include location of facilities, selection of a candidate and choice of a new product or process technology. For numerous applications see Keeney and Raiffa (1976) or the comprehensive Multiple Criteria Decision Aid Bibliography at http://www.lamsade.dauphine.fr/mcda/biblio/index.html. Many such decisions are group decisions. Benbasat and Lim (1993) and Dennis et al. (1996) perform a meta-analysis of 29 and 64 studies, respectively, and report the pros and cons of using group support systems for these decisions.

In Section 3, we assume that the individual multiattribute utility functions are additive, i.e., they can be expressed as the weighted sum of single-attribute evaluation functions (attribute utilities). A surprising result is that the group utility can be decomposed into a weighted sum of group attribute utilities. Clearly, a series of bilateral agreements are needed to derive group utility from individual utilities. Interestingly, agreements regarding all attribute weights and attribute utilities are not necessary. In fact, an agreement on the weight of any one chosen attribute, or on the trade-off between two attributes, may be sufficient to derive the multiattribute pair utility. For example, suppose time to market entry, product features, product performance, product reliability and cost are relevant attributes for a new product introduction decision. Marketing is willing to pay ten million dollars to accelerate market entry by six months; whereas R&D prefers that the additional resources be spent on enhancing product features and is willing to pay only five million dollars to facilitate early market entry. A bilateral agreement between the two departments may be to pay eight million dollars for an earlier introduction of the product. Given individual utilities of Marketing and R&D, the one bilateral agreement on trade-offs is sufficient to derive their joint utility. Consistency with the Extended Pareto Rule and the additive structure of the multiattribute utility ensures that the agreement on the trade-offs among the other remaining attributes (say cost and product features) cannot be arbitrary.

If a large number of individuals are involved in a group decision, pairwise comparisons may be too onerous. Section 4 shows how our framework can be extended to consider coalition agreements. Finally, in Section 5, we discuss some possible extensions of the present work. We begin by laying out the formal framework for our approach.

2 The Formal Framework

2.1 Setting

Consider a set of individuals $N = \{1, 2, ..., n\}$, with $n \geq 2$, who must jointly choose from a set of alternatives. The outcome of a decision is evaluated on m attributes and therefore an outcome $x = (x_1, ..., x_m)$ is a point in the outcome space $X = X_1 \times X_2 \times ... \times X_m$, where X_a, $a = 1$ to m, is the set of possible outcomes on the a^{th} attribute. We assume X is finite. Each individual i's utility u_i is assumed to satisfy von Neumann-Morgenstern (1947) rationality requirements, $u_i : x \to \mathrm{Re}$. The choice set $\widetilde{X}$ from which the individuals and the group choose a preferred course of action is the set of all probability distributions over X. We will use symbols x, y, z, etc. to denote both the deterministic outcomes as well as probability distributions over X. The expected value of u_i is used to determine individual i's preference over probability distributions or lotteries over X. We require that there are two consequences x and y so that all individuals strictly prefer x to y. For simplicity of exposition, we will assume throughout that there is the least preferred consequence, x^0, and the most preferred consequence, x^*, for all individuals and we can therefore set $u_i(x^0) = 0$, $u_i(x^*) = 1$, $i = 1$ to n.

It is clear that we have assumed that the preference relation $\succsim_i$ for individual i is complete and satisfies von Neumann-Morgenstern axioms. We will assume that preferences of each coalition $S \subseteq N$ also satisfy von Neumann-Morgenstern axioms, except completeness ($x \succsim y$ or $y \succsim x$). Thus, a coalition will have a partial ordering or incomplete preferences over X. Incomplete preferences merely imply that the coalition is sometimes unable to express the direction of preference for certain pairs of alternatives.

A collection of preferences $\succsim_S$, $S \subseteq N$, satisfies the Extended Pareto Rule (EPR) if for all disjoint coalitions A and B, and for all $x, y \in \widetilde{X}$,

$$x \succsim_A y,\ x \succsim_B y \Longrightarrow x \succsim_{A \cup B} y,\ \textit{and} \tag{1}$$

$$x \succ_A y,\ x \succsim_B y \Longrightarrow x \succ_{A \cup B} y. \tag{2}$$

It is clear that for $n = 2$, the EPR reduces to exactly the well-known Pareto Rule. An implication of the EPR is that if we break a group into subgroups and if each subgroup prefers x to y, then the group as a whole should prefer x to y. We will use the EPR to derive group preferences from pair preferences.

2.2 Pair Preferences

Consider two individuals i and j who are endowed with utility functions u_i and u_j, respectively. As shown in Harsanyi (1955) if the pair preference $\succsim_{ij}$ is complete and the Pareto Rule holds, then the joint utility u_{ij} takes the form $u_{ij} = \alpha_i^j u_i + (1 - \alpha_i^j) u_j$, for some $\alpha_i^j \in (0, 1)$. Baucells and Shapley (2000) extend this result to two disjoint coalition A and B using EPR:

$$u_{A \cup B} = \alpha u_A + (1 - \alpha) u_B \tag{3}$$

We now present an example of how α_i^j can be elicited. Choose a consequence x so that $u_i(x) \neq u_j(x)$ and without loss of generality let $u_i(x) < u_j(x)$. The usual von Neumann-Morgenstern interpretation of utility implies that if individual i is indifferent between x for sure and the binary lottery yielding x^* (the best outcome) with a probability p_i and yielding x^0 (the worst outcome) with a probability $(1 - p_i)$, then $u_i(x) = p_i$. Similar interpretation holds for determining $u_j(x) = p_j$. To elicit $u_{ij}(x)$ through a bilateral agreement, find a probability $p_{ij} \in (p_i, p_j)$ so that, as a pair, i and j are indifferent between x for sure and the lottery yielding x^* with a p_{ij} chance and x^0 with a $(1 - p_{ij})$ chance. A compromise between individuals i and j is needed to reach the common indifference probability, p_{ij}. We note that p_{ij} cannot take the extreme values of the interval because such an assignment ($p_{ij} = p_i$ or $p_{ij} = p_j$) would violate strong condition 2 of the Extended Pareto Rule. To derive α_i^j from p_{ij}, observe that $p_{ij} = u_{ij}(x) = \alpha_i^j u_i(x) + (1 - \alpha_i^j) u_j(x)$, so that

$$\alpha_i^j = \frac{p_j - p_{ij}}{p_j - p_i}. \tag{4}$$

It may not be immediately clear why a compromise on the utility of one outcome predetermines the compromise on the utility of other outcomes. We show by means of an example that the Extended Pareto Principle will be violated if the compromise on the utility of another outcome y is not restricted in accord with the original compromise on the utility of the outcome x.

Consider a setting with four monetary consequences of \$0, \$200, \$500 and \$1,000. The utilities of individuals 1 and 2, respectively, for these four consequences are $(0, .2, .5, 1)$ and $(0, .4, .75, 1)$. Assume that individuals 1 and 2 agree that $u_{12}(\$500) = 0.55 \in (.5, .75)$. Using (4) yields $\alpha_1^2 = 0.8$. Now, suppose we choose the outcome \$200 and seek a compromise to determine $u_{12}(200)$. Notice that in order to be consistent with the original compromise ($u_{12}(500) = 0.55$), $u_{12}(200)$ must be $0.24 = 0.8u_1(200) + 0.2u_2(200)$: any other agreement, say $u_{12}(200) = 0.32$, produces a violation of the Pareto Principle. To show this, we will construct two lotteries L_1 and L_2 such that $L_1 \succ_1 L_2$, $L_1 \succ_2 L_2$, but $L_2 \succ_{12} L_1$ - a violation of the Extended Pareto Principle. The lotteries L_1 and L_2 and the corresponding expected utilities are given below.

	\$0	\$200	\$500	\$1,000
L_1	40%	0%	60%	0%
L_2	13%	75%	0%	12%

	EU_1	EU_2	EU_{12}
L_1	0.30	0.45	0.33
L_2	0.27	0.42	0.36

Thus, in the above example both individuals 1 and 2 prefer L_1 to L_2. If, however, we choose $u_{12}(200) = 0.32$ and $u_{12}(500) = 0.55$, then the pair $\{1, 2\}$ prefers L_2 to L_1. The violation of the Pareto Principle vanishes if $u_{12}(200)$ is chosen to be 0.24 as implied by the original agreement on $u_{12}(500)$.

In theory, agreement on the utility of *one* outcome is sufficient to completely specify the pair utility. In practice, and assuming that we agree with the normative/prescriptive appeal of the axioms, one should seek compromises on additional points and revisit the agreements to ensure consistency. Further, the ranking of alternatives implied by the pair utility should be displayed. If the individuals do not find the ranking satisfactory, the agreement point should be revisited. Empirically, many experiments report violation of expected utility at the individual level (Kahneman and Tversky 1979, Kagel and Roth 1995). It is an appealing empirical question to explore how these violations may extend to the pair and group level.

2.3 Three Individuals And "No Arbitrage" In The Utility Comparison Rates

A bilateral agreement between individuals 1 and 2 yields u_{12}. Similarly, a bilateral agreement between individuals 2 and 3 yields u_{23}. We now show that these two bilateral agreements are sufficient to derive u_{123} (group utility for the case $n = 3$). We could of course choose either u_{12} and u_{13} or u_{13} and u_{23} to derive u_{123}. Recall that $u_{ij} = \alpha_i^j u_i + (1 - \alpha_i^j) u_j$. Alternatively, we may write $u_{ij} = (u_i + \delta_i^j u_j)/(1 + \delta_i^j)$, where the "utility comparison rate" $\delta_i^j = (1 - \alpha_i^j)/\alpha_i^j \in (0, \infty)$. In the alternative expression for u_{ij}, the parameter δ_i^j is the utility comparison rate between i and j: δ_i^j units of i's utility are expressed in the same units as one unit of j's utility. The order of the subscripts and superscripts is important because $\delta_j^i = 1/\delta_i^j$. Henceforth, we use δ_i^j to denote the utility comparison rate associated with a bilateral agreement between individuals i and j.

For a geometric illustration of the assertion that u_{12} and u_{23} are sufficient to determine u_{123}, refer to Figure 1. If the outcomes in X are finite, then u_i can be thought of a vector of utilities or a point in a vector space. Under this interpretation, note that u_{12} is a convex combination of u_1 and u_2 and, therefore, lies on the line segment connecting u_1 and u_2. Similarly, u_{23} lies on the line segment connecting u_2 and u_3. Now apply (3) to the partition $\{12, 3\}$. So u_{123} must lie somewhere on the line segment connecting u_{12} and u_3. Apply (3) again to the partition $\{1, 23\}$ to conclude that u_{123} must lie somewhere on the line segment connecting u_1 and u_{23}. There is only one point that lies on both the line segments $\overline{u_{12}u_3}$ and $\overline{u_1u_{23}}$ - the point u_{123} where these two line segments intersect. Thus from the two bilateral agreements u_{12} and u_{23}, a complete preference $\succsim_{123}$with utility u_{123} emerges. Figure 1 also illustrates that there is a unique utility candidate for the remaining bilateral agreement u_{13} - the point where the line connecting u_2 and u_{123} intersects line segment $\overline{u_1u_3}$. Otherwise, the Extended Pareto Rule applied to $\{2, 13\}$ would not hold.

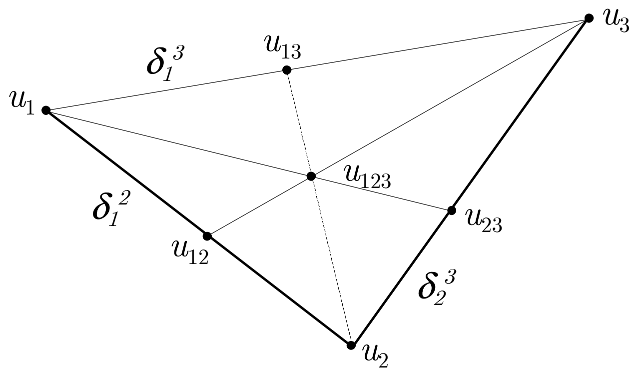

Figure 1: Deriving u_{123} from u_{12} and u_{23}.

The following result, proven in Baucells and Shapley (2000), provides the utilities that obtain from this construction.

Proposition 1 *Consider bilateral agreements* u_{12} *and* u_{23} *with utility comparison rates* δ_1^2 *and* δ_2^3*, respectively. If the Extended Pareto Rule holds, then* $\succsim_{123}$ *is complete and has utility* $u_{123} \equiv (u_1 + \delta_1^2 u_2 + \delta_1^3 u_3)/(1 + \delta_1^2 + \delta_1^3)$*. Moreover, if* $\succsim_{13}$ *is a complete preference, then there is a unique utility comparison rate given by* $\delta_1^3 \equiv \delta_1^2 \delta_2^3$ *so that* $u_{13} \equiv (u_1 + \delta_1^3 u_3)/(1 + \delta_1^3)$.

2.4 Group Preferences Based On Bilateral Agreements

For a group of n individuals, the group utility, u_N, is obtained by eliciting $(n-1)$ bilateral agreements. For an example, suppose one obtains $u_{12}, u_{13}, ..., u_{1n}$. We now know $\delta_1^2, \delta_1^3, ..., \delta_1^n$; that is, the utility comparison rates between Individual 1 and Individual i, $i = 2$ to n. The EPR permits us to write

$$u_N = (u_1 + \delta_1^2 u_2 + ... + \delta_1^n u_n)/(1 + \delta_1^2 + ... + \delta_1^n). \tag{5}$$

There are many possible ways (spanning trees) to obtain $(n-1)$ bilateral agreements. For example, another possibility is to obtain $u_{12}, u_{23}, ..., u_{(n-1)n}$. Such a comparison or spanning tree will yield $\delta_1^2, \delta_2^3, ..., \delta_{(n-1)}^n$. We can easily obtain δ_1^i in (5) by the multiplication $\delta_1^2 \times \delta_2^3 \times ... \times \delta_{(i-1)}^i$. In general, however, we need to first specify the spanning tree. It is an empirical question whether the choice of a spanning tree systematically influences the derived utility comparison rates or utility weights. The current research makes us presume that such influences may exist. Fisher et al. (1987), (2000a), (2000b), and von Nitzsch and Weber (1993) report several difficulties and inconsistencies in eliciting attribute weights at the individual level. We expect these inconsistencies to affect the pair behavior too. Recently, Keeney (2002) reviews these and other more qualitative common mistakes in making value trade-offs.

If more than $n-1$ bilateral agreements are elicited, then the group has to identify $n-1$ of them to be used in the calculations of the group preference that follow. The other bilateral agreements can be used as consistency checks. For example, in the case of $n = 3$, if we ask the pair $\{1, 3\}$ to elicit δ_1^3 and this value differs from $\delta_1^2 \delta_2^3$, then the inconsistency has to be resolved until equality holds.[1] This is not different from other

[1] Current research on invariant measures of incomplete preferences (Baucells and Shapley 2001) suggest that if two or more comparison rates are to be averaged out, then it is advisable to use geometric means, i.e., use $\delta = (\delta_1 \delta_2 ... \delta_k)^{1/k}$. Here, δ_r are different estimates of some utility comparison rate δ_i^j .

areas of decision analysis and applied economics where the normative/prescriptive model creates a consistency burden on the descriptive/behavioral reality. Olson et al. (1995) conclude, in the context of decision support systems, that "feedback concerning the consistency of decision maker responses should be regularly provided to make users comfortable and ... yield results valuable to users." Sengupta and Te'eni (1993) found that cognitive feedback facilitates group convergence as well. Descriptive research on the influence of the actual spanning tree is critical to incorporate the behavioral and psychological insights in the design of decision support system (Dyer et al. 1992). In what follows, we assume that the decision context and practical considerations will dictate the choice of a spanning tree.

Fix a spanning tree $\mathcal{T}$ on the set of individuals, or a collection of pairs that contains precisely $n-1$ elements and has no cycles. Consider complete preferences or bilateral agreements for all pairs in $\mathcal{T}$ and let δ_i^j, $ij \in \mathcal{T}$, be the corresponding utility comparison rates. To obtain the weights λ_i that apply to u_i in the expression for the group (or coalition) utility, we have to multiply the δ_i^j along the branches of the tree that connect some chosen base agent and individual i. Formally, choose an arbitrary *base agent*, say $i = 1$, as the "root" of the tree and let $\delta_1^1 \equiv 1$. For $i \neq 1$, let $\mathcal{P}_1^i$ be the collection of pairs in the unique path between 1 and i in $\mathcal{T}$, and define

$$\lambda_i \equiv \delta_1^i = \prod_{jk \in \mathcal{P}_1^i} \delta_j^k \tag{6}$$

The following Proposition in Baucells and Shapley (2000) provides the main result upon which the subsequent developments are based.

Proposition 2 *Assume that any three utilities are linearly independent and that every pair ij in N has complete (but undetermined) preferences. Let $\mathcal{T}$ be a spanning tree of bilateral agreements (determined pair preferences) and let $(\lambda_1, ..., \lambda_n)$ be given as in (6). Then, the Extended Pareto Rule holds if and only if for all coalition S in N, $\succsim_S$ is complete and has utility*

$$u_S \equiv \frac{\sum_{i \in S} \lambda_i u_i}{\sum_{i \in S} \lambda_i}. \tag{7}$$

We remark that it is not immediately clear why the λ's used in u_N have to agree with the λ's used in u_S. In fact, Harsanyi (1963), Hart and Mas-Colell (1996), Maschler and Owen (1992) propose models in cooperative game theory where the λ's vary with the coalition. Propositions 1 and 2 show that uniqueness of the λ's for all subsets of N is a consequence of expected utility and EPR.

The interest of Proposition 2 resides in its usefulness to devise a practical procedure for preference aggregation. We will exploit several nice features of Proposition 2. First, it just requires the determination of $n-1$ parameters, namely, the $n-1$ utility comparison rates between pairs of individuals. Second, the choice of the $n-1$ pairs is rather flexible in that we can choose any $n-1$ pairs that form a spanning tree. Additional criteria may be used so that a particular spanning tree offers more advantages. Third, simple manipulations allow us to obtain an alternative representation of the group utility as a weighted sum of utilities of disjoint coalitions (see Section 4).

3 Multiattribute (Additive) Group Decisions

We now examine the case where the consequence of a decision is evaluated on multiple attributes. We assume that the members of the group have agreed on the attributes $X_1, ..., X_m$ and each individual is able to specify his own utility function for these multiple attributes. An arbitrary multidimensional consequence is denoted $x = (x_1, ..., x_m)$, and x_a^* and x_a^0 are respectively the most preferred and least preferred consequences on attribute a that are agreed upon by all individuals. We further assume that each individual's utility function over the multiple attributes is additively separable. Thus, we can write

$$u_i(x) = \sum_{a=1}^{m} \beta_{i,a} v_{i,a}(x), \;\; i = 1 \text{ to } n, \tag{8}$$

where $\beta_{i,a}$ is the importance weight (scaling constant) that individual i attaches to attribute a, $0 < \beta_{i,a} < 1$, $\sum_{a=1}^{m} \beta_{i,a} = 1$ and $v_{i,a}$ is individual i's component utility function (attribute evaluation function) for attribute a which, is normalized by $v_{i,a}(x_a^*) = 1$, $v_{i,a}(x_a^0) = 0, i = 1$ to n, $a = 1$ to m.

In general, the attribute weights and the component utilities in (8) will differ across individuals. To obtain the pair utility u_{ij}, however, an agreement on the weight and the component utility for each attribute is not required. The pair utility u_{ij} can be constructed by seeking a compromise on a part of the problem (e.g. on the weight that individuals i and j assign to an attribute a). The Extended Pareto Rule restricts the compromises on other parts of the problems if a compromise on one part of the problem is reached.

We will discuss three different methods for eliciting bilateral agreements to specify pair utilities which, in turn, are used to derive the group utility. The group utility so constructed will have $n \times m$ terms. It is interesting to examine whether the group utility can be represented as the weighted sum of group attribute utilities (i.e., $u_N = \beta_1 v_1 + ... + \beta_n v_m$, where β_a is the "group attribute weight" for the attribute a and v_a is the "group attribute utility" for the attribute a). Surprisingly, as Proposition 4 shows, we are indeed able to express the group utility as the weighted sum of group attribute utilities. From the narrow perspective of simply being able to rank alternatives, it may not seem important how the group utility is represented so long as it can somehow be assessed. For practical considerations, however, the decomposition of the group utility into "group attribute weights" and "group attribute utilities" may facilitate communication and implementation. For example, a higher level policy maker who has delegated the responsibility of the decision to the committee may wish to interpret the group attribute weights as the weights he will assign to the attributes if he were to make the decision on his own. In design of group decision support systems this representation is an additional output that can be given to the group to increase the understanding of the aggregation procedure and provide further feedback on their inputs (Limayem and DeSanctis 2000).

The basic idea is to impose the Extended Pareto Rule to a larger set of preferences, say $\succsim_Q$, for some $Q \subseteq N \times M$ (N is the set of individuals $\{1...n\}$ and M is the set of attributes $\{1...m\}$). Intuitively, we may think that each individual has multiple preferences, one for each attribute and that individual aggregates these preferences using weights $\beta_{i,a}$. Thus, we may enlarge the set of "basic" individuals to $n \times m$. The enlarged coalition $N \times M$ corresponds to the group utility, and the enlarged coalition $\{i\} \times M$ corresponds to individual i, with utility u_i. Similarly, $v_{i,a}$, the attribute evaluation function of individual i, represents the preference of the coalition $\{i\} \times \{a\}$; and the group attribute evaluation function v_a is the preference of the coalition $N \times \{a\}$.

For example, application of the Extended Pareto Rule to all the disjoint subsets of $\{i, j\} \times M$ permits us to increase the number of linearity conditions. The preferences of the subset $\{i,j\} \times \{a\}$ is interpreted as the pair attribute evaluation; and $\{i\} \times M \cup \{j\} \times \{a\}$ corresponds to i taking into account j's opinion regarding attribute a; and $\{i\} \times M \backslash \{a\}$, is i's preferences without taking into account attribute a, etc... We obtain that for all consequences x the following relationships hold

$$
\begin{aligned}
u_{ij} &= \alpha_i^j u_i + (1-\alpha_i^j) u_j, \quad \textit{for some } \alpha_i^j \in (0,1) && (9) \\
v_{ij,a} &= \alpha_{i,a}^{j,a} v_{i,a} + (1-\alpha_{i,a}^{j,a}) v_{j,a}, \quad \textit{for some } \alpha_{i,a}^{j,a} \in (0,1) && (10) \\
u_{ij} &= \sum_{a=1}^{m} \beta_{ij,a} v_{ij,a}, \quad \textit{for some } \beta_{ij,a} \textit{ with } \sum_{a=1}^{m} \beta_{ij,a} = 1. && (11)
\end{aligned}
$$

As before, α_i^j and the utility comparison rate $\delta_i^j = (1-\alpha_i^j)/\alpha_i^j$ determine pair utility $u_{ij} = \alpha_i^j u_i + (1-\alpha_i^j) u_j$. The weights $\alpha_{i,a}^{j,a}$ represent the trade-off between $v_{i,a}$ and $v_{j,a}$ to form the pair attribute evaluation function $v_{ij,a} = \alpha_{i,a}^{j,a} v_{i,a} + (1-\alpha_{i,a}^{j,a}) v_{j,a}$, where $v_{ij,a}$ is the utility of the coalition $\{i,j\} \times \{a\}$, expressed as the weighted sum of the utilities of $\{i\} \times \{a\}$ and $\{j\} \times \{a\}$. Finally, Proposition 5 allows us to express u_{ij} as the weighted sum of utilities that can be formed in any partition of $\{i,j\} \times M$. One such partition takes the form $\{i,j\} \times \{a\}$, for $a \in M$ and produces (11).

Before proceeding with different ways to elicit the previous parameters, we now present a result that links their values leaving only one degree of freedom for each pair of individuals, and $n-1$ degrees of freedom for the group. The intuition behind this result comes from our idea of the spanning tree. If we view $v_{i,a}$ as the nodes of the spanning tree, then the fact that u_i is a linear combination of the $v_{i,a}$ for all $a \in M$ tells us that all the bilateral agreements between any two nodes $v_{i,a}$ and $v_{i,b}$ are already in place with $\delta_{i,a}^{i,b} = \beta_{i,b}/\beta_{i,a}$. Thus, once we establish one link between i and j, then our consistency condition already fills the gaps and establishes all the utility comparison rates between individuals and between attribute evaluation functions. Having imposed the Extended Pareto Rule to the attribute evaluation function will allow us to determine some additional parameters that are linked to α_i^j.

Proposition 3 *If the Extended Pareto Rule holds among the disjoint subsets of* $\{i,j\} \times M$*, then the parameters* $\delta_i^j = (1-\alpha_i^j)/\alpha_i^j$, $\delta_{i,a}^{j,a} = (1-\alpha_{i,a}^{j,a})/\alpha_{i,a}^{j,a}$*, and* $\beta_{ij,a}$ *in (9), (10), and (11) are related as follows*

$$
\begin{aligned}
\delta_{i,a}^{j,a} &= \frac{\beta_{j,a}}{\beta_{i,a}} \delta_i^j && (12) \\
\beta_{ij,a} &= \alpha_i^j \beta_{i,a} + (1-\alpha_i^j) \beta_{j,a} && (13)
\end{aligned}
$$

for all $i, j \in N$ and all $a \in M$. Moreover

$$v_{ij,a} = \frac{\alpha_i^j \beta_{i,a} v_{i,a} + (1-\alpha_i^j)\beta_{j,a} v_{j,a}}{\beta_{ij,a}}. \tag{14}$$

Proof. Using (9), (10), and (11), we write

$$\begin{aligned} u_{ij} &= \sum_{a=1}^{m} \beta_{ij,a} v_{ij,a} = \sum_{a=1}^{m} \beta_{ij,a} [\alpha_{i,a}^{j,a} v_{i,a} + (1-\alpha_{i,a}^{j,a}) v_{j,a}] \\ &= \alpha_i^j u_i + (1-\alpha_i^j) u_j = \alpha_i^j \sum_{a=1}^{m} \beta_{i,a} v_{i,a} + (1-\alpha_i^j) \sum_{a=1}^{m} \beta_{j,a} v_{j,a}. \end{aligned}$$

Because this holds for all $x \in X$, linear independence of the attribute utilities only holds if the coefficients of the $v_{i,a}$'s are the same. Evaluate this expression at $(x_1^o, ..., x_{a-1}^o, x_a^*, x_{a+1}^o ..., x_m^o)$ for all $a \in M$ to arrive at

$$\begin{aligned} \beta_{ij,a} \alpha_{i,a}^{j,a} &= \beta_{i,a} \alpha_i^j \\ \beta_{ij,a}(1-\alpha_{i,a}^{j,a}) &= \beta_{j,a}(1-\alpha_i^j) \end{aligned}$$

Equation (12) follows from dividing the two expressions above, and (13) from adding them. Equation (14) follows from replacing $\alpha_{i,a}^{j,a} = \beta_{i,a}\alpha_i^j / \beta_{ij,a}$ in (10) and using (13). ■

With this result in mind we now present three alternative ways to elicit bilateral agreements among $(n-1)$ pairs of individuals.

3.1 Method I: Compromise On Attribute Evaluation Functions

For a given pair of individuals i and j, and some choice of an attribute a, let $v_{i,a}$ and $v_{j,a}$ be the corresponding attribute evaluation functions. Next, we fix alternatives $x^h = (x_1^o, ..., x_{a-1}^o, x_a^*, x_{a+1}^o ..., x_m^o)$ and $x^\ell = (x_1^o, ..., x_{a-1}^o, x_a^o, x_{a+1}^o ..., x_m^o)$ and find some intermediate alternative $x = (x_1^o, ..., x_{a-1}^o, x_a, x_{a+1}^o ..., x_m^o)$ such that $v_{i,a}(x) \neq v_{j,a}(x)$. Thus, $u_i(x) = \beta_{i,a} v_{i,a}(x_a)$, $u_i(x^h) = \beta_{i,a}$ and $u_i(x^\ell) = 0$ so that individual i is indifferent between x and a probability mixture $p_i = v_{i,a}(x)$ between x^h and x^ℓ. Then, we ask individuals i and j to find a compromise probability $p_{ij} \in (p_i, p_j)$. This says that $u_{ij}(x) = \beta_{ij,a} p_{ij}$, or

$$v_{ij,a}(x) = p_{ij} = \alpha_{i,a}^{j,a} p_i + (1-\alpha_{i,a}^{j,a}) p_j = \alpha_{i,a}^{j,a} v_{i,a}(x) + (1-\alpha_{i,a}^{j,a}) v_{j,a}(x)$$

so that

$$\alpha_{i,a}^{j,a} = \frac{p_j - p_{ij}}{p_j - p_i} \quad \text{and} \quad \delta_{i,a}^{j,a} = \frac{p_{ij} - p_i}{p_j - p_{ij}}. \tag{15}$$

Utility comparison rates for individuals i and j for attribute a can also be obtained by seeking a compromise on individuals' certainty equivalents of $50 - 50$ gamble between x^h and x^ℓ. If x is the compromise certainty equivalent; then in (15), $p_{ij} = 0.5$, $p_i = v_{i,a}(x)$, and $p_j = v_{j,a}(x)$. The certainty equivalent method is widely used in decision analysis.

Using (12) and (13) we derive the value of the other parameters. Specifically we obtain that $u_{ij} \equiv \alpha_i^j u_i + (1 - \alpha_i^j)u_j$, where $\alpha_i^j = 1/(1 + \delta_i^j)$ and $\delta_i^j = \beta_{i,a}\delta_{i,a}^{j,a}/\beta_{j,a}$. Alternatively, the group utility is represented by

$$u_{ij} \equiv \sum_{a=1}^{m} \beta_{ij,a} v_{ij,a},$$

where $\beta_{ij,a} = \alpha_i^j \beta_{i,a} + (1 - \alpha_i^j)\beta_{j,a}$. Finally, for all $b \in M$, $v_{ij,b}$ can be expressed as in (14). This possibility of finding pair attribute evaluation functions is the result that later we will extend to u_N.

3.2 Method II: Compromise On Attribute Weights

An alternative way to elicit the bilateral agreement is by means of comparing the weights given to the different attributes. For a given pair of individuals i and j and some choice of an attribute a, let $\beta_{i,a}$ and $\beta_{j,a}$ be the corresponding weights given to attribute a. Recall that $\beta_{i,a}$ is interpreted as the utility of alternative $x^h = (x_1^o, ..., x_{a-1}^o, x_a^*, x_{a+1}^o, ..., x_m^o)$. Alternatively, individual i is indifferent between x^h and a mixture that with probability $\beta_{i,a}$ produces x^* or x^o. If $\beta_{i,a} \neq \beta_{j,a}$, then we ask the pair to reach a bilateral agreement $\beta_{ij,a}$ that falls strictly between $\beta_{i,a}$ and $\beta_{j,a}$ that will be interpreted as the pair utility of x^h, or

$$u_{ij}(x^h) = \beta_{ij,a} = \alpha_i^j u_i(x^h) + (1 - \alpha_i^j)u_j(x^h) = \alpha_i^j \beta_{i,a} + (1 - \alpha_i^j)\beta_{j,a}$$

so that

$$\alpha_i^j = \frac{\beta_{j,a} - \beta_{ij,a}}{\beta_{j,a} - \beta_{i,a}} \quad \text{and} \quad \delta_i^j = \frac{\beta_{ij,a} - \beta_{i,a}}{\beta_{j,a} - \beta_{ij,a}}. \tag{16}$$

Using α_i^j, δ_i^j, (13), and (14), we can compute $\beta_{ij,b}$ and $v_{ij,b}$ for all $b \in M$, and write u_{ij} as a linear combination of $v_{ij,b}$.

3.3 Method III: Compromise On The Willingness To Pay

A common procedure widely used in multiattribute applications to elicit the individual weights $\beta_{i,a}$ is the following. Assuming attribute $a = 1$ is the most important, we seek x_1 so that

$$(x_1, x_2^o, x_3, ..., x_m) \sim_i (x_1^o, x_2^*, x_3, ..., x_m)$$

This implies that $\beta_{i,1} v_{i,1}(x_1) = \beta_{i,2}$. Now, we seek x_1' so that

$$(x_1', x_2, x_3^o, x_4, ..., x_m) \sim_i (x_1^o, x_2, x_3^*, x_4, ..., x_m),$$

which implies $\beta_{i,1} v_{i,1}(x_1') = \beta_{i,3}$, and so on. This method relates all the attribute weights to $\beta_{i,1}$, and we elicit $\beta_{i,1}$ by finding the probability p so that a mixture of x^* and x^o is indifferent to $(x_1^*, x_2^o, ..., x_m^o)$. This implies that $\beta_{i,1} = p$. The idea is to get one weight by the lottery method and the others by trade-offs. Notice that if the first attribute is money, then we derive $\beta_{i,1}$ by finding out how much individual i is willing to pay to raise the value of x_2 from x_2^o to x_2^*, and so on. Let x_1^i be i's willingness to pay, or

$$(x_1^i, x_2^o, x_3, ..., x_m) \sim_i (x_1^o, x_2^*, x_3, ..., x_m)$$

Similarly, let x_1^j be j's willingness to pay. If $x_1^i \neq x_1^j$, then we can ask the pair to reach a compromise and seek a pair willingness to pay $x_1^{ij} \in (x_1^i, x_1^j)$. The next step, of course, is to find the α_i^j and δ_i^j that follow from x_1^{ij}. They are given by:

$$\alpha_i^j = \frac{\beta_{j,1}[v_{j,1}(x_1^j) - v_{j,1}(x_1^{ij})]}{\beta_{i,1}[v_{i,1}(x_1^{ij}) - v_{i,1}(x_1^i)] + \beta_{j,1}[v_{j,1}(x_1^j) - v_{j,1}(x_1^{ij})]}, \textit{ and} \tag{17}$$

$$\delta_i^j = \frac{\beta_{i,1}}{\beta_{j,1}} \frac{[v_{i,1}(x_1^{ij}) - v_{i,1}(x_1^i)]}{[v_{j,1}(x_1^j) - v_{j,1}(x_1^{ij})]}.$$

To derive these expressions, just notice that the agreement in willingness to pay x_1^{ij} says

$$(x_1^{ij}, x_2^o, x_3, ..., x_m) \sim_{ij} (x_1^o, x_2^*, x_3, ..., x_m), \textit{ or}$$
$$u_{ij}(x_1^{ij}, x_2^o, x_3, ..., x_m) = u_{ij}(x_1^o, x_2^*, x_3, ..., x_m).$$

Using $u_{ij} = \alpha_i^j u_i + (1 - \alpha_i^j) u_j$, and cancelling the terms involving $x_3, ..., x_m$, produces

$$\alpha_i^j \beta_{i,1} v_{i,1}(x_1^{ij}) + (1 - \alpha_i^j) \beta_{j,1} v_{j,1}(x_1^{ij}) = \alpha_i^j \beta_{i,2} + (1 - \alpha_i^j) \beta_{j,2}$$

and the following alternative expression for α_i^j,

$$\alpha_i^j = \frac{\beta_{j,2} - \beta_{j,1} v_{j,1}(x_1^{ij})}{\beta_{j,2} - \beta_{i,2} + \beta_{i,1} v_{i,1}(x_1^{ij}) - \beta_{j,1} v_{j,1}(x_1^{ij})}.$$

Plugging $\beta_{i,2} = \beta_{i,1} v_{i,1}(x_1^i)$ and $\beta_{j,2} = \beta_{j,1} v_{j,1}(x_1^j)$ yields (17).

Again, using α_i^j, δ_i^j, (13), and (14), we can compute $\beta_{ij,b}$ and $v_{ij,b}$ for all $b \in M$, and write u_{ij} as a linear combination of the $v_{ij,b}$.

We hasten to emphasize that the three methods are supposed to yield the same parameter values of interpersonal comparison. Further, for any given method, the choice of alternatives should yield the same utility comparison rates. Introducing alternative methods has a practical motivation. Several studies in multicriteria decision making have found that subjects prefer transparent and relatively unsophisticated decision models (Buchanan and Daellenbach 1987 and Narasimhan and Vickery 1988). Whereas a probabilistic pair agreement as in (4) would be sufficient in theory, to provide alternative, more intuitive, elicitation procedures may enhance the reliability of the outcomes and/or manifest certain inconsistencies that otherwise would have passed unnoticed. This normative requirement of individual and pair consistency is unlikely to be met empirically, so that the question of which method yields more reliable results is to be elucidated empirically. See Harte and Koele (1995) and Zanakis et al. (1998) for current work on this area.

3.4 The Main Result

After repeating any of these procedures for $n-1$ pairs forming a spanning tree, the final results are weights α_i^j and utility comparison rates δ_i^j, for $ij \in \mathcal{T}$. To extend the aggregation procedure we apply EPR to preferences of rectangular subsets $S \times M$, $S \subseteq N$ and $E \subseteq M$. The interpretation of such preferences is straightforward: the preference of the members of S if they only consider attributes $a \in E$.[2] We use (6) to calculate $\lambda_i = \delta_1^i$. Finally, we simplify the notation by writing $v_a \equiv v_{N,a}$ and $\beta_a \equiv \beta_{N,a}$. The result we obtain is:

Proposition 4 *Let λ_i be the weights obtained as in (6) by means of $n-1$ bilateral agreements forming a spanning tree of N. Then, group preference is complete and is represented by*

$$u_N \equiv \frac{\sum_{i=1}^n \lambda_i u_i}{\sum_{i=1}^n \lambda_i}. \tag{18}$$

[2] We agree with two referees, and thank them for pointing this out, that preferences for general subsets of $N \times M$ are difficult to interpret, and that it is sufficient to consider rectangular subsets.

Alternatively, the group utility is equally represented by

$$u_N \equiv \sum_{a=1}^{m} \beta_a v_a, \tag{19}$$

where $\beta_a = \sum_{i=1}^n \lambda_i \beta_{i,a} / \sum_{i=1}^n \lambda_i$ *and each* v_a *is a group attribute preference given by*

$$v_a \equiv \frac{\sum_{i=1}^n \lambda_i \beta_{i,a} v_{i,a}}{\sum_{i=1}^n \lambda_i \beta_{i,a}}. \tag{20}$$

Proof. (18) is just a direct application of Proposition 2. By substituting (8) into (18), we obtain a sum of $n \times m$ terms of the form $v_{i,a}$, with weights of the form $\lambda_i \times \beta_{i,a}$. (19) follows from forming a partition of $N \times M$ in sets $A_a = N \times \{a\}$, for $a \in M$, and applying Proposition 5. Indeed, $\delta_{1,1}^{A_a} = \sum_{i=1}^n \lambda_i \beta_{i,a}/(\lambda_1 \beta_{1,1})$. Here the subscript $(1,1)$ indicates that the utilities are expressed in units of $v_{1,1}$. By normalizing so that $\sum_{a=1}^m \beta_a = 1$, we obtain

$$\beta_a = \frac{\delta_{1,1}^{A_a}}{\sum_{a=1}^m \delta_{1,1}^{A_a}} = \frac{\delta_{1,1}^{A_a}}{\sum_{a=1}^m \sum_{i=1}^n \lambda_i \beta_{i,a}} = \frac{\sum_{i=1}^n \lambda_i \beta_{i,a}}{\sum_{i=1}^n \lambda_i \sum_{a=1}^m \beta_{i,a}} = \frac{\sum_{i=1}^n \lambda_i \beta_{i,a}}{\sum_{i=1}^n \lambda_i}.$$

The attribute evaluation function v_a corresponds to the utility of the "coalition" $A_a = N \times \{a\}$. To express v_a in terms of the $v_{i,a}$'s suffices to break A_a into their components $\{i\} \times \{a\}$. Then, $\delta_{1,a}^{i,a} = (\lambda_i \beta_{i,a})/(\lambda_1 \beta_{1,1})$. Of course, the denominator cancels out once we write

$$v_a = \frac{\sum_{i=1}^n \delta_{1,a}^{i,a} v_{i,a}}{\sum_{i=1}^n \delta_{1,a}^{i,a}} = \frac{\sum_{i=1}^n \lambda_i \beta_{i,a} v_{i,a}}{\sum_{i=1}^n \lambda_i \beta_{i,a}}.$$

■

3.5 An Example

To illustrate the use of the three elicitation methods, we consider an example with $n = 2$ individuals and $m = 3$ attributes. The three attributes of an alternative $x = (x_1, x_2, x_3)$ may represent a monetary outcome (say a discounted cash flow) x_1, some indicator of environmental impact x_2, and some measure of health and safety x_3. Let the worst outcome be $x^o = (0,0,0)$ and the best be $x^* = (100, 100, 100)$. The individual utilities are additive, separable in the following attribute evaluation functions

$$u_1(x) = 0.4 \frac{1 - e^{-x_1/100}}{1 - e^{-1}} + 0.35 \frac{x_2}{100} + 0.25 \frac{x_3}{100}$$

$$u_2(x) = 0.5 \frac{1 - e^{-x_1/200}}{1 - e^{-1/2}} + 0.3 \frac{x_2}{100} + 0.2 \frac{x_3}{100}$$

Notice that $v_{i,a}(x^o) = 0$ and $v_{i,a}(x^*) = 1$. The first component reveals that both individuals are risk averse with respect to monetary consequences with exponential utility functions. Regarding the other two attributes, we assume for simplicity that their attribute evaluation function is linear in x_2 and x_3.

To elicit a bilateral agreement applying **Method I**, we choose the monetary attribute x_1. Accordingly, we let $x^h = (100, 0, 0)$, $x^\ell = (0, 0, 0)$ and fix some intermediate alternative, say $x = (40, 0, 0)$. Thus, $v_{1,1}(x) = 0.52 \neq v_{2,1}(x) = 0.46$ so that Individual 1 is indifferent between x and a probability mixture $p_1 = 0.52$ between x^h and x^ℓ. $p_2 = 0.46$ has the same interpretation for Individual 2. Now, individuals 1 and 2 elicit a bilateral agreement by finding a compromise probability $p_{12} \in (0.46, 0.52)$, say $p_{12} = 0.51$. Using (15) we find that

$$\alpha_{1,1}^{2,1} = \frac{p_2 - p_{12}}{p_2 - p_1} = \frac{0.46 - 0.51}{0.46 - 0.52} = 0.81 \quad \text{and} \quad \delta_{1,1}^{2,1} = \frac{0.51 - 0.52}{0.46 - 0.51} = 0.23$$

From $\delta_{1,1}^{2,1}$ and (12) we derive $\delta_1^2 = \beta_{1,1}\delta_{1,1}^{2,1}/\beta_{2,1} = 0.4 \times 0.23/0.5 = 0.19$ and $\alpha_1^2 = 1/(1 + \delta_1^2) = 0.84$. This allows us to write the group utility as

$$u_{12}(x) = 0.84 u_1(x) + 0.16 u_2(x).$$

Alternatively, the group utility can be represented by

$$u_{12} \equiv \sum_{a=1}^{m} \beta_{12,a} v_a = 0.42 v_1 + 0.34 v_2 + 0.24 v_3$$

where $\beta_{12,a} = \alpha_1^2 \beta_{1,a} + (1 - \alpha_1^2)\beta_{2,a}$ and $v_a \equiv v_{12,a} = \sum_{i=1}^{n} \lambda_i \beta_{i,a} v_{i,a} / \sum_{i=1}^{n} \lambda_i \beta_{i,a}$. Because $\lambda_1 = 1$ and $\lambda_2 = \delta_1^2 = 0.19$, we have that

$$\begin{aligned}
v_1 &= \frac{1 \times 0.4 \times v_{1,1} + 0.19 \times 0.5 \times v_{2,1}}{0.4 + 0.19 \times 0.5} = 0.81 v_{1,1} + 0.19 v_{2,1} \\
v_2 &= \frac{1 \times 0.35 \times v_{1,2} + 0.19 \times 0.3 \times v_{2,2}}{0.35 + 0.19 \times 0.3} = 0.86 v_{1,2} + 0.14 v_{2,2} \\
v_3 &= \frac{1 \times 0.25 \times v_{1,3} + 0.19 \times 0.2 \times v_{2,3}}{0.25 + 0.19 \times 0.2} = 0.87 v_{1,3} + 0.13 v_{2,3}
\end{aligned}$$

Alternatively, individuals 1 and 2 could use **Method II** and compare the weights given to some attribute, say $a = 2$. In this case, $\beta_{1,2} = 0.35$ and $\beta_{2,2} = 0.3$ are the weights, and a bilateral agreement is some $\beta_{12,2} \in (0.3, 0.35)$. If this bilateral agreement is to be consistent with the previous one, then $\beta_{12,2} = 0.342$ so that

$$\alpha_1^2 = \frac{\beta_{2,2} - \beta_{12,2}}{\beta_{2,2} - \beta_{1,2}} = 0.84 \quad \text{and} \quad \delta_1^2 = \frac{\beta_{12,2} - \beta_{1,2}}{\beta_{2,2} - \beta_{12,2}} = 0.19.$$

Finally, **Method III** is based on willingness to pay. Thus, we begin finding the monetary outcomes that individuals 1 and 2 are willing to pay to move x_2 from 0, its worst level, to 100, its best level. By setting

$$(x_1^i, 0, x_3) \sim_i (0, 100, x_3)$$

we find that $x_1^1 = 80.5$ and $x_1^2 = 53.8$. Now a bilateral agreement takes the form of a compromise in this willingness to pay, $x_1^{12} \in (53.8, 80.5)$. If this bilateral agreement is to be consistent with the previous one, then $x_1^{12} = 74.4$ and (17) produces

$$\begin{aligned} \alpha_1^2 &= \frac{\beta_{2,1}[v_{2,1}(x_1^2) - v_{2,1}(x_1^{12})]}{\beta_{1,1}[v_{1,1}(x_1^{12}) - v_{1,1}(x_1^1)] + \beta_{2,1}[v_{2,1}(x_1^2) - v_{2,1}(x_1^{12})]} = 0.84, \text{ and} \\ \delta_1^2 &= \frac{\beta_{1,1}}{\beta_{2,1}} \frac{[v_{1,1}(x_1^{12}) - v_{1,1}(x_1^1)]}{[v_{2,1}(x_1^j) - v_{2,1}(x_1^{12})]} = \frac{0.4}{0.5} \frac{[0.83 - 0.875]}{[0.6 - 0.79]} = 0.19. \end{aligned}$$

Of course, in both Method II and Method III the group utility can also be expressed in terms of group attribute evaluation functions v_a upon computing $\beta_{12,a}$ and using Equation (20).

4 Coalition Agreements

So far, we have emphasized bilateral agreements and the resulting pairwise utilities as the building blocks for the group utility. If a large number of individuals are involved in a group decision, pairwise comparisons may be too onerous. A more practical procedure may be to divide the group into small coalitions $A_1, A_2, ..., A_q$. Then, representatives of each coalition will meet and try to reach an agreement. This is analogous to multiattribute utility theory where trade-offs are generally sought among individual attributes, but sometimes also among subgroups of attributes.

Because our setting treats individuals and coalitions alike, once we have a partition of the group and utilities for each of these partitions, we can proceed as if these coalitions were the "original" individuals. For example, let u_A and u_B be the utilities of two disjoint coalitions A and B. Both u_A and u_B are weighted averages of the individual utilities of the members of A and B, respectively. Now, instead of some pair agreement between some $i \in A$ and some $j \in B$ of the form $u_{ij} = (u_i + \delta_i^j u_j)/(1 + \delta_i^j)$, let the agreement take place at a coalition level. Thus, two representatives from A and B will now try to reach a bilateral agreement at the level of coalitions of the form

$u_{A\cup B} = (u_A + \delta_A^B u_B)/(1+\delta_A^B)$. It is immediately clear that if $q-1$ such agreements take place among a spanning tree of the partition $A_1, A_2, ..., A_q$ of N, then group utility u_N is obtained as before.

The question remains of how such group agreements relate to the hypothetical pair agreements that individuals i and j could have made on their own and that produced the same group utility $u_{A\cup B}$. First, the weight of i in u_A is $\lambda_i/\sum_{k\in A}\lambda_k$ and the weight of j in u_B is $\lambda_j/\sum_{k\in B}\lambda_k$. Because the utility comparison rate between u_A and u_B is δ_A^B, or

$$u_{A\cup B} = \frac{u_A + \delta_A^B u_B}{1+\delta_A^B} = \frac{\sum_{k\in A}\lambda_k u_k/\sum_{k\in A}\lambda_k + \delta_A^B\sum_{k\in B}\lambda_k u_k/\sum_{k\in B}\lambda_k}{1+\delta_A^B}$$

the ratio of weights between u_i and u_j in $u_{A\cup B}$ is

$$\delta_i^j = \delta_A^B \frac{\lambda_j/\sum_{k\in B}\lambda_k}{\lambda_i/\sum_{k\in A}\lambda_k}.$$

Of course, we don't have to restrict ourselves to one partition. We could imagine that an intermediate partition may form (filtrations of N), and agreements are formed at higher levels. We now see a picture where individuals may act as portrayers of their own preferences or become representatives of some coalition (say the different departments in a corporation). In the latter case, the individual should reflect the coalition preferences. We leave the investigation of a more general hierarchical coalition structure, appropriate for organizational decision making, as a future research question.

Conversely, having obtained a group utility u_N, we could derive the hypothetical utility comparison rates between disjoint coalitions implied by the bilateral agreements.

Proposition 5 *Let u_N be a group utility as in (7). If A and B are disjoint coalitions such that $A\cup B = N$, then $\delta_A^B \equiv (\sum_{i\in B}\lambda_i)/(\sum_{i\in A}\lambda_i)$ is the utility comparison rate between coalitions A and B, and $u_{A\cup B} = (u_A + \delta_A^B u_B)/(1+\delta_A^B)$. If A_1, A_2,..., A_q is a partition of N and let $\delta_1^{A_i} \equiv (\sum_{i\in A_i}\lambda_i)$, then $u_N = \sum_{i=1}^q \delta_1^{A_i} u_{A_i}/\sum_{i=1}^q \delta_1^{A_i}$.*

Proof. Simple manipulations of expression (7) yield

$$u_{A\cup B} = \frac{\sum_{i\in A}\lambda_i u_i + \sum_{i\in B}\lambda_i u_i}{\sum_{i\in A\cup B}\lambda_i} = \frac{\left(\sum_{i\in A}\lambda_i\right) u_A + \left(\sum_{i\in B}\lambda_i\right) u_B}{\sum_{i\in A\cup B}\lambda_i} = \frac{u_A + \delta_A^B u_B}{1+\delta_A^B}.$$

■

5 Discussion

The key building block in our theory for aggregating individuals' preferences is the bilateral agreement on a chosen parameter (utility of a consequence, weight of an attribute, willingness to pay, etc.) between two individuals or two coalitions. To reach the bilateral agreement, both parties must compromise. In committee decisions, such compromises are common. Nevertheless, bilateral agreements can be difficult to achieve in some circumstances. An interesting research question is to investigate the case where pairs are unable to reach a complete agreement. Technically, one must relax the completeness assumption at the pair level (Baucells and Shapley 2001). It is quite possible that the preferred set of alternatives is narrowed down considerably even when a consensus cannot be reached on the chosen parameter. The presence of a higher level decision maker or an arbitrator helps a great deal in narrowing down the agreement zone. In a practical application of choosing a new product design for introduction where Marketing assigned a greater weight to early market entry, but R&D assigned a greater weight to product features and favored a somewhat delayed market entry, the presence of the Vice President of the division greatly facilitated an agreement on the importance weights assigned to these attributes (see Sarin, 1993).

We note that our approach may be applicable to decisions under certainty. In this case, individual and group preferences are defined through a measurable value function (Krantz et al., 1971; Dyer and Sarin, 1979a, 1979b). Since the use of lotteries for eliciting utilities and weights may be artificial in decisions under certainty, alternative methods to operationalize our theory need to be explored. One such method may be to seek equal difference points (preference difference between x^h and x is the same as that between x and x^ℓ) and seek compromise on these.

Several authors have noted that many multicriteria problems are resolved in group settings and that these require aggregating individual weights and preferences (Lewis and Butler 1993, Korhonen et al. 1993, Harte et al. 1996, Xanthopulos et al. 2000). Although applied formal analysis faces many practical difficulties (Kasanen et al. 2000), our theory provides a systematic way to elicit group preference from individual preferences. In the widely used additive case, Results (19) and (20) of Proposition 4 provide an interesting alternative representation of the group preference.

Decision analysis has a strong tradition of breaking down complex problems into simple parts and then combining the information collected on these parts to reach a decision. In multiattribute decisions, for example, the multiattribute preference function is built from attribute weights and attribute evaluation functions. The attribute weights, for example, are elicited by restricting attention to trade-offs between two attributes at a time. Independence conditions (utility independence, for example) are used to justify a particular decomposition. In a similar vein, our approach uses the Extended Pareto Rule to build the group preference from individual and pair preferences. Since the aggregation of preferences is at the heart of a group decision problem, we have provided a theoretically sound way to approach the preference aggregation issue at the simplest level.

Finally, once the Extended Pareto Rule is firmly adhered to, our theory can easily accommodate other forms of multiattribute utility functions (multilinear or more general forms). We could consider the case where individual multiattribute utilities are multiplicative (Keeney and Kirkwood 1975, Keeney and Raiffa 1976). In a so called homogeneous case where individuals agree on attribute utilities, but not on weights, the group utility takes the multiplicative form and the group multiplicative scaling constant is a weighted sum of individuals' multiplicative constants. In the heterogeneous case (individuals disagree on both attribute utilities and on weights), a group utility can be obtained through bilateral agreements. The structure of the group utility, however, takes a more complex form and does not simplify to an easy-to-interpret multiplicative form. Further research is needed to generalize and interpret our theory to nonadditive multiattribute utility functions.

We believe that the Extended Pareto Rule is a much more appealing normative requirement than the exact form of the multiattribute utility function - additive, multiplicative or some other. It would, however, be interesting to explore the extensions of our theory to cases where the Pareto Rule is relaxed and the group utility is nonlinear. We hope that our work will spark additional theoretical and applied research into the somewhat neglected, but important, area of group decision making.

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