

# Structural Models of Default and the Cross-Section of Corporate Bond Yield Spreads

Jack Bao\*

January 8, 2009

Job Market Paper

## Abstract

This paper tests the ability of structural models of default to price corporate bonds in the cross-section. I find that the Black-Cox model can explain 45% of the cross-sectional variation in observed yield spreads. The unexplained variation cannot be attributed solely to non-credit components. Specifically, unexplained yield spreads are related to credit risk proxies such as recent equity volatility, ratings, and option expensiveness, suggesting that the model does not fully capture credit risk in the cross-section. Further suggesting that the Black-Cox model does not fully capture credit risk in the cross-section, I find that unexplained yield spreads are related to unexplained CDS spreads. Based on the relation of unexplained yield spreads with option expensiveness and recent equity volatility, I then calibrate a jump diffusion model and a stochastic volatility model, finding that the jump diffusion model weakly improves cross-sectional explanatory power while the stochastic volatility model does not. As the jump diffusion model relies on jump risk premia estimates from equity options, this suggests that the corporate bond and equity option markets price a common risk. However, much of the cross-sectional variation in yield spreads remains unexplained by structural models.

---

\*MIT Sloan School of Management, jackbao@mit.edu. I am grateful for the comments and encouragement provided by my dissertation committee, Gustavo Manso, Jun Pan (chair) and Jiang Wang. I also thank Jason Abaluck, Manuel Adelino, D.H. Bao, Nittai Bergman, Tara Bhandari, Hui Chen, Serdar Dinc, Carola Frydman, Apurv Jain, Scott Joslin, Leonid Kogan, Jiro E. Kondo, Jonathan Lewellen, Stewart Myers, Dimitris Papanikolaou, Neil Pearson, Weiyang Qiu, Antoinette Schoar, Mary Tian, Ngoc-Khanh Tran, and Jialan Wang and participants at the MIT Finance Lunch and the MIT Finance Seminar for helpful comments and discussions. All remaining errors are my own.

# 1 Introduction

The literature on structural models of default, beginning with Merton (1974), has sought to price corporate debt by modeling firm fundamentals and combining equity and debt information to capture the default processes of firms. Since Merton’s work, numerous structural models of default incorporating different assumptions have been developed.<sup>1</sup> Though structural models are appealing from an economic perspective, Huang and Huang (2003) argue that a broad group of structural models underpredict the levels of empirically observed yield spreads.<sup>2</sup> Proposed solutions have included both models that generate larger yield spreads and also non-credit solutions such as liquidity and the choice of benchmark security. It remains unclear how important each component is.

Structural models of default provide predictions about the level of yield spreads as well as the relative yield spreads of different bonds. While there is a large literature on the level of yield spreads, the implications of structural models in *relative* pricing have largely been ignored. This paper focuses on the cross-section to better understand the disconnect between observed and model yield spreads. Cross-sectional tests provide an important metric for evaluating structural models beyond whether they are able to generate larger model yield spreads than previous models. While some structural models cannot generate large levels of spreads, it is possible that they contain mechanisms explaining the cross-section of yield spreads. Furthermore, by examining relations between unexplained yield spreads and credit risk proxies, cross-sectional tests can determine whether a structural model’s unexplained yield spreads are solely due to non-credit components in bond yields or are due to the model failing to fully capture credit risk. Correlations between unexplained yield spreads and credit risk proxies may suggest structural models of default that contain dynamics which can further explain the cross-section of yield spreads.

My cross-sectional tests begin with the Black and Cox (1976) model as the base case. The Black-Cox model captures two fundamental determinants of the likelihood of default, leverage and asset volatility. To calibrate the Black-Cox model, I focus on matching firm-level parameters such as market leverage, equity volatility, and payout ratio. From this balance sheet and equity information, I am able to construct a panel of model yield spreads which can be compared to observed yield spreads. As an initial examination, I consider the cross-sectional explanatory power of the Black-Cox model spread, finding a within-group  $R^2$

---

<sup>1</sup>See for example Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), Anderson and Sundaresan (1996), Collin-Dufresne and Goldstein (2001), and Goldstein, Ju, and Leland (2001).

<sup>2</sup>See also studies by Eom, Helwege, and Huang (2004), Ericsson, Reneby, and Wang (2005), Cremers, Driessen, and Maenhout (2006), and Huang and Zhou (2007).

of 44.9% when regressing observed yield spreads on Black-Cox yield spreads. Thus, I find that over half of the cross-sectional variation in yield spreads remains unexplained by the Black-Cox model. This could be for two reasons. First, the remaining unexplained yield spread could be due to elements that structural models are not designed to capture such as liquidity, transitory price movements, and other non-credit components. Alternatively, the unexplained portion of yield spreads could reflect the Black-Cox model's inability to fully capture credit risk in the cross-section.<sup>3</sup>

To better understand the cross-sectional variation of observed yield spreads, and particularly, the variation that is not explained by the Black-Cox model, I examine the relation between unexplained yield spreads and credit risk proxies such as recent equity volatility and ratings. After controlling for Black-Cox yield spreads, observed yield spreads are related to these proxies of credit risk. The cross-sectional effects of these credit risk variables are economically large. For example, a move from the 25th to 75th percentile in recent equity volatility accounts for a 48 basis point difference in unexplained yield spreads. There is also evidence that unexplained yield spreads are related to option expensiveness, a proxy for credit risk premia. These cross-sectional relations suggest different structural models which may potentially improve our understanding of the cross-section of yield spreads. I explore this in two directions, through a jump diffusion model and also through a stochastic volatility model.

First, the relation between unexplained yield spreads and option expensiveness suggests that there may be a common risk premium priced in the equity option and corporate bond markets beyond a standard diffusion risk premium. I calibrate a jump diffusion model to address this relation. I use individual equity option expensiveness to infer jump risk premia firm-by-firm and then use this information to construct model yield spreads.<sup>4</sup> My results indicate that there is weak evidence that this model helps to explain the cross-section of yield spreads above and beyond the Black-Cox model. Constructing a residual from a regression of jump model spreads on Black-Cox spreads, I find that a one standard deviation change in this residual corresponds to a 16 basis point difference in observed yield spreads. However, the improvement in the within-group  $R^2$  when including jump residuals in a regression of observed yield spreads on Black-Cox yield spreads is only from 43.4% to 45.3%. Despite this limited improvement in cross-sectional explanatory power, it is still interesting that the corporate bond and equity option markets seem to price a common risk.

The limited ability of the jump model to explain the cross-section above and beyond a

---

<sup>3</sup>Of course, these two explanations are not mutually exclusive.

<sup>4</sup>Cremers, Driessen, Maenhout, and Weinbaum (2006) document a relation between equity option implied volatilities and skews and yield spreads.

Black-Cox model presents an important contrast to its ability to explain the level of yield spreads over the Black-Cox model. In a calibration using ratings-level information, Cremers, Driessen, and Maenhout (2006) find that a jump model with an equity index option-implied jump risk premium greatly increases the level of yield spreads relative to a diffusion-only model. Using individual equity options, I also find that the jump model helps to explain the level of yield spreads, though the improvement is somewhat smaller in my study due to the use of individual equity options rather than index options.

The relation between unexplained yield spreads and recent equity volatility suggests a second potential solution to unexplained yield spreads, a stochastic volatility (Heston (1993)) model. Such a model has richer volatility dynamics that incorporate both recent and long-run volatility. This contrasts with the Black-Cox model which uses only a constant long-run volatility. In addition, the stochastic volatility model can generate further cross-sectional variation in yield spreads through across-firm differences in the correlation between asset return and variance shocks. This correlation is typically thought to be negative, meaning that asset variance is high exactly when asset returns are low. Thus, larger shocks to asset value are more likely to occur when asset value is low, increasing the probability of left-tail events. For firms sufficiently far from default, this will generate a greater yield spread through a greater probability of default.

I find that the stochastic volatility model cannot solve the puzzle of strong correlation between yield spreads and recent equity volatility. The mean estimate of the mean-reversion parameter of asset variance is 11.7, indicating a half-life of 0.71 months. Since bond maturities are typically multiple years, it will be the long-run asset variance that has a strong effect on bond pricing rather than short-run variance. In addition, the correlation between asset returns and variance,  $\rho$ , has little explanatory power for the cross-section of observed yield spreads. Estimates of  $\rho$  are generally negative with a mean of -0.15, a magnitude that is significantly smaller in economic terms than for equity indices.<sup>5</sup> Controlling for Merton model spreads<sup>6</sup>, firms with more negative values of  $\rho$  have bonds with larger stochastic volatility model spreads, but these bonds do not have higher observed yield spreads. I further explore time-varying volatility by pricing bonds with a slower variance mean-reversion parameter and also by calibrating a Black-Cox model using only recent equity volatility. The former calibration does not help explain the cross-section of observed yield spreads while the latter does provide some additional explanatory power.

I also consider the cross-sectional pricing of credit default swaps (CDS). CDS have

---

<sup>5</sup>Pan (2002) estimates this correlation to be -0.57 for the S&P 500 and a stochastic volatility model without a volatility premium.

<sup>6</sup>For the stochastic volatility model, the Merton model is the correct benchmark as the Heston model does not capture a first passage time default.

similar credit risk exposures as corporate bonds, but are thought to have different non-credit risk exposures. Whereas the primary non-credit component in the corporate bond market is liquidity, the primary non-(firm)credit component in the CDS market is counterparty risk. The results for CDS are similar to corporate bonds: unexplained CDS spreads are related to recent equity volatility, ratings, and option expensiveness. In addition, I find that unexplained CDS spreads are significantly related to unexplained corporate bond yield spreads. The within-group  $R^2$  of a regression of unexplained CDS spreads on unexplained corporate bond yield spreads is 49.6%, smaller than the  $R^2$  of 70.2% for a regression of CDS spreads on corporate bond yield spreads, but still quite economically significant. Since the most important common exposure of credit default swaps and corporate bonds is credit risk, this commonality provides further evidence that the Black-Cox model does not fully capture credit risk in the cross-section. Though part of this commonality could be due to commonality in liquidity, the magnitude of the commonality suggests that it is at least in part due to common unexplained credit risk.

My focus on the cross-section of yield spreads is related to the reduced-form regressions framework used by Collin-Dufresne, Goldstein, and Martin (2001) and Campbell and Taksler (2003). CDGM find a common component in yield spread changes that they are unable to relate to standard macroeconomic and financial variables.<sup>7</sup> Campbell and Taksler document a relation between observed yield spreads and equity volatility and argue that the relation is too strong to be explained by a structural model such as Merton's (1974). A reduced-form regression framework is parsimonious and allows researchers to determine what yield spreads are related to. However, my focus is on testing whether a theoretically-founded measure of risk – a structural model of default – can explain the cross-section of empirically observed yield spreads.<sup>8</sup>

Another related strand of literature is on the relation between yield spreads and liquidity. Since structural models of default are not designed to capture liquidity, it is likely that unexplained yield spreads are related to liquidity proxies. Multiple studies have shown that corporate bond yield spreads are negatively related to proxies for liquidity.<sup>9</sup> In particular, most studies have focused on age and issued amount as proxies for liquidity. Consistent with this literature, I find that observed yield spreads are negatively related to liquidity in the cross-section even after controlling for Black-Cox yield spreads. In particular, moving from

---

<sup>7</sup>See Avramov, Jostova, and Philipov (2007) for a follow-up which argues that yield spread changes are indeed related to changes in firm characteristics.

<sup>8</sup>Bharath and Shumway (2008) examine the default prediction of Moody's KMV model, a model based on the Merton model. Though this is a theoretically-founded measure of risk, it only reflects one-year P-measure default probabilities and is not designed to price corporate bonds.

<sup>9</sup>See Houweling, Mentink, and Vorst (2005) and references therein.

the 25th to the 75th percentile of bond age corresponds to a 10 to 15 basis point difference in unexplained yield spreads. Recently, Longstaff, Mithal, and Neis (2005) and Nashikkar, Subrahmanyam, and Mahanti (2007) have studied the relation between yield spreads and liquidity by first using CDS to control for credit risk and find evidence that the non-default component of yield spreads is related to liquidity. Their motivation for using CDS to control for credit risk is based on the fact that CDS are considered to be very liquid and is similar to my motivation for considering commonality in unexplained CDS and corporate bond spreads above.

The remainder of the paper is organized as follows. Section 2 describes the data and provides summary statistics. Section 3 details the base case calibration and the relation between unexplained yield spreads and firm- and bond-level characteristics. Section 4 presents calibrations for a jump diffusion model. Section 5 presents calibrations for a stochastic volatility model. Section 6 briefly discusses endogenous default models. Section 7 presents results on credit default swaps and commonality between CDS and corporate bonds. Section 8 concludes.

## 2 Data and Summary Statistics

### 2.1 Data Sources

The bond pricing data used in this paper is obtained from FINRA's TRACE (Transaction Reporting and Compliance Engine). Originally introduced by the National Association of Securities Dealers (NASD), TRACE was instituted to provide price transparency in the corporate bond market.<sup>10</sup> Beginning in July 2002, there has been a gradual increase in reporting requirements in the secondary corporate bond market that has required the reporting of approximately 99% of all public transactions starting on February 7, 2005. From TRACE, I am able to construct quarterly observed bond yields for 2003 to 2007. For my sample, I filter out canceled, corrected, and special trades and also drop cases where prices are obviously misreported. In addition, I only use pricing data from trades that are at least \$100k in face value as both Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2008) find that smaller trades are subject to larger transitory price movements. From the Fixed Income Securities Database (FISD), I obtain bond characteristics that include flags for callability, putability, and convertibility along with various other characteristics such as issuance, offering date, and coupons. I drop callable, putable, and convertible bonds and

---

<sup>10</sup>Corporate bonds are largely traded via a non-centralized OTC system. See Biais and Green (2007) for a detailed discussion of the evolution of bond trading.

also bonds with variable coupons from my sample. From MarketAxcess, I obtain the characteristics of benchmark treasuries and from Datastream, the prices of these treasuries. From the TRACE, FISD, MarketAxcess, and Datastream data, I am able to compute observed yield spreads. For each issuer, I keep one bond that is close to four-years maturity and one bond that is close to ten-years maturity. This is done so that the results below are not due to a handful of firms that have an extremely large number of issues.

The remaining data used consists mostly of firm-level characteristics used to construct model yield spreads and other firm and equity characteristics.<sup>11</sup> These are obtained from standard sources such as CRSP and Compustat.

## 2.2 Variable Construction: Model Inputs

In calibrating a structural model, the basic inputs are the same as that of options,  $\frac{K}{V}$ ,  $T$ ,  $r$ ,  $\delta$ , and  $\sigma_v$ . As will be described below,  $\frac{K}{V}$ , the default boundary over the total value of the firm and  $\sigma_v$ , the asset volatility, will be calibrated to match market leverage and equity volatility, respectively. Market leverage is constructed as the market value of debt divided by the sum of the market value of debt and the market value of equity. The market value of equity is constructed as the product of share price and shares outstanding from CRSP. It should be noted from Table 1 that my sample contains firms with larger than average equity market capitalization. The median of \$13.27 billion is larger than the 80th percentile cutoff of \$9 billion for NYSE firms as of December 2007. Since over 70% of the firms in my sample are S&P 500 firms, the large mean market capitalization is unsurprising. Market value of debt is constructed as the sum of long-term debt and debt in current liabilities multiplied by a firm's average market value of debt to face value of debt ratio calculated from TRACE data. Equity volatility is constructed using the volatility of daily log equity returns. The maturity of a firm,  $T$ , is constructed as the average duration of a firm's outstanding bond issues where the appropriate Lehman rating index is used as the interest rate for discounting purposes. The mean firm maturity in my sample is slightly more than six years. The interest rate,  $r$ , is calculated as the average interest rate over the last ten years. Finally, the firm payout ratio,  $\delta$ , is calculated as a firm's annual dividends plus annual coupon payments divided by total firm value.

---

<sup>11</sup>Following Fama and French (2001), I drop utilities (SIC codes 4900-4949) and financial firms (SIC codes 6000-6999).

## 2.3 Firm and Bond Characteristics

Table 1 also contains summary statistics for a number of additional firm and bond characteristics. The average implied volatility of a short-term out-of-the-money (OTM) put option and a short-term at-the-money (ATM) put option minus recent realized equity volatility,  $IV - \text{equity volatility}$ , is meant to capture option expensiveness. The average implied volatility tends to be greater than recent realized volatility which is unsurprising given results that options, and in particular, OTM puts tend to be overpriced (relative to realized equity volatility and Black-Scholes). Firm age is calculated as the number of years since the firm's BEGDT obtained from CRSP's msfhdr file. Firms in my sample are mostly mature firms with an average age over 40 years. Return on assets for a quarter are calculated as income before extraordinary items divided by the mean of quarter-start and quarter-end total assets. The firms in my sample tend to be profitable with a mean quarterly ROA of 1.35% as compared to an average of -0.9% for the full Compustat universe during the time period I study. Equity beta is calculated using a standard 60-month rolling window and the estimates for my sample do not exhibit unusual properties. Asset tangibility is constructed using the estimates from Berger, Ofek, and Swary (1996) as  $0.715 \text{ Receivables} + 0.547 \text{ Inventory} + 0.535 \text{ Capital} + \text{Cash Holdings}$ , scaled by total book assets. Interest coverage is calculated as earnings before interest and taxes divided by the interest expense. Finally, deviations from historical leverage are calculated as the difference between a firm's mean leverage over the 10 years prior to my sample minus its current leverage.

Most of the bonds in my sample have an A or Baa rating, with the average rating being Baa1. The average amount outstanding is approximately \$300 million in face value, which is larger than the average bond in FISD which has an average face value of approximately \$170 million.<sup>12</sup> Mean trade sizes are over half a million dollars in face value and trades on average occur on less than half the days in which the bonds are in the sample.

## 3 Base Case and Cross-Sectional Tests

As a base case, I calibrate a Black-Cox model. This model has the advantage of capturing both leverage and asset volatility while still having closed-form solutions for derivative prices and probabilities of default. It also improves on the Merton model in that default occurs the first time that asset value falls below a boundary rather than only if firm value is below the face value of debt at maturity. Such a framework is perhaps more intuitive for pricing coupon bonds as a firm's value can be below  $K$  at time  $t$  and above  $K$  at time  $s$  where  $s >$

---

<sup>12</sup>See Bao and Pan (2008).

t in the Merton model. This would allow a firm to be in default at one point in time and solvent at a future point in time.

### 3.1 Asset Value Process and Claims on Cashflow

The asset value process is a standard Geometric Brownian Motion,

$$\frac{dV_t}{V_t} = (\pi^v + r - \delta)dt + \sigma_v dW_t^v, \quad (1)$$

where  $\pi^v$  is the asset risk premium,  $r$  is the risk-free rate,  $\delta$  is the payout ratio, and  $\sigma_v$  is the constant asset volatility. Under the risk-neutral measure, the asset value process is a Geometric Brownian Motion with  $\pi^v = 0$ . If the total firm value falls below  $K$ , the face value of debt, the firm defaults and  $(1 - R_{firm})K$  is lost in bankruptcy. Firm-level recovery is set at 80% to be consistent with Kaplan and Andrade's (1998) finding that the cost of financial distress is approximately 15 to 20% of firm value. For each firm, the issuance-weighted average duration of its outstanding debt is taken to be the firm's "maturity",  $T$ . If the firm is solvent at  $T$ , debtholders receive  $K$  and equityholders receive the remaining firm value. Given these claims to cashflows, modeling the firm involves pricing three components, (1) equity, (2) debt, and (3) bankruptcy costs.

#### 1. Equity

At maturity, equity is the residual claimant if the firm is still solvent. With a first-passage time model and the face value of debt used as the default boundary, equity is always in the money if the firm is solvent and is paid  $V_T - K$ . This piece of equity is equivalent to a down-and-out call option. Equity is also a claim on dividends that the firm pays out.

#### 2. Debt

At maturity, debt receives  $K$  if the firm is solvent and  $R_{firm}K$  if the firm has defaulted. Additionally, debt is also a claim on coupon payments.

#### 3. Bankruptcy costs

If the firm defaults, there is a sunk bankruptcy cost of  $(1 - R_{firm})K$ .

Equity and debt at maturity and bankruptcy costs have closed form solutions.<sup>13</sup> Dividends and coupons are claims on the remaining asset value. To divide the remaining asset value between equity and debt, I construct the total payout ratio,  $\delta$ , and the equity and debt

---

<sup>13</sup>See Appendix B and also Bjork (2004).

payout ratios,  $\delta_e$  and  $\delta_b$ , respectively.  $\delta_e$  is the value of dividends paid divided by firm value while  $\delta_b$  is equal to coupon payments divided by firm value.  $\frac{\delta_e}{\delta}$  is then the proportion of the remaining firm value attributed to equity.

### 3.2 Calibration

For each firm, the values of  $\sigma_v$  and  $(\frac{K}{V})_t$  are determined by matching model-implied values of equity volatility and market leverage to observed values:

$$\sigma_E^2 = \left( \frac{\partial \log E}{\partial \log V} \right)^2 \sigma_v^2 \quad (2)$$

$$\text{Market Leverage}_{\text{empirical}} = \frac{\text{Model Debt}(\frac{K}{V}, \sigma_v, T, r, \delta)}{\text{Model Debt}(\frac{K}{V}, \sigma_v, T, r, \delta) + \text{Model Equity}(\frac{K}{V}, \sigma_v, T, r, \delta)}$$

In the Black-Cox model,  $\sigma_v$  is constant while  $\frac{K}{V}$  is time-varying due to its denominator. This, along with the time-varying nature of  $\frac{\partial \log E}{\partial \log V}$ , leads to two important modeling choices. First, equity volatility is time-varying in the model even though asset volatility is constant. Thus, it is necessary to calculate equity volatility using short enough time periods so that  $\frac{\partial \log E}{\partial \log V}$  does not change much. Changes in  $\frac{\partial \log E}{\partial \log V}$  are largely driven by changes in the firm's market leverage, which typically does not change much at short horizons. Thus, I calculate equity volatility quarterly using daily data. An alternative strategy would be to estimate equity volatility at even shorter horizons such as by using minute-by-minute data. However, as documented in Appendix A, even small bid-ask bounces can generate large equity volatility when such high frequency data is used.

The second important modeling choice I make is to aggregate the volatility equation over time. In the simplest scheme, the two equations could be simultaneously solved period-by-period.<sup>14</sup> However, this would give a time-varying asset volatility, which would be an inconsistent application of the Black-Cox model. In addition, period-by-period estimates may be imprecise due to noisy estimates of equity volatility. Thus, I also extend the volatility data back to 1993 for 15 years of total data, so that estimates of  $\sigma_v$  will be more precise. I solve for asset volatility so that the volatility equation holds on average:

$$\sigma_v^2 = \frac{\sum_t \sigma_{E,t}^2}{\sum_t \left( \frac{\partial \log E}{\partial \log V} \right)_t^2}$$

---

<sup>14</sup>In section 5.3.2, I explore such an estimation strategy.

Since  $\frac{K}{V}$  is time-varying in the model, I allow for one market leverage equation per quarter. I then simultaneously solve for  $\sigma_v$  and a time-series of  $(\frac{K}{V})_t$  through a single volatility equation and a market leverage equation for each quarter.

Important distinctions between my calibration methodology and that of Huang and Huang (2003) are that I calibrate firm-by-firm and that I calibrate to match equity volatilities rather than historical probabilities of default.<sup>15</sup> Because they are concerned with the level of the yield spread for typical firms, Huang and Huang calibrate by rating to the average firm balance sheet information within a rating and the historical probability of default for the rating. I substitute firm-level balance sheet information and equity volatility. Huang and Huang's choice to match historical default probabilities is based on generating model-implied credit spreads with empirically reasonable parameters. Matching equity volatilities is consistent with the spirit of this goal. In addition, there are two important reasons to match equity volatility rather than the historical probability of default. First, it is not necessarily true that historical default probabilities reflect forward-looking default probabilities. In fact, Huang and Huang indicate that they would ideally calibrate their models at each time period separately to reflect each period's default probability, but are constrained by the limited data on defaults. Second, from a practical perspective, default probabilities, by definition, are not available on a firm-by-firm basis. Since Huang and Huang calibrate by ratings, they are able to match the historical default probability for each rating. By matching each individual firm's default probability to the historical probability of default for its rating, I would prevent structural models from explaining cross-sectional variation in yield spreads within a rating. Matching historical default probabilities would also hard-code model yield spreads to be sorted cross-sectionally by rating. One possible method to estimate firm-by-firm default probabilities would be to use the logit method in Campbell, Hilscher, and Szilagyi (2007). However, they already use equity volatility and leverage as inputs and thus, do not provide a theoretical improvement over simply using a structural model's predicted defaults.

After firm-level parameters are calculated, pricing a coupon bond is an application of risk-neutral pricing. Specifically, a T-year bond with semi-annual coupons  $c$  and face value of \$1 has a model price of:

$$B_t = \sum_{i=1}^{2T} \frac{c}{2} e^{-ri/2} P^{i/2} + e^{-rT} P^T + \sum_{i=1}^{2T} R_{bond} e^{-ri/2} (P^{(i-1)/2} - P^{i/2}) \quad (3)$$

---

<sup>15</sup>In Appendix C, I consider the default probabilities implied by the model in my calibrations.

where  $P^t$  is the risk-neutral probability that a firm is still solvent at  $t$  and  $R_{bond}$  is the recovery rate of the bond.<sup>16</sup> The first term in the bond pricing equation is the value of coupon payments and the second term is the value of the face value. The third term is the value of the recovery given default.  $P^{(i-1)/2} - P^{i/2}$  represents the risk-neutral probability of default between times  $\frac{i-1}{2}$  and  $\frac{i}{2}$ . The cumulative risk-neutral probability of default is:

$$1 - P^t = N \left( \frac{-\log \left( \frac{V}{K} \right) - \left( r - \delta - \frac{\sigma_v^2}{2} \right) t}{\sigma_v \sqrt{t}} \right) \quad (4)$$

$$+ \exp \left( \frac{-2 \log \left( \frac{V}{K} \right) \left( r - \delta - \frac{\sigma_v^2}{2} \right)}{\sigma_v^2} \right) N \left( \frac{-\log \left( \frac{V}{K} \right) + \left( r - \delta - \frac{\sigma_v^2}{2} \right) t}{\sigma_v \sqrt{t}} \right)$$

### 3.3 Calibration Results and Cross-Sectional Tests

I assess the performance of the Black-Cox model by first testing whether the credit spread puzzle on levels established by Huang and Huang (2003) holds in my sample of bonds. Because I construct a panel of model-implied yield spreads, my exercise is an empirical exercise in which I am able to provide t-stats.<sup>17</sup> In Table 2, I confirm that observed yield spreads are too low to be explained solely by the Black-Cox model. The mean difference between observed and model spreads is 90 basis points for four-year bonds and 88 basis points for ten-year bonds. Compared to mean observed yield spreads of 160 basis points and 176 basis points, respectively, the results are very economically significant. In addition, both differences have t-statistics that are significant at the 1% level when standard errors are clustered by firm and time. Differences are also significant for each rating with the exception of four-year poorer-rated bonds for which the sample size is small.

The results in Table 2 suggest that either (1) the model does not properly account for the credit risk inherent in corporate bonds or (2) part of the level of yield spreads is due to liquidity or some other non-credit risk factor. Simply comparing the levels of observed and model yield spreads does not allow one to distinguish between these two competing, but not mutually exclusive hypotheses. However, focusing on the cross-section, I can assess the

---

<sup>16</sup>Note that the bond recovery rate and the firm recovery rate do not have to be the same. In fact, Andrade and Kaplan (1998) find that the cost of financial distress is approximately 15-20% of firm value at bankruptcy. Carey and Gordy (2007) find that recovery for senior unsecured debt is slightly above 50%. The disparity between overall firm recovery and senior debt recovery is largely due to bank debt having priority over public debt. Here, I set the bond-level recovery equal to 50%, but consider setting recovery rates by industry in section 3.6.

<sup>17</sup>Huang and Huang (2003) calibrate by ratings using the mean firm fundamentals for a rating. Thus, their conclusion is about the economic significance of the difference between model and observed yield spreads.

ability of the Black-Cox model to capture credit risk in the cross-section through a regression framework. To test the model in the cross-section, I use the framework:

$$\text{observed yield spread}_{it} = \alpha_t + \alpha_1 \text{model yield spread}_{it} + \varepsilon_{it} \quad (5)$$

From this regression, I examine how much of the cross-section of observed yield spreads is explained by the model yield spread and also construct the unexplained yield spread,  $\hat{\varepsilon}_{it}$ . This residual is orthogonal to the model yield spread by construction and can be thought of as the portion of the observed yield spread unexplained by the model. I examine the relation between the unexplained yield spread and various credit risk proxies, liquidity proxies, and firm-level characteristics in this and the following sections. This methodology is similar in spirit to the cross-sectional anomalies literature for equity returns.<sup>18</sup> In the literature on equity returns, stocks are typically sorted on a characteristic into portfolios. If there is a spread in the return based on the characteristic and the portfolios have similar risk (typically measured by  $\beta$ ), an anomaly has been discovered (and is potentially an indication that the underlying risk model does not fully capture risk in the cross-section). Here, I explicitly control for credit risk by first partialing it out before considering relations with firm-level and bond-level characteristics. An alternative methodology would be to include characteristics in the left-hand side of equation (5). However, given the large correlations of the model yield spread with credit risk proxies, I instead first partial out the model yield spread to allow it the best chance to succeed.

Running the regression from equation (5), I find that the coefficient of the model yield spread is 0.58 with a robust t-stat of 8.00. It is statistically different from both 0 (the case where the model contains no information) and 1 (the case in which the model yield spread and observed yield spread move one-for-one). I find a within-group  $R^2$  of 44.9%, indicating that the model yield spread explains almost half of the cross-sectional variation in the observed yield spread. Though this  $R^2$  does provide a sense of the ability of the model to explain the observed yield spread in the cross-section, it does not on its own allow a determination of whether the model successfully explains the cross-section. The remaining unexplained observed yield spread could potentially be due to liquidity, transitory price movements, and other non-credit components that structural models are not designed to capture.

To examine whether the model can fully capture credit risk in the cross-section, I regress the unexplained yield spread on proxies for credit risk and credit risk premia in the final four columns of Table 3, Panel A. The residuals from a regression of the observed yield spread on the model yield spread are, by construction, orthogonal to the model yield spread. These

---

<sup>18</sup>See Lakonishok, Shleifer, and Vishny (1994) for a typical example.

unexplained yield spreads should be unrelated to risk proxies if the model has sufficiently controlled for credit risk in the cross-section. The unexplained spreads are significantly related to leverage, recent equity volatility, and ratings, though the relation to leverage is statistically insignificant once recent equity volatility is included. Using the first partial derivative of the Merton model with respect to equity volatility and average firm parameters, Campbell and Taksler (2003) find that the theoretical sensitivity of yield spreads to equity volatility is smaller than suggested by their regression framework. The results in Table 3 formalize this finding for firm-by-firm model-implied yield spreads. It is important to note that this relation holds even after controlling for market leverage and the model estimate of asset volatility, the two major theoretical determinants of credit risk in the Black-Cox model. In unreported results, I also find that unexplained spreads are related to the difference between recent equity volatility and the mean of historical equity volatility. A move from the 25th to 75th percentile in recent equity volatility in the sample corresponds to a 48 basis point difference in the unexplained yield spread. A move of three points in rating (which would be equivalent to moving from A1 to Baa1) corresponds to a 50 basis point difference in the unexplained yield spread. Both of these quantities are very significant economically compared to the average observed yield spread of approximately 165 basis points. Finally, once recent equity volatility is controlled for, the unexplained yield spread is significantly related to option expensiveness as proxied for by option-implied volatility minus equity volatility.

Table 4 contains a further examination of the relation between observed yield spreads and recent equity volatility. Bonds are first sorted by their Black-Cox yield spread into quintiles and quintiles 1 and 2 are combined as both groups have small model yield spreads. Within each group, bonds are then sorted by past three months' equity volatility. Thus, within each model yield group, I can examine the difference in observed yield spread with the model yield spread reasonably well controlled for. For model yield spread quintiles 1 & 2, 3, and 4, there is a monotonic trend of increasing observed yield spread with increasing recent equity volatility while the model yield spread remains relatively flat. The magnitudes of the differences between quintile 5 and 1 of equity volatility are economically large as the observed yield spread for quintile 5 is close to twice as large as the observed spread for quintile 1.

The overall conclusion from tests of the Black-Cox model is that while the model does capture a significant amount of cross-sectional variation in yield spreads, it does not fully capture cross-sectional differences in credit risk. Even after controlling for the model, observed yield spreads are related to recent equity volatility, equity option expensiveness, and ratings. The relation to recent equity volatility suggests that the use of a stochastic

volatility model might be useful while the relation to option expensiveness suggests the use of a model with an additional, non-diffusion, risk premium.

### 3.4 Additional Firm Characteristics

I examine the relation between the unexplained yield spread and additional firm characteristics in Panel B of Table 3. My focus is on seven firm characteristics in addition to the credit risk proxies considered previously: firm size (measured by equity market capitalization or total firm value), firm age, return on assets, equity beta, asset tangibility, interest coverage, and deviations from historical leverage. The relations between these variables and unexplained yield spreads may suggest further modeling assumptions needed in a structural model to explain the cross-section of yield spreads. In addition, the relations between these variables and ratings may also shed light on the relation between unexplained yield spreads and ratings.

Firm size potentially affects yield spreads through multiple channels. First, there may be less asymmetric information about accounting information for large firms, leading to lower yield spreads. This would be consistent with Duffie and Lando's (2001) incomplete accounting information model. Second, larger firms may have better reputations in the debt market, decreasing their cost of borrowing. Diamond (1989) formalizes the relation between reputation and borrowing costs in a model of project selection. Finally it is possible that the debt market simply exhibits a size effect similar to the equity market.

Firm age is also a potential proxy for reputation as older firms that continue to borrow have presumably established a positive reputation. Profitability, as measured by ROA, is included as Moody's explicitly acknowledges<sup>19</sup> using profitability in assigning ratings. Equity beta is included as a proxy for how much the firm moves with the market and would be expected to be positively related to yield spreads. Chen (2008) constructs a model in which the (endogenous) default boundary is higher in bad times. This is driven by higher risk premia and lower expected growth rates in bad times. Equity beta is used to capture the former effect as firms which move more closely with the market have higher risk premia exactly when the aggregate risk premium is higher. In addition, I also construct a downside beta,  $\beta^-$ , as in Ang, Chen, and Xing (2006). Downside beta reflects the co-movement of a firm's equity with the market, conditional on a below average market return. Asset tangibility is included as an explanatory variable as I assume 50% recovery in my calibrations as a simplification. If there is in fact cross-sectional variation in expected recovery rates, yield spreads should be negatively related to tangibility. Interest coverage is included as

---

<sup>19</sup>See Fons, Cantor, and Mahoney (2002).

a proxy for corporate liquidity. While firms can theoretically pay interest expenses by liquidating assets, frictions make the ability for firms to pay interest expenses from earnings important. Kim, Ramaswamy, and Sundaresan (1993) present a model in which bankruptcy occurs when a firm's net cashflow cannot cover interest expenses. Finally, the deviation from historical leverage is included as firms may have mean-reverting leverage. A firm with leverage lower than its historical average might be expected to increase its leverage. Thus, just using its current leverage in pricing would generate a model yield spread that is too low. Collin-Dufresne and Goldstein (2001) present a model with a mean-reverting leverage ratio.

When ratings are not controlled for (columns 1 to 4 of Table 3, Panel B), unexplained yield spreads are negatively related to firm size and asset tangibility. Both relations are economically significant as moving from the 25th percentile in equity market capitalization to the 75th percentile results in a 32 basis point decrease in unexplained yield spreads and a similar move for asset tangibility corresponds to an 11 basis point decrease in unexplained yield spreads. The result for asset tangibility indicates that there is some cross-sectional variation in recovery rates that my analysis does not capture. Firm age, return on assets, equity beta, interest coverage, and deviations from historical leverage are statistically insignificant. In unreported results, I also find that substituting downside beta for beta does not change these findings.

In columns 5 and 6 of Panel B, I include ratings as a control and find that unexplained yield spreads are no longer related to firm size and asset tangibility. As shown in the final two columns of Panel B, ratings are related to both firm size and asset tangibility. This relation is particularly strong for firm size as a move from the 25th to 75th percentile in equity market capitalization corresponds to a two point improvement in rating (i.e. A3 to A1). These results suggest that at least in part, the relation between unexplained yield spreads and ratings can be explained by firm size and the ability of ratings to capture expected recoveries in the case of default. Interestingly, when I regress unexplained yield spreads on the firm characteristics considered in this section and the credit risk proxies considered previously, the only statistically significant variables are return on assets (at the 10% level with the wrong sign), ratings, recent equity volatility, and option expensiveness. This further affirms the results in section 3.3 that the credit risk proxies considered are important in understanding unexplained yield spreads.

### 3.5 Liquidity<sup>20</sup>

In Panel C of Table 3, I consider the cross-sectional variation in the unexplained yield spread due to liquidity variables. As structural models of default are designed to capture credit risk and not liquidity, unexplained yield spreads should be positively related to illiquidity. Older bonds are thought to be less liquid because a larger fraction of their issuance is likely to have been acquired by buy-and-hold investors. The results indicate that older bonds indeed have larger yield spreads. A move from the 25th percentile to the 75th percentile of age corresponds to approximately a 10 to 15 basis point difference in the unexplained yield spread. Larger issues are thought to be more liquid and this is consistent with the finding that larger issues have lower yield spreads. The difference in spreads between a bond at the 25th percentile and a bond at the 75th percentile of amount outstanding is approximately 10 basis points.

In addition to characteristic-based liquidity variables, I also consider trading-based variables. Total volume traded is positively related to yield spreads, but is insignificant. The number of trades is positively related to yield spreads, a surprising result if one believes that more liquid bonds trade less often. A move from the 25th percentile to 75th percentile in number of trades is associated with an unexplained yield spread that is 12 basis points higher. However, traders are more likely to break-up trades for less liquid issues, resulting in smaller trading sizes and a larger number of trades. This is consistent with the negative (albeit insignificant) sign on the average trade size. Finally, turnover and percent of days traded are positively related to yield spreads. Overall, it seems that part of the unexplained cross-sectional variation in observed yield spreads is due to liquidity effects, particularly those effects measured by bond age and amount outstanding.

### 3.6 Recovery Rates

My calibration of the Black-Cox model is able to capture cross-sectional variation in leverage and asset volatility, but does not capture cross-sectional variation in recovery rates. I have set firm-level recovery rates to 80% to be consistent with Andrade and Kaplan's (1998) estimate of the cost of financial distress being between 15 and 20% and bond-level recovery at 50% to be roughly consistent with Carey and Gordy (2007). The assumption about firm-level recovery has little effect in the calibration as it enters only in the first stage when firm-level parameters are inferred from equity and balance sheet information. Since this part of the calibration hinges largely on equity information and equity has no recovery in default, the impact of firm-level recovery is minimal. However, the latter assumption may

---

<sup>20</sup>See Houweling, Mentink, and Vorst (2005) for an examination of different liquidity proxies. Issued amount and age are the most popular liquidity proxies in the literature that the authors survey and also the most applicable here.

potentially be important in bond pricing if there is significant cross-sectional variation in expected recovery rates.

Schuermann (2004) provides a survey of the literature on loss given default ( $= 1 - \text{recovery rate}$ ). He finds three main factors that drive differences in loss given default: (1) seniority and collateral, (2) the business cycle, and (3) the industry of the firm. Priority is important as bank loans have higher recovery rates than corporate bonds and senior bonds have higher recovery rates than subordinated and junior bonds. In addition, senior secured bonds have higher recovery rates than senior unsecured bonds. The vast majority of my sample ( $\approx 97\%$ ) is classified as senior and unsecured by FISD, indicating that it is unlikely that the cross-sectional results presented above are driven solely by differences in seniority or collateral. A caveat is that without full information about the debt structure of companies in my sample, I cannot distinguish between companies with high or low levels of bank loans. Companies with high levels of bank loans would potentially have lower recovery rates on corporate bonds as their corporate bonds are junior to a larger fraction of the firm's debt.

The second determinant of recovery rates, the business cycle, is a variable that predicts time-variation in recovery rates. Altman, Resti, and Sironi (2004) find that recovery rates are lower in bad times. Acharya, Bharath, and Srinivasan (2007) find a fire-sales effect. Specifically, the recovery rate for a firm is lower if its industry is in distress. These effects suggest that in bond pricing equation (3), the risk-neutral recovery rate should in fact be lower than the P-measure recovery rate, further complicating the treatment of the recovery rate. Calculating a risk-neutral expected recovery rate requires the assumption of a model and also the use of a credit-sensitive security for calibration purposes. This implicitly chooses a risk-neutral expected recovery rate for which the model's pricing is exactly correct, effectively imposing that all remaining variation in yield spreads is due to differences in recovery rates. Since I am examining the ability of these models to price corporate bonds in the cross-section, I do not adopt this methodology.

Altman and Kishore (1996) find that the recovery rate of corporate bonds is related to the industry of the underlying firm. Using the mean industry-level recovery rates from Altman and Kishore's paper, I re-calculate model yield spreads for the Black-Cox model. As shown in Panel A of Table 5, the cross-sectional results for risk and risk premia proxies are largely unchanged. In unreported results, I find that the results for liquidity proxies and firm characteristics are also similar.

As described above, an ideal calibration of a structural model would involve bond-by-bond, risk-neutral expected recovery rates. However, without more detailed information about firm-level debt structure and the assumption of an underlying model to infer risk-neutral recovery rates, this cannot be done. Thus, I instead consider the possible effect

of the recovery rate by varying recovery rates based on observed yield spreads. The main concern when using the same recovery rate for all bonds is that for two bonds with similar model-implied yield spreads, the bond with the higher observed yield spread is exactly the bond with a lower recovery rate and that this explains exactly what the model is missing in the cross-section. To explore this possibility, I first sort issuers into deciles by the average model yield spread of their bonds. Within each decile, I then sort into terciles by the average observed yield spread of each issuer. I assign a 15.44% recovery to the high observed yield spread tercile, 41% to the middle tercile, and 66.56% to the low tercile<sup>21</sup> and then re-estimate Black-Cox yield spreads. This effectively assumes that bonds with high observed yield spreads compared to the population of bonds with similar model yield spreads have higher yield spreads at least in part because they have lower recovery rates. Mechanically, this improves the cross-sectional explanatory power of the model. However, as shown in Panel B of Table 5, this improvement is limited as the within-group  $R^2$  only improves to 50.49%. More importantly, the unexplained yield spread remains related to recent equity volatility, ratings, and option expensiveness. Therefore, it is unlikely that cross-sectional variation in recovery rates can explain the cross-sectional variation in observed yield spreads that the Black-Cox model does not capture.

## 4 Jump Diffusion Model

Making use of the results in Kou and Wang (2003) and the calibration in Huang and Huang (2003), I now calibrate a double-exponential jump diffusion model (henceforth referred to as the jump model). Such a model has the potential to explain the levels of yield spreads as well as the cross-section through changes in the distribution of firm value and an additional source of risk premia, the jump risk premia. Cremers, Driessen, and Maenhout (2006) calibrate a similar model in the aggregate by matching equity index options and find that the incorporation of a jump risk premia greatly reduces the levels puzzle. Here, I calibrate firm-by-firm to individual equity options to examine if a jump model can help to explain the cross-section of yield spreads above and beyond the Black-Cox model. Under the P-measure, asset value is assumed to follow the process:

$$\frac{dV_t}{V_{t-}} = (\pi^v + r - \delta)dt + \sigma_v dW_t^v + d \left[ \sum_{i=1}^{N_t} (Z_i - 1) \right] - \lambda \xi dt \quad (6)$$

where  $Y \equiv \log(Z)$  and  $f_Y(y) = p_u \eta_u e^{-\eta_u y} 1_{\{y \geq 0\}} + p_d \eta_d e^{-\eta_d y} 1_{\{y < 0\}}$

---

<sup>21</sup>These recovery rates are chose to match the mean recovery rate reported by Altman and Kishore  $\pm$  one standard deviation.

The mean percentage jump size is:

$$\xi = E(e^Y - 1) = \frac{p_u \eta_u}{\eta_u - 1} + \frac{p_d \eta_d}{\eta_d + 1} - 1$$

The probability of up and down jumps are  $p_u$  and  $p_d$ , respectively and  $\frac{1}{\eta_u}$  and  $\frac{1}{\eta_d}$  are the mean up and down jump sizes, respectively.

I follow Kou (2002) and Huang and Huang (2003) in defining the transformation from  $\mathbb{P}$  to  $\mathbb{Q}$ , the risk-neutral measure, via a single parameter,  $\gamma$ . Under  $\mathbb{Q}$ , the asset value process is:

$$\frac{dV_t}{V_{t-}} = (r - \delta)dt + \sigma_v dW_t^{v\mathbb{Q}} + d \left[ \sum_{i=1}^{N_t^{\mathbb{Q}}} (Z_i^{\mathbb{Q}} - 1) \right] - \lambda^{\mathbb{Q}} \xi^{\mathbb{Q}} dt \quad (7)$$

where  $Y^{\mathbb{Q}} \equiv \log(Z^{\mathbb{Q}})$  and  $f_{Y^{\mathbb{Q}}}(y) = p_u^{\mathbb{Q}} \eta_u^{\mathbb{Q}} e^{-\eta_u^{\mathbb{Q}} y} 1_{\{y \geq 0\}} + p_d^{\mathbb{Q}} \eta_d^{\mathbb{Q}} e^{-\eta_d^{\mathbb{Q}} y} 1_{\{y < 0\}}$

$$\begin{aligned} \xi^{\mathbb{Q}} &= E(e^{Y^{\mathbb{Q}}} - 1) = \frac{p_u^{\mathbb{Q}} \eta_u^{\mathbb{Q}}}{\eta_u^{\mathbb{Q}} - 1} + \frac{p_d^{\mathbb{Q}} \eta_d^{\mathbb{Q}}}{\eta_d^{\mathbb{Q}} + 1} - 1 \\ \eta_u^{\mathbb{Q}} &= \eta_u + \gamma; \eta_d^{\mathbb{Q}} = \eta_d + \gamma; p_u^{\mathbb{Q}} = \frac{\frac{p_u \eta_u}{\eta_u}}{\frac{p_u \eta_u}{\eta_u} + \frac{p_d \eta_d}{\eta_d}}; \lambda^{\mathbb{Q}} = \frac{p_u \eta_u}{\eta_u + \gamma} + \frac{p_d \eta_d}{\eta_d - \gamma} \end{aligned}$$

The jump risk premium is  $\lambda \xi - \lambda^{\mathbb{Q}} \xi^{\mathbb{Q}}$ .

## 4.1 Calibration Methodology

In calibrating the model, I make a number of simplifications for tractability. First, I start with the asset volatility calculated for the Black-Cox model. This assumption is equivalent to accepting that the mapping from equity variance to asset variance given by the Black-Cox model is reasonable.<sup>22</sup> When incorporating jumps, the total asset variance is no longer just variance due to the diffusion component in the asset value process as the jumps also contribute to the total asset variance. The total asset variance is:

$$\sigma_{v,total}^2 = \sigma_v^2 + \lambda \left[ \frac{2p_u}{\eta_u^2} + \frac{2p_d}{\eta_d^2} \right] \quad (8)$$

The second assumption that I make is regarding the choice of jump parameters. Huang and Huang (2003) choose  $\lambda$ , the jump frequency, to equal 3, and  $\eta_u$  and  $\eta_d$ , the inverse of the mean up and down-jump sizes, respectively, to equal 30. They argue that these parameter

---

<sup>22</sup>Previous papers have considered different methods of mapping equity variance to asset variance. Eom, Helwege, and Huang (2004) use the delta of a call option from the Merton model. Bao and Pan (2008) follow a similar procedure, but allow for stochastic interest rates. Schaefer and Strebulaev (2004) use a leverage-weighted average of equity and debt return variance.

values are roughly consistent with the results from Anderson, Benzoni, and Lund (2001). Cremers, Driessen, and Maenhout (2006) find  $\lambda$  to be much smaller and jump sizes to be much larger, though their jump sizes seem much larger than what is empirically observed. I choose  $\lambda = 1$ , an average of one jump per year, and  $\eta_u = \eta_d \equiv \eta$ . The mean absolute jump size,  $\frac{1}{\eta}$ , is then calibrated by matching the fourth moment of equity returns in a similar manner to how asset volatility is calculated for the base case. As a simplification, I use partial derivatives from the Black-Cox model and the equation,<sup>23</sup>

$$\frac{1}{n} \sum \text{fourth moment}_t = \frac{1}{n} \sum \left( \frac{\partial \log E}{\partial \log V} \right)^4 E[(\sigma_v \sqrt{dt} Z + d \sum Y_i)^4] \quad (9)$$

To be precise, the transformation from a function of asset value to a function of equity value requires an application of Ito's Lemma with jumps and not a simple application of first partials. The disparity when using the above simplification is particularly severe if jump sizes are large. For my sample, estimated jump sizes turn out to be relatively small (with an average  $\eta$  around 25). In addition, sensitivity analysis shows that yield spreads are not very sensitive to the size of the jump<sup>24</sup> and are instead much more sensitive to the transformation from P to Q-dynamics.

Finally, I estimate  $\gamma$ , the parameter to transform the P-measure jump parameters to Q-measure jump parameters. To estimate  $\gamma$ , I use equity option prices with the intuition that if there is indeed a jump risk premium (or any other non-diffusion risk premium), it should be reflected in the prices of both corporate bonds and equity options. Since equity options are now compound options on asset value with an underlying process that is a double-exponential jump diffusion, I calculate approximate option prices by calculating the risk-neutral probability of asset value falling between discrete levels at option maturity, given  $\gamma$ . Equity options are then priced by inferring the value of equity from assets, calculating option payoffs, and using payoffs along with risk-neutral probabilities. The parameter  $\gamma$  is estimated by minimizing the sum of percentage squared pricing errors of a group of options:

$$SSPE = \min_{\gamma} \sum \left( \frac{\text{actual price} - \text{model price}}{\text{actual price}} \right)^2 \quad (10)$$

I use four options in calculating  $\gamma$ : (1) a short-horizon (< 3 month) OTM put, (2) a short-horizon ATM put, (3) a short-horizon OTM call, and (4) a longer-horizon (> 6 month) OTM put. Because individual equity options are American while pricing is European, I choose

<sup>23</sup>In some cases, fourth moments are too small to back out any jump size. In these cases,  $\eta$  is set equal to 100, a small jump size that has a trivial effect on default probabilities.

<sup>24</sup>Note that the estimate of jump size alone has little effect on yield spreads because large jump sizes will cause a large downward adjustment in  $\sigma_v$  while small jump sizes will cause a small downward adjustment in  $\sigma_v$ .

options for which the pricing difference between American and European options is likely to be small. The longer-horizon OTM put is included due to possible differences in the volatility surface across maturities. Among each subgroup of options, I choose the option with the greatest trading volume during the most recent month. Bakshi, Cao, and Chen (1997) minimize the sum of squared dollar pricing errors rather than percentage pricing errors. However, this underweights OTM options which should provide important information about jump risk premia.

One important consideration in calculating the distribution of asset value is the choice of asset volatility. Under jump model dynamics, asset volatility is constant. This is, of course, a simplification of reality. Using the above procedure with a constant asset volatility may generate positive results that are due to the strong relation between yield spreads and recent realized equity volatility rather than a risk premium. Suppose that implied volatility exactly equals recent equity volatility. Some firms have a higher recent equity volatility than their asset volatility (calculated for the full time period) and current leverage can explain. If the full sample asset volatility is used to calculate the distribution of asset value in the next couple of months, it will be exactly these high recent equity volatility firms that will be deemed to have expensive equity options and large jump risk premia. Thus, these firms will have a larger model yield spread. Since high recent equity volatility firms have higher observed yield spreads, the model would be deemed an improvement despite the fact that all equity options are actually of equal expensiveness. To prevent my results from being driven solely by the recent equity volatility effect, I use recent equity volatility to calibrate a short-term asset volatility and use this asset volatility to determine the distribution of asset value at option maturity. Using the full sample asset volatility generates results that suggest that observed yield spreads are strongly related to jump spreads even when Black-Cox spreads are controlled for.

For the firm-level calibration above to be entirely precise would require joint estimation of  $\sigma_v$ ,  $\eta$ , and  $\gamma$  as the calibration of  $\sigma_v$  and  $\eta$  requires functions of equity value. Equity value contains a call option on firm value and is, thus, dependent on the risk-neutral distribution of asset value. The risk-neutral distribution of asset value depends on  $\gamma$  which cannot be calculated without  $\sigma_v$  and  $\eta$ . An alternative to simultaneously solving variance, fourth moment, and option valuation equations would be to follow a procedure like Huang and Zhou (2007). They use the difference between empirical and model CDS prices as moment restrictions in a GMM framework to back-out parameters.

In my calibration exercise, I avoid using a series of defaultable securities to estimate parameters or using a simultaneous calibration and instead try to calibrate to equity and equity options data through the sequential process described above. My calibration does

capture two essential elements: (1) Asset variance should be related to leverage and the equity variance and (2) jump model spreads should be larger for firms with expensive equity options, holding asset volatility and leverage constant. A sanity check for whether my calibration methodology has reasonably captured that firms with expensive equity options should have higher jump risk premia and thus higher yield spreads is a regression of the jump model spread on the Black-Cox model spread and option expensiveness as measured by the difference between implied and recent equity volatility. I find that the jump model spread is indeed strongly positively related to both the Black-Cox spread and option expensiveness.

Probabilities of default are calculated using the results in Kou and Wang (2003) and previously applied by Huang and Huang (2003). Kou and Wang (2003) find an analytical solution for the Laplace transform of survival probabilities in the jump diffusion model. This transform can be inverted using the Gaver-Stehfest algorithm. Interested readers are referred to Kou and Wang’s original work for details. With risk-neutral default probabilities, bonds can be priced using equation (3).

## 4.2 Calibration Results

As expected, incorporating jump risk premia decreases the difference between observed and model yield spreads as compared to the Black-Cox model. As shown in Table 6, the difference between observed and model yield spreads decreases for all ratings and for both the four-year and ten-year bonds. The mean difference drops from 54 to 32 basis points for four-year bonds and from 79 to 57 basis points for ten-year bonds.<sup>25</sup> These drops are unsurprising given that the jump diffusion model incorporates an additional risk premium which increases Q-measure default probabilities. These results using individual equity option-implied jump risk premia are consistent with the finding in Cremers, Driessen, and Maenhout (2006) that index equity option-implied jump risk premia can generate higher model-implied yield spreads in ratings-level calibrations than for diffusion-only models.<sup>26</sup>

Since I calibrate at a firm-by-firm level rather than at the ratings level, I can further test whether the jump model can help to explain the cross-section of yield spreads in addition to the level of yield spreads. In Table 7, I examine whether the same firms that have high jump risk premia (as inferred from equity options) also have higher observed spreads, with Black-Cox spreads held constant. I find weak evidence that this is indeed the case. In Panel A, I first sort on Black-Cox yield spreads and then jump risk premia within each Black-Cox

---

<sup>25</sup>The reader may notice that the reported difference for the Black-Cox model here does not correspond to the numbers reported in Table 6. This is due to the fact that the sample for the jump model is a subset of the sample for the Black-Cox model.

<sup>26</sup>They argue that once jumps are incorporated and yield spreads are tax-adjusted to reflect the fact that treasuries are exempt from state taxes, the level credit spread puzzle disappears.

group. It appears that observed spreads do increase with jump risk premia for jump risk premia quintiles 2 to 5. However, differences between quintile 5 and quintile 1 observed spreads tend to be statistically insignificant, largely due to the high observed spreads of the low jump risk premia quintile. Many of the firms in this quintile are in fact high recent equity volatility firms. As shown in Section 3, such firms tend to have high unexplained yield spreads. Thus, the recent equity volatility effect tends to blunt the results for the jump model.

In Panel B, I use a regression framework to study the ability of the jump model to explain the cross-section of yield spreads above and beyond a Black-Cox model. For the full sample and for four-year, ten-year, and investment grade subsamples, the portion of the jump yield spread orthogonal to the Black-Cox yield spread (labeled the jump residual) is positively related to observed yield spreads. The economic significance of the jump residual, however, is relatively small. Moving from the 25th to 75th percentile of the jump residual is approximately a 15 basis point move which then translates to a less than 4 basis point move in observed yield spreads. A move from the 10th to 90th percentile equates to approximately a 12 basis point move in observed yield spreads. A two standard deviation move in the jump residual is much larger at 140 basis points and corresponds to a 33 basis point move in observed yield spreads. Economically, this is still a much smaller move than the 255 basis point difference in observed yield spreads for a two standard deviation difference. Thus, it appears that incorporating a jump risk premium calibrated from equity options does improve the cross-sectional explanatory power over a Black-Cox model, but the economic significance of this improvement is limited. In particular, the jump model is a much greater success in explaining the level of yield spreads than the cross-section.

## 5 Stochastic Volatility

As a potential solution to the fact that observed yield spreads are strongly related to recent equity volatility, I calibrate a Heston (1993) model<sup>27</sup> where the underlying firm value process has mean-reverting variance. The modeling of claims on firm cashflow is similar to the base case with one important difference. Equity at maturity is a Heston call option rather than a down-and-out call option. The major distinction between the two options besides the way volatility is modeled, is that the Heston option does not capture a first-passage time default. Thus, the proper baseline model to compare the Heston model to is the Merton model and

---

<sup>27</sup>In estimating call option prices for the Heston (1993) model, I follow the Duffie, Pan, and Singleton (2000) formulation. Lord and Kahl (2008) find that the formulation in Heston (1993) is sometimes inaccurate because of a discontinuous characteristic function when only the principal branch of logarithms are used. See appendix D for the option pricing formula used.

not the Black-Cox model. Numerically, the major difference is that the  $\Delta$  of the barrier option is much higher when  $K$ , the face value of debt, is near  $V$ , the total value of the firm.

As in Heston's (1993) stochastic volatility model, the processes of the underlying and of the variance are:

$$\begin{aligned}\frac{dV_t}{V_t} &= (\pi^v + r - \delta)dt + \sqrt{H_t}(\rho dW_t^1 + \sqrt{1 - \rho^2}dW_t^2) \\ dH_t &= \kappa_H(\theta_H - H_t) + \sigma_H\sqrt{H_t}dW_t^1\end{aligned}\quad (11)$$

where  $V_t$  is asset value and  $H_t$  is asset variance.  $\kappa_H$  is the mean-reversion parameter for asset variance,  $\theta_H$  is the long-run average asset variance, and  $\sigma_H$  is the volatility of variance term. Besides incorporating mean-reverting dynamics for asset variance, the above specification also allows for correlation between firm value and variance shocks,  $\rho$ .<sup>28</sup>

Allowing for stochastic variance, the relevant asset variance for pricing bonds is some weighting of the current asset variance and the long-run average asset variance,  $\theta_H$ . If  $\kappa_H$ , which measures the speed of the mean-reversion in asset variance, is sufficiently small, current asset variance is important in pricing corporate bonds and this could explain the relation between unexplained yield spreads and recent equity volatility. In addition, the correlation parameter,  $\rho$ , could potentially further explain the cross-sectional dispersion of yield spreads. If firm value shocks are negatively correlated to asset variance shocks, asset variance tends to be high exactly when firm value is low. For most firms, this increases the probability of default and drives up yield spreads. Thus, cross-sectional variation in  $\rho$  would lead to cross-sectional variation in model yield spreads.

## 5.1 Estimation procedure

Given the above processes for asset value and variance, I can calculate model-implied relations between equity variance and asset variance and also the model-implied correlation between asset return shocks and variance shocks. First, denote  $E_t = f(t, V_t, H_t)$ , where  $E_t$  is the value of equity. Then, the aforementioned relations are:

$$\sigma_{E,t}^2 = f_v^2 \left( \frac{V_t}{E_t} \right) H_t + \frac{f_H^2}{E_t^2} \sigma_H^2 H_t + 2 \frac{V_t f_v f_H}{E_t^2} \sigma_H H_t \rho \quad (12)$$

$$E_t[(d \log V_t - (\pi^v + r - \delta - \frac{1}{2}H_t)dt)(dH_t - \kappa_H(\theta_H - H_t)dt)] = \sigma_H H_t \rho dt \quad (13)$$

---

<sup>28</sup>The model calibrated in this section does not have a volatility risk premium.

Since asset value is not observed, this requires backing out asset returns from equity returns. I estimate  $d \log V_t$  from  $d \log E_t$  (log equity returns without dividends). The model provides the estimate,  $d \log V_t \approx \frac{d \log E_t}{f_v} \frac{E_t}{V_t}$ , where  $d \log E_t$  is estimated as log equity returns minus  $\delta_e \frac{V_t}{E_t} dt$  and the other terms are estimated through the model.

To estimate  $[\kappa_H, \theta_H, \sigma_H, \rho]$ , I adopt the following iterative procedure:

1. Start with observed equity volatility and an initial guess of  $[\kappa_H, \theta_H, \sigma_H, \rho]$  and use equation (12) to estimate a time-series of  $H_t$ .
2. Using the  $H_t$  estimated in step 1, estimate  $[\kappa_H, \theta_H, \sigma_H]$  using maximum likelihood.
3. Using the  $H_t$  and  $[\kappa_H, \theta_H, \sigma_H]$  calculated in the previous two steps along with equation (13), estimate  $\rho$ .
4. Return to step 1 using the  $[\kappa_H, \theta_H, \sigma_H, \rho]$  estimated from the previous two steps as the initial guess. Repeat until convergence.

In the Merton model, the risk-neutral probability that asset value is above the face value of debt at time  $t$  is  $N(d_2)$ , where  $d_2 = \frac{\ln(\frac{V}{K}) + (r - \delta - \frac{\sigma_v^2}{2})t}{\sigma_v \sqrt{t}}$ . For a Heston model, it is similarly,

$$e^{rt} G_{0,-1} = e^{rt} \left( \frac{\psi(0, H_0, t)}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im} [\psi(-iu, H_0, t) e^{iu(\log k)}]}{u} du \right) \quad (14)$$

where  $\psi$  is defined as in Duffie, Pan, and Singleton (2000), but is scaled by  $e^{-uy}$  in my application.<sup>29</sup> With the risk-neutral probability of survival, bonds can be priced using equation (3).

## 5.2 Calibration Results

In Table 8, I present calibration results for the stochastic volatility model. One important difference that arises between these results and the Black-Cox results is that the stochastic volatility model is able to generate larger yield spreads for poorer-rated bonds. This is due to the modeling convention that is used. In the stochastic volatility model, the sensitivity of equity to underlying asset value is smaller than in a Black-Cox model for firms that are near the default boundary.<sup>30</sup> It is exactly firms with higher leverage (typically firms with junk

<sup>29</sup>A stochastic volatility model is a special case of the example in Section 4 of Duffie, Pan, and Singleton (2000). See also Appendix B in Pan (2002) and section 8F of Duffie (2001) for details of pricing options via transform analysis.

<sup>30</sup>This is due to the fact that equity in a Black-Cox model is based on a down-and-out call option which has a larger  $\Delta$  near the boundary than a vanilla call option.

ratings) for which this is true. Since asset volatility is inversely related to this sensitivity, the stochastic volatility model infers larger asset volatilities which then lead to larger yield spreads. Thus, the Merton model actually overestimates yield spreads for short-maturity junk debt. However, the results for investment grade and longer maturity bonds remain unchanged as the stochastic volatility model cannot generate sufficiently high yield spreads to match observed yield spreads for these bonds.

To test the cross-sectional explanatory power of the stochastic volatility model, I compare the ability of the model to improve on yield spread estimates from the Merton model. Comparing the model to a Black-Cox model would potentially lead to results that are driven by the difference between barrier options and vanilla options. In Panel A of Table 9, I sort first by the Merton model yield spread as a control and then sort within each group by the difference between stochastic volatility and Merton yield spreads. If the additional elements in the stochastic volatility model indeed help to explain yield spreads, the observed yield spread should increase across quintiles while the Merton yield spreads remain relatively flat. The stochastic volatility yield spread should also increase across quintiles. Empirically, observed spreads do not increase across quintiles, suggesting that the stochastic volatility model does not help to explain the cross-section of yield spreads.

Using regressions with time fixed-effects in Panel B, I confirm that the stochastic volatility model has little explanatory power above and beyond the Merton model. The first two columns of Panel B show that regressing observed yield spreads on stochastic volatility yield spreads actually results in a slightly lower within-group  $R^2$  than regressing on Merton model spreads. In the final four columns of Panel B, I first orthogonalize the stochastic volatility model to the Merton model. Then, I regress the observed yield spread on the Merton spread and the residual stochastic volatility spread. For the full sample and for four-year, ten-year, and investment grade subsamples, the stochastic volatility model residual adds little additional explanatory power as the coefficient on the residual stochastic volatility spread is statistically and economically insignificant.

The inability of the stochastic volatility model to improve cross-sectional predictions of yield spreads stems from two sources. First, estimated values of  $\kappa_H$ ,<sup>31</sup> the mean-reversion parameter, are large (averaging 11.7) and thus, the effect of recent volatility is muted. Second, *ceteris paribus*, firms with a more negative correlation between return shocks and variance shocks do not seem to have larger observed yield spreads even though they generally have larger stochastic volatility model yield spreads.

---

<sup>31</sup>See Panel C of Table 5 for summary statistics for parameter estimates.

## 5.3 Alternative Specifications

In this subsection, I further address the relation between unexplained yield spreads and recent equity volatility. The stochastic volatility model is unable to address the relation between unexplained yield spreads and recent equity volatility largely because of the large estimated mean-reversion parameter for asset variance. Here, I examine whether the relation between the unexplained yield spread and recent equity volatility can be explained if a slower mean-reversion parameter is imposed. I also examine whether allowing asset volatility to be inferred solely from recent equity volatility improves the cross-sectional explanatory power of the Black-Cox model.

### 5.3.1 Stochastic Volatility with Slower Mean-Reversion

In this section, my calibration methodology follows that of section 5.1, except that I price bonds as if the mean-reversion parameter for asset variance,  $\kappa_H$ , is one. The mean-reversion parameter used here is much smaller than the average P-measure estimates in section 5.1. In a model with a volatility risk premium, the Q-measure mean-reversion parameter is typically much smaller than the P-measure parameter. In such a model<sup>32</sup>,  $\theta_H^Q$ , the Q-measure long-run mean variance, is also greater than  $\theta_H^P$ , but I do not impose this restriction here as my goal is solely to determine the effect of slower mean reversion on model yield spreads.

I examine the cross-sectional explanatory power of this model through the same regression framework as in section 5.2. The results in Table 10 indicate that the stochastic volatility model does not help to explain the cross-section of yield spreads above and beyond the Merton model. Even for four-year bonds where a smaller mean-reversion parameter should have the largest effect, the SV residual is insignificant. Surprisingly, the SV residual is statistically significant and *negative* for ten-year bonds at the 10% level.

At first glance, these results seem surprising given the intuition that a slower mean-reversion weights recent volatility more and yield spreads are related to recent volatility more than constant volatility models can explain. However, the volatility of variance parameter,  $\sigma_H$ , complicates this interpretation. With deterministic changes in variance, having a slower mean-reversion parameter when the current variance is lower than the mean historical variance will decrease yield spreads. However, when  $\sigma_H$  is important, this is not necessarily the case. For firms that are unlikely to default, a larger than average asset variance is needed to increase default probabilities while lower than average asset variances have little effect on already low default probabilities. A slower mean-reversion parameter allows for positive shocks in variance to be more persistent and hence, have a greater effect on yield

---

<sup>32</sup>See Pan (2002) for an example.

spreads. It seems unlikely that a stochastic volatility model with mean-reverting variance can explain the relation between yield spreads and recent equity volatility.

### 5.3.2 Black-Cox with Recent Equity Volatility

A method of purely examining whether bonds are priced with recent volatility being more important than P-measure estimates would suggest is to calibrate the Black-Cox model using only recent equity volatility and calibrating equation (2) period-by-period. While such a scheme is inconsistent with the Black-Cox modeling assumption of constant asset volatility, it does capture a scenario in which bonds are priced using current volatility as an input for the probability of default.

In Panel B of Table 10, I find evidence that observed yield spreads are indeed related to these period-by-period estimates above and beyond the Black-Cox estimates from Section 3. A move from the 25th to 75th percentile in the period yield spread residual corresponds to a change in observed yield spread of 16 basis points. A move from the 10th to 90th percentile corresponds to a 54 basis point change in the observed yield spread. Thus, it appears that recent volatility is an important determinant of yield spreads. Interestingly, I find that even when this period-by-period yield spread is partialled out from observed yield spreads, unexplained observed yield spreads are related to recent equity volatility as shown by the statistically significant coefficient for the last column of Panel B.

## 6 Endogenous Default Models

Throughout this paper, I have focused on exogenous default models, largely because the default barrier is unobservable. This ignores an important literature on endogenous default models typified by the optimal capital structure and endogenous boundary model of Leland (1994) and the Anderson and Sundaresan (1996) strategic debt service model. Here, I discuss these models and present some preliminary evidence.

In Leland's model, firms optimally choose the level of debt (effectively through the coupon rate) and the default boundary. These endogenous choices generate two comparative statics that can be examined. First, the relation between coupons and asset volatility is U-shaped. Second, the relation between yield spreads and asset volatility is positive for investment grade bonds, but negative for junk grade bonds. A standard exogenous default boundary model makes no predictions about the relation between coupons and asset volatility and predicts a positive relation between yield spreads and asset volatility, regardless of rating.

Examining the two aforementioned implications of the Leland model requires estimates of the asset volatility. While the asset volatility has been estimated earlier for the Black-

Cox model, I do not use these estimates here and instead use a model-free estimate. In particular, I first estimate period-by-period asset variance using:

$$\sigma_v^2 = (1 - L)^2 \sigma_E^2 \quad (15)$$

where  $L$  is a firm's leverage.

Then, I take the time series mean of  $\sigma_v^2$  for each firm as an estimate of the firm's (constant) asset variance. With estimates of the asset variance, I can now examine the implications of the Leland model.

To examine the relation between coupons and asset volatility, I run a regression of the average coupon rate of a firm on asset volatility and asset variance. I plot the results of this regression in Figure 1 along with the theoretical relation derived by Leland. The empirical relation between coupons and asset volatility is indeed U-shaped, but is less convex than the Leland model suggests.

To further examine the implications of the Leland model, I regress yield spreads on market leverage, asset volatility, and an interaction between asset volatility and an investment grade dummy. The implication from the Leland model is that the coefficient on asset volatility should be negative, reflecting higher prices (lower yield spreads) for junk debt when the asset volatility is higher, and the sum of the asset volatility and interaction coefficients should be positive. However, I find that the coefficient on asset volatility is positive and significant, indicating that yield spreads are positively related to asset volatility, even for junk debt. Overall, it seems unlikely that a Leland-type model can fully explain the cross-section of yield spreads.

The second major type of endogenous default model, Anderson and Sundaresan's (1996) strategic debt service model, hinges largely on the bargaining power of equityholders. If recovery rates are lower, equityholders know that debtholders will receive little in default and can strategically refuse to pay debtholders the full promised amount. This predicts that yield spreads are negatively related to recovery rates. In Section 3, it is established that unexplained yield spreads from the Black-Cox model are negatively related to asset tangibility (a proxy for recovery rate), but this relation is statistically insignificant. In addition, it cannot be determined whether this negative relation is due to strategic debt service or simply pricing a lower payoff in case of default.

From the evidence presented in this section, it seems unlikely that endogenous default models can fully explain the cross-section of observed yield spreads as their predicted comparative statics are not strongly supported in the data. However, to truly determine if some of these elements marginally add to explaining the cross-section of yield spreads requires a full calibration.

## 7 Credit Default Swaps

Credit default swaps are insurance contracts in which the buyer pays a (typically) quarterly payment, the CDS spread, to a seller until the maturity of the contract or when the underlying firm (also known as the reference entity) defaults. In the case of default by the underlying, the seller pays the difference between the notional amount and the recovery to the buyer. Conceptually, CDS are very similar to corporate bonds. The seller of CDS contracts is long credit risk, much like the buyer of corporate bonds while the buyer of CDS contracts is short credit risk, like an investor who is short corporate bonds. The pricing of credit default swaps using structural models also follows the pricing of corporate bonds very closely. The CDS spread is determined by calculating the spread such that the present value of the risk-neutral expected value of the series of spread payments is equal to the present value of the risk-neutral expected value of the credit-event payment. A discretized version of this pricing formula is:

$$\begin{aligned} \sum_{i=1}^{4T} \left(1 - q\left(\frac{i}{4}\right)\right) \frac{s}{4} e^{-r\frac{i}{4}} + \sum_{i=1}^{48T} \left(q\left(\frac{i}{48}\right) - q\left(\frac{i-1}{48}\right)\right) \frac{\text{mod}(i-1, 12) + 1}{12} \frac{s}{4} e^{-r\frac{i}{48}} \quad (16) \\ = \sum_{i=1}^{48T} \left(q\left(\frac{i}{48}\right) - q\left(\frac{i-1}{48}\right)\right) (1 - R) e^{-r\frac{i}{48}} \end{aligned}$$

where  $q(t)$  is the cumulative risk-neutral probability of default,  $s$  is the CDS spread, and  $R$  is the recovery rate. The maturity of the CDS contract,  $T$ , is five years in my sample.

The left-hand side is the value of the quarterly payments by the buyer plus the accrued swap spread if the reference entity defaults between two swap spread payment dates while the right-hand side is the value of the payment given default.<sup>33</sup>

Though corporate bonds and credit default swaps are similar in their exposures to credit risk, their non-credit components are thought to be very different. As of the fourth quarter of 2007, there were \$5.8 trillion of corporate bonds outstanding according to the Securities Industry and Financial Markets Association. In contrast, Christopher Cox, the SEC Chairman, cited the size of the CDS market as \$58 trillion in testimony to the United States Senate in 2008. Also, it is generally believed that the CDS market is more liquid than the corporate bond market, particularly for five-year CDS contracts. Thus, Longstaff, Mithal, and Neis

---

<sup>33</sup>I have made the simplification that the parties determine whether or not the reference entity has defaulted every  $\frac{1}{48}$  years and that CDS premia also accrue at this horizon. In contrast, if the parties continuously monitor whether the reference entity has defaulted, the summations in the equation should be replaced by integrals.

(2005) and Nashikkar, Subrahmanyam, and Mahanti (2007) use CDS as proxies for credit risk and generate CDS-implied corporate bond yield spreads. In contrast to the corporate bond market, a major source of risk in the CDS market is counterparty risk, the likelihood that the counterparty in a CDS contract will be unable to pay its side of the contract. Since corporate bonds and CDS of the same underlying entity directly share credit risk, but not necessarily liquidity and counterparty risk, it is interesting to examine the performance of structural models of default in the CDS market. In particular, if structural models fail to fully capture credit risk in the cross-section, it is likely that unexplained spreads should be correlated for corporate bonds and CDS of the same company.

## 7.1 Calibration and Cross-Sectional Results

In Panel A of Table 11, I examine the magnitudes of observed and model CDS spreads for the Black-Cox model, finding that the mean difference between observed and model spreads for five-year CDS contracts is 42 basis points in my sample and statistically significant. The magnitude of this difference is smaller than for both four-year corporate bonds (90 basis points) and for ten-year corporate bonds (88 basis points).<sup>34</sup> In addition, the difference between observed and model CDS spreads is significant for all ratings groups, though it is larger for firms with poorer ratings.

The cross-sectional performance of structural models in the CDS market is similar to that of the bond market. Following the same procedure as for corporate bonds, I first regress the observed CDS spread on the model CDS spread, finding a coefficient of 0.59 and a within-group  $R^2$  of 36.58%. Interestingly, this  $R^2$  is *lower* than the  $R^2$  from an analogous regression for corporate bonds. I then construct the residual from this regression as the unexplained CDS spread and test whether the unexplained spread is related to credit risk proxies and firm-level characteristics. In Panel B, I find that the unexplained CDS spread is strongly related to recent equity volatility and ratings, much like corporate bonds. Unexplained CDS spreads are also related to firm size as in the corporate bond market. Thus, the cross-sectional performance of the Black-Cox model is similar for both the corporate bond and CDS market.

---

<sup>34</sup>This is consistent with findings by Hull, Predescu, and White (2004) that the benchmark risk-free rate in the CDS market is close to the swap rate rather than the Treasury rate. During the 2003 to 2007 period, the mean difference between swap rates and Treasury rates was approximately 45 basis points.

## 7.2 Commonality Between CDS and Corporate Bonds

The common credit risk component in corporate bonds and CDS suggests a natural test of whether unexplained corporate bond yield spreads are related to unexplained CDS spreads. For each firm-quarter, I choose the bond for a firm that is closest to five years to maturity to compare to my sample of five-year CDS contracts. As a benchmark regression, I first regress the observed CDS spread on the observed corporate bond yield spread with time fixed-effects. As shown in Panel C of Table 11, CDS spreads and corporate bond yield spreads are strongly related with a coefficient of 1.05 and a robust t-stat of 23.61. The within-group  $R^2$  is large at 70.16%. I then run a similar regression for the unexplained CDS spread and unexplained corporate bond yield spread, finding a coefficient of 0.98 and a robust t-stat of 14.78. The within-group  $R^2$  drops to 49.62%, indicating that the Black-Cox model has captured some common component between corporate bonds and CDS, but that there is also a remaining common component. Since unexplained corporate bond yield spreads and CDS spreads are related to recent equity volatility and ratings, I further purge out recent equity volatility and ratings from the unexplained corporate bond and CDS spreads. The still unexplained spreads are related, though the within-group  $R^2$  declines further to 37.67%.

The results above indicate that a Black-Cox model is able to capture some component of credit risk as the relation between corporate bonds and CDS weaken once the model is controlled for. However, there remains an important relation between unexplained corporate bond yield spreads and CDS spreads. An important caveat is that liquidity has thus far been treated as security-specific (and only for corporate bonds) rather than firm-specific and it is possible that a common liquidity component across markets could account for a portion of this commonality. Empirical evidence about CDS liquidity and in particular, the relation between CDS and corporate bond liquidity, is limited. Nashikkar, Subrahmanyam, and Mahanti (2007) find that less liquid CDS contracts (ones with greater bid-ask spreads) have more expensive corporate bonds (bonds with lower yield spreads). Tang and Yan (2007) find that less liquid CDS contracts have larger CDS spreads. Together, these results suggest that companies with less liquid CDS contracts should have higher CDS spreads and *lower* corporate bond yield spreads. This actually works against my finding of a positive commonality between unexplained corporate bond and CDS spreads, though the current understanding of liquidity commonality between corporate bond markets and CDS markets is limited and I cannot completely dismiss the possibility of a positive common liquidity component. However, the magnitude of commonality that remains and the previous results on the corporate bond and CDS cross-sections suggest that there is some component of credit risk that the Black-Cox model cannot capture in the cross-section.

## 8 Conclusion

In this paper, I test the ability of structural models of default to explain the cross-section of corporate bond yield spreads. Though structural models present predictions about both the levels of yield spreads and also the relative yield spreads of different bonds, the literature has thus far focused on levels. Huang and Huang (2003) find that for a broad group of structural models matched to historical default probabilities, yield spreads are too high to be explained solely by credit risk. Much of the literature that has followed involves examining model mechanisms that can generate model yield spreads that are closer to the levels of historically observed yield spreads. Instead, I focus on an alternative test of structural models, their cross-sectional explanatory power, rather than solely asking if they can generate large model yield spreads. This allows me to directly assess the determinants of the disconnect between observed and model yield spreads.

My base case is a Black-Cox model. Regressing observed yield spreads on Black-Cox yield spreads, I find that the Black-Cox model is able to explain 44.9% of the variation in observed yield spreads. I then construct unexplained yield spreads as the residuals from this regression. As expected, unexplained yield spreads are related to proxies for liquidity, as structural models are designed to capture credit risk and not illiquidity. More importantly, I find a significant cross-sectional relation between unexplained yield spreads and proxies for credit risk such as recent equity volatility and ratings. This indicates a failure of the Black-Cox model to fully capture credit risk in the cross-section. In addition, unexplained yield spreads are related to equity option expensiveness, suggesting an additional risk premium.

The relations between unexplained yield spreads and recent equity volatility and option expensiveness suggest that models with stochastic volatility and a non-diffusion risk premium may help to explain cross-sectional yield spreads. Calibrations based on a Heston (1993) model do not improve cross-sectional explanatory power as estimates of the asset variance mean-reversion parameter are high. A double-exponential jump diffusion model with jump risk premia inferred from individual equity options does improve cross-sectional explanatory power, suggesting that there is a risk premium that is priced in both the corporate bond and equity option markets. However, the economic significance of this relation is limited as compared to the Cremers, Driessen, and Maenhout (2006) finding that a model with index equity option-implied jump risk premia can explain much of the levels puzzle.

In addition to examining the cross-sectional explanatory power of structural models for corporate bonds, I examine whether the Black-Cox model can explain the cross-section of CDS spreads. Much like the results for corporate bonds, unexplained CDS spreads are related to proxies for credit risk in the cross-section. I also find that unexplained CDS

spreads are related to unexplained corporate bond yield spreads of the same firm, consistent with structural models being unable to fully capture credit risk in the cross-section. Though I cannot completely rule out that this commonality is a liquidity commonality, the magnitude of the relation suggests that it is at least in part due to credit risk that the model has not captured.

In this paper, I do not address the relation between unexplained spreads and ratings through a structural model of default. I do find that ratings are significantly related to the size of a firm and that firm size is related to unexplained spreads when ratings are not controlled for. A further examination of what ratings truly capture would be useful as they are intended to represent forward-looking predictions of credit-worthiness. Current studies of ratings have generally focused on market reactions to rating announcements. Hand, Holthausen, and Leftwich (1992) find significant bond price reactions to unexpected bond ratings changes. There are potentially two interpretations for this result. First, ratings agencies receive private information from the firms that they rate and bond price reactions could reflect this information. Second, it is possible that changes in ratings elicit changes in bond prices simply because market participants price bonds based on ratings, regardless of how much information ratings capture. A more recent study by Hull, Predescu, and White (2004) finds that the credit default swap market anticipates ratings announcements, casting doubt on the second explanation. This is consistent with ratings reflecting something fundamental that markets use to price bonds rather than ratings leading bond prices. Though ratings may be slower than the market to incorporate information, a potential avenue for future research would be to examine the information that ratings agencies use and to see how this information affects bond pricing above and beyond what structural models of default suggest. These inputs may then be a guide for future work in structural models.

The tests in this paper present an additional challenge to researchers in credit risk modeling. Most of the previous work on structural models of default has been focused on constructing a model to match the level of yield spreads. I argue that explaining the cross-section of observed yield spreads through a theoretically-founded model is an equally important and difficult task that should be the focus of future research.

# Appendix

## A Volatility Due to Bid-Ask Spreads

In equation (2),  $\frac{\partial \log E}{\partial \log V}$  and  $\sigma_E$  are not constant over time, but I only update these values quarterly due to the fact that Compustat data is updated quarterly. An alternative specification is to update  $\frac{\partial \log E}{\partial \log V}$  more frequently by using a linear interpolation of Compustat values between quarterly reports and then use higher frequency updates of  $\sigma_E$ . This, however, creates problems in volatility estimates due to bid-ask spread. Using a framework similar to Roll (1984), I examine the amount of annualized equity volatility that can be generated solely through bid-ask bounce depending on the sampling horizon. Suppose that all changes in equity price are due to whether the transaction is at the bid or ask price, denoted  $b$  and  $a$ , respectively. Also, denote the bid-ask spread as  $s = a - b$ . If transactions at bid and ask are equally likely, the distribution of log returns is:

$$\log(R) = \begin{cases} 0 & \text{w.p. } 0.5, \\ \log(1 + \frac{s}{b}) & \text{w.p. } 0.25, \\ \log(1 - \frac{s}{a}) & \text{w.p. } 0.25 \end{cases}$$

Suppose  $\frac{b+a}{2} = 100$ , then the annualized volatility generated solely by bid-ask spreads for 5-minute, 30-minute, and daily sampling (in %) are:

Bid-Ask Spread	Vol (5 min)	Vol (30 min)	Vol (daily)
0.1	9.91	4.05	1.12
0.2	19.83	8.09	2.24
0.3	29.74	12.14	3.37
0.4	39.65	16.19	4.49
0.5	49.57	20.24	5.61

As can be seen above, sampling at too high a frequency can generate large equity volatility even when there are no changes in firm fundamentals. As a baseline for comparison, the median spreads as a percentage of stock value for IBM, GE, GM, JNJ, and WMT in January 2003 were 0.18, 0.16, 0.36, 0.30, and 0.29, respectively.<sup>35</sup>

<sup>35</sup>Quote-by-quote spreads from the NYSE TAQ data are used.

## B Claims on the Firm in the Black-Cox Model

Following Bjork (2004), the value of a down-and-out call option where the barrier equals the strike price is:

$$\begin{aligned}
 Call &= V e^{-\delta T} N(d_1) - K e^{-rT} N(d_2) - \left(\frac{K}{V}\right)^{2(r-\delta-\frac{\sigma_v^2}{2})/\sigma_v^2} \left[\frac{K^2}{V} e^{-\delta T} N(\tilde{d}_1) - K e^{-rT} N(\tilde{d}_2)\right] \quad (17) \\
 d_1 &= \frac{\ln(\frac{V}{K}) + (r - \delta + \frac{\sigma_v^2}{2})T}{\sigma_v \sqrt{T}}, \quad d_2 = d_1 - \sigma_v \sqrt{T} \\
 \tilde{d}_1 &= \frac{\ln(\frac{K}{V}) + (r - \delta + \frac{\sigma_v^2}{2})T}{\sigma_v \sqrt{T}}, \quad \tilde{d}_2 = \tilde{d}_1 - \sigma_v \sqrt{T}
 \end{aligned}$$

Defining  $Q$  as the risk-neutral probability of default (see equation (4), the value of debt at maturity and the value of bankruptcy costs are:

$$\begin{aligned}
 \text{Debt at Maturity} &= K e^{-rT} - K e^{-rT} Q(1 - R_{firm}) \quad (18) \\
 \text{Bankruptcy Costs} &= K e^{-rT} Q(1 - R_{firm})
 \end{aligned}$$

Thus, the remaining value of the firm is equal to  $V - Call - K e^{-rT}$ , and is attributed to equity and debt in a proportion equal to  $\frac{\delta_e}{\delta}$  and  $\frac{(\delta - \delta_e)}{\delta}$ , respectively.

## C Model-Implied Default Probabilities

In contrast to Huang and Huang (2003), I match equity volatilities rather than probabilities of default when calculating firm parameters. This is done both because historical default probabilities do not necessarily reflect forward-looking default probabilities and because there is no data on firm-by-firm default probabilities. Here, I examine model-implied default probabilities for the Black-Cox model and compare them to historical default probabilities.

Huang and Huang use average firm parameters within a rating and use the Bhandari (1988) estimates to generate an equity risk premium to match along with historical probabilities of default. From this, they generate implied asset volatilities and implied asset risk premia. As my calculations in Section 3 infer an asset volatility from equity volatility, I only need to infer an asset risk premium to be able to calculate probabilities of default. I follow the Huang and Huang procedure in using the Bhandari estimates (columns labeled with Lev) and also use equity risk premia calculated from the CAPM. In the third and fourth columns of Panels A and B, I report the model-implied default probabilities for four-year and

Panel A: 4yr Default Probabilities

Rating	Historical	Average Firm		Firm-by-Firm CAPM			Firm-by-Firm Lev		
		CAPM Used	Lev Used	Mean	Std	Med	Mean	Std	Med
Aaa	0.04	0.01	0.01	0.47	0.92	0.00	0.45	0.88	0.00
Aa	0.23	0.00	0.00	0.18	0.85	0.00	0.14	0.64	0.00
A	0.35	0.07	0.12	1.12	4.59	0.01	0.83	3.25	0.01
Baa	1.24	0.38	0.62	1.97	7.30	0.04	1.49	4.90	0.04
Ba	8.51	0.89	1.41	5.07	14.49	0.38	2.96	6.50	0.50
B	23.32	6.43	8.14	9.78	15.81	1.98	6.97	11.20	2.02
C		20.80	19.90	21.76	21.12	18.20	17.70	19.41	10.02

Panel B: 10yr Default Probabilities

Rating	Historical	Average Firm		Firm-by-Firm CAPM			Firm-by-Firm Lev		
		CAPM Used	Lev Used	Mean	Std	Med	Mean	Std	Med
Aaa	0.77	0.88	0.91	2.45	3.89	0.01	2.40	3.85	0.01
Aa	0.99	0.70	0.97	1.87	4.73	0.32	1.60	3.61	0.34
A	1.55	1.78	3.10	3.98	8.70	0.69	3.23	6.27	0.82
Baa	4.39	3.77	6.40	6.80	12.76	1.60	5.56	9.52	1.73
Ba	20.63	5.02	8.36	12.01	21.19	3.71	10.33	14.63	5.25
B	43.91	17.83	23.04	24.49	28.98	11.01	20.32	22.90	12.44
C		36.93	35.19	36.74	28.19	38.35	28.20	25.78	27.27

ten-year horizons, respectively. For four-year horizons, the model tends to underestimate default probabilities for all ratings. At the ten-year horizon, the model actually does a reasonable job for investment grade bonds, but vastly underestimates default probabilities for junk debt.

In addition to using average firm parameters, I also calculate model-implied default probabilities firm-by-firm for comparison with historical default probabilities. I find that the mean model-implied default probabilities tend to be higher than historical default probabilities for investment grade firms at both the four-year and ten-year horizon. However, median default probabilities are much lower, indicating that these results are driven by cases where a few firms have very high model-implied default probabilities.

## D Option Pricing Formula for the Stochastic Volatility Model

In this section, I present call pricing formulas for a stochastic volatility model. This formulation is a special case of Duffie, Pan, and Singleton (2000) and also of Pan (2002).

$$\begin{aligned}
 \frac{Call}{V_t} &= G_{1,-1}(-\log k, H_0, T) - kG_{0,-1}(-\log k, H_0, T) & (19) \\
 G_{1,-1} &= \frac{\psi(1, H_0, T)}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im}[\psi(1 - iu, H_0, T)e^{iu(\log k)}]}{u} du \\
 G_{0,-1} &= \frac{\psi(0, H_0, T)}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im}[\psi(-iu, H_0, T)e^{iu(\log k)}]}{u} du \\
 \psi(s, H_0, T) &= \exp(\alpha(T, s) + \beta(T, s)H_0) \\
 \alpha(T, s) &= -rT + (r - \delta)sT - \kappa_H \theta_H \left( \frac{\gamma + b}{\sigma_H^2} T + \frac{2}{\sigma_H^2} \log \left[ 1 - \frac{\gamma + b}{2\gamma} (1 - e^{-\gamma T}) \right] \right) \\
 \beta(T, s) &= -\frac{a(1 - e^{-\gamma T})}{2\gamma - (\gamma + b)(1 - e^{-\gamma T})} \\
 a &= s(1 - s) \\
 b &= \sigma_H \rho s \\
 \gamma &= \sqrt{b^2 + a\sigma_H^2}
 \end{aligned}$$

## References

- [1] Acharya, V. V., S. T. Bharath, and A. Srinivasan (2007). Does Industry-wide Distress Affect Defaulted Firms? Evidence from Creditor Recoveries. *Journal of Financial Economics* 85, 787-821.
- [2] Altman, E. I. and V. M. Kishore (1996). Almost Everything You Wanted to Know about Recoveries on Defaulted Bonds. *Financial Analysts Journal* 52, 57-64.
- [3] Altman, E., A. Resti, and A. Sironi (2004). Default Recovery Rates in Credit Risk Modelling: A Review of the Literature and Empirical Evidence. *Economic Notes* 33, 183-208.
- [4] Anderson, R. and S. Sundaresan (1996). Design and Valuation of Debt Contracts. *Review of Financial Studies* 9, 37-68.
- [5] Anderson, T. G., L. Benzoni, and J. Lund (2002). An Empirical Investigation of Continuous-Time Models for Equity Returns. *Journal of Finance* 57, 1239-1284.
- [6] Andrade, G. and S. N. Kaplan (1998). How Costly is Financial (not Economic) Distress? Evidence from Highly Levered Transactions that Became Distressed. *Journal of Finance* 53, 1443-1493.
- [7] Ang, A., J. Chen, and Y. Xing (2006). Downside Risk. *Review of Financial Studies* 19, 1191-1239.
- [8] Avramov, D., G. Jostova, and A. Philipov (2007). Understanding Changes in Corporate Credit Spreads. *Financial Analysts Journal* 63, 90-105.
- [9] Bakshi, G., C. Cao, and Z. Chen (1997). Empirical Performance of Alternative Option Pricing Models. *Journal of Finance* 52, 2003-2049.
- [10] Bao, J. and J. Pan (2008). Excess Volatility of Corporate Bonds. Working Paper, MIT.
- [11] Bao, J., J. Pan, and J. Wang (2008). Liquidity of Corporate Bonds. Working Paper, MIT.
- [12] Berger, P. G., E. Ofek, and I. Swary (1996). Investor Valuation of the Abandonment Option. *Journal of Financial Economics* 42, 257-287.
- [13] Bhandari, L. C. (1988). Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence. *Journal of Finance* 43, 507-528.
- [14] Bharath, S. T. and T. Shumway (2008). Forecasting Default with the Merton Distance to Default Model. *Review of Financial Studies* 21, 1339-1369.
- [15] Biais, B. and R. C. Green (2007). The Microstructure of the Bond Market in the 20th Century. Toulouse University and Carnegie Mellon University Working Paper.

- [16] Bjork, T. (2004). *Arbitrage Theory in Continuous Time*, 2nd ed. Oxford University Press.
- [17] Black, F. and J. C. Cox (1976). Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *Journal of Finance* 31, 351-367.
- [18] Campbell, J. Y. and G. B. Taksler (2003). Equity Volatility and Corporate Bond Yields. *Journal of Finance* 58, 2321-2349.
- [19] Campbell, J. Y. , J. Hilscher, and J. Szilagyi (2007). In Search of Distress Risk. *Journal of Finance*, forthcoming.
- [20] Carey, M. S. and M. Gordy (2007). The Bank as Grim Reaper: Debt Composition and Recoveries on Defaulted Debt. Working Paper, Federal Reserve Board.
- [21] Chen, H. (2008). Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure. Working Paper, MIT.
- [22] Collin-Dufresne, P. and R. S. Goldstein (2001). Do Credit Spreads Reflect Stationary Leverage Ratios?. *Journal of Finance* 56, 1929-1957.
- [23] Collin-Dufresne, P., R. S. Goldstein, and S. Martin (2001). The Determinants of Credit Spread Changes. *Journal of Finance* 56, 2177-2207.
- [24] Cremers, M., J. Driessen, and P. Maenhout (2006). Explaining the Level of Credit Spreads: Option-Implied Jump Risk Premia in a Firm Value Model. *Review of Financial Studies*, forthcoming.
- [25] Cremers, M., J. Driessen, P. Maenhout, and D. Weinbaum (2006). Individual Stock-Price Implied Volatility and Credit Spreads. *Journal of Banking and Finance*, forthcoming.
- [26] Diamond, D. W. (1989). Reputation Acquisition in Debt Markets. *Journal of Political Economy* 97, 828-862.
- [27] Duffie, D. (2001). *Dynamic Asset Pricing Theory*, 3rd ed. Princeton University Press.
- [28] Duffie, D. and D. Lando (2001). Term Structures of Credit Spreads with Incomplete Accounting Information. *Econometrica* 69, 633-664.
- [29] Duffie, D., J. Pan, and K. Singleton (2000). Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica* 68, 1343-1376.
- [30] Edwards, A. K., L. E. Harris, and M. S. Piwowar (2007). Corporate Bond Market Transaction Costs and Transparency. *Journal of Finance* 62, 1421-1451.
- [31] Eom, Y. H., J. Helwege, and J. Z. Huang (2004). Structural Models of Corporate Bond Pricing: An Empirical Analysis. *Review of Financial Studies* 17, 499-544.

- [32] Ericsson, J., J. Reneby, and H. Wang (2005). Can Structural Models Price Default Risk? Evidence from Bond and Credit Derivative Markets. Working Paper, McGill University and Stockholm School of Economics.
- [33] Fama, E. F. and K. R. French (2001). Disappearing Dividends: Changing Firm Characteristics or Lower Propensity to Pay? *Journal of Financial Economics* 60, 3-43.
- [34] Fons, J. S., R. Cantor, and C. Mahoney (2002). Understanding Moody's Corporate Bond Ratings and Rating Process. *Moody's Investors Service*.
- [35] Goldstein, R. S., N. Ju., and H. E. Leland (2001). An EBIT-Based Model of Dynamic Capital Structure. *Journal of Business* 74, 483-512.
- [36] Hand, J. R. M., R. W. Hothausen, and R. W. Leftwich (1992). The Effect of Bond Rating Agency Announcements on Bond and Stock Prices. *Journal of Finance* 47, 733-752.
- [37] Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies* 6, 327-343.
- [38] Houweling, P., A. Mentink, and T. Vorst (2005). Comparing Possible Proxies of Corporate Bond Liquidity. *Journal of Banking & Finance* 29, 1331-1358.
- [39] Huang, J. Z. and M. Huang (2003). How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk. Working Paper, Penn State and Stanford.
- [40] Huang, J. Z. and H. Zhou (2007). Specification Analysis of Structural Credit Risk Models. Working Paper, Penn State and Federal Reserve Board.
- [41] Hull, J., M. Predescu, and A. White (2004). The Relationship Between Credit Default Swap Spreads, Bond Yields, and Credit Rating Announcements. *Journal of Banking & Finance* 28, 2789-2881.
- [42] Kim, J., K. Ramaswamy, and S. Sundaresan (1993). Does Default Risk in Coupons Affect the Valuation of Corporate Bonds?: A Contingent Claims Model. *Financial Management* 22, 117-131.
- [43] Kou, S. G. (2002). A Jump-Diffusion Model for Option Pricing. *Management Science* 48, 1086-1101.
- [44] Kou, S. G. and H. Wang (2003). First Passage Times of a Jump Diffusion Process. *Advanced Applied Probability* 35, 504-531.
- [45] Lakonishok, J., A. Shleifer, and R. W. Vishny (1994). Contrarian Investment, Extrapolation, and Risk. *Journal of Finance* 49, 1541-1578.
- [46] Leland, H. E. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *Journal of Finance* 49, 157-196.

- [47] Leland, H. E. and K. Toft (1996). Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. *Journal of Finance* 51, 987-1019.
- [48] Longstaff, F. A. and E. S. Schwartz (1995). A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. *Journal of Finance* 50, 789-819.
- [49] Longstaff, F. A., S. Mithal, and E. Neis (2005). Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit-Default Swap Market. *Journal of Finance* 60, 2213-2253.
- [50] Lord, R. and C. Kahl (2008). Complex Logarithms in Heston-like Models. Working Paper, Rabobank International and ABN-Amro.
- [51] Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29, 449-470.
- [52] Nashikkar, A., M. Subrahmanyam, and S. Mahanti (2007). Latent Liquidity and Corporate Bond Yield Spreads. Working Paper, NYU and State Street Global Markets.
- [53] Pan, J. (2002). The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study. *Journal of Financial Economics* 63, 3-50.
- [54] Roll, R. (1984). A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. *Journal of Finance* 39, 1127-1139.
- [55] Schuermann, T. (2004). What Do We Know About Loss Given Default? Working Paper, Federal Reserve Bank of New York.
- [56] Tang, D. Y. and H. Yan (2007). Liquidity and Credit Default Swap Spreads. Working Paper, Kennesaw State University and University of South Carolina.

Table 1: Summary Statistics

	Firm Summary Statistics					
	Obs	Mean	Std Dev	25th	50th	75th
Market Leverage	3,250	30.61	23.46	13.83	23.59	39.98
Equity Volatility	3,250	25.60	12.11	17.86	22.71	30.26
IV - Equity Volatility	2,364	6.73	6.88	2.95	6.33	9.94
Equity Market Cap	3,250	31.22	53.90	4.37	13.27	33.04
Firm Age	3,250	40.73	25.37	17.42	36.66	62.91
Return on Assets	2,874	1.35	1.19	0.65	1.29	1.97
Equity Beta	3,227	0.95	0.75	0.49	0.81	1.24
Avg Bond Duration	3,250	6.07	2.66	4.14	5.81	7.60
Asset Tangibility	3,220	40.57	12.01	33.09	41.97	49.87
Interest Coverage	3,249	8.87	9.90	2.95	5.89	11.37
Hist - Current Lev	3,249	0.89	13.02	-5.79	0.92	7.89
S&P	3,250	71.23				

	4yr Bonds					
	Obs	Mean	Std Dev	25th	50th	75th
Maturity	2,540	3.36	1.36	2.24	3.44	4.19
Rating	2,493	8.16	3.66	6.00	7.00	10.00
Age	2,540	6.94	3.99	4.33	6.79	8.56
Amount Outstanding	2,540	297.45	377.21	100.00	200.00	325.00
Volume	2,540	47.10	132.98	3.50	12.60	37.72
Trades	2,540	146.51	436.43	16.00	45.00	129.50
Turnover	2,532	13.36	19.42	3.15	7.87	15.83
Avg Trade Size	2,540	520.24	763.91	118.90	271.14	585.61
% Days Traded	2,540	41.36	31.77	14.06	32.26	67.74

	10yr Bonds					
	Obs	Mean	Std Dev	25th	50th	75th
Maturity	1,687	12.83	8.12	8.90	10.70	15.29
Rating	1,650	8.30	3.71	6.00	8.00	10.00
Age	1,687	8.69	4.90	4.18	9.49	11.97
Amount Outstanding	1,687	358.39	478.95	150.00	250.00	350.00
Volume	1,687	124.00	593.67	4.95	15.15	45.11
Trades	1,687	203.96	774.65	15.00	41.00	109.00
Turnover	1,680	15.08	22.32	3.08	7.71	16.98
Avg Trade Size	1,687	644.04	791.17	154.57	390.61	803.34
% Days Traded	1,687	40.15	31.25	12.90	32.26	61.90

Observations are quarterly. Market leverage is the ratio of market value of debt to the sum of market value of debt plus market value of equity. Equity volatility is the annualized volatility of daily log equity returns for a quarter. IV - equity volatility is the mean of the implied volatilities of a short-term OTM put and a short-term ATM put minus recent equity volatility. Equity market capitalization is the product of share price and shares outstanding in \$B. Firm age is the number of years since a firm's BEGDT in CRSP. Return on Assets is a firm's income before extraordinary items divided by the mean of total assets at the start and end of the quarter. Equity beta is a firm's CAPM beta using a 60-month rolling window. Avg Bond Duration is the issuance-weighted average of the duration of a firm's outstanding bonds. Asset tangibility is calculated using the estimates in Berger, Ofek, and Swary (1996). Interest coverage is EBIT divided by the interest expense. Hist - Current Lev is the difference between a firm's mean leverage from 1993 to 2002 and its current leverage. S&P equals 1 if a firm is an S&P firm. Maturity is the number of years to a bond's maturity and age is the number of years since a bond's issuance. Rating is coded as 1 for Aaa and 21 for C with intermediate ratings coded. Amount Outstanding is the face value outstanding in \$mm. Volume is the quarterly trading volume in \$mm face value. Trades is the number of trades in a quarter. Turnover is the volume scaled by the amount outstanding. Avg Trade Size is the average trade size in \$. %Days Traded equals the number of days a bond was traded divided by the number of trading days in the quarter.

Table 2: Level of Yield Spreads, Black-Cox Model

## Panel A: 4yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		BC Yield	
				mean	med	mean	med
aaa	83	21.21	22.42	3.88	4.17	5.12	4.94
aa	206	16.63	22.69	4.08	4.13	4.91	4.89
a	973	25.80	22.45	4.22	4.37	5.19	4.89
baa	778	32.75	22.98	5.06	4.72	5.23	4.93
ba	214	42.36	21.05	6.49	6.59	5.58	5.47
b	154	55.64	20.52	7.55	7.09	7.75	6.93
c	91	70.12	18.36	20.06	10.74	9.91	9.77
full	2,546	31.90	22.24	5.46	4.52	5.56	4.89

Rating	Observed Spread		BC Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
aaa	35.03	33.70	12.85	5.42	22.17	19.15	2.94
aa	36.19	38.32	0.63	0.01	35.56	37.61	9.53
a	61.99	57.89	24.62	0.15	37.37	51.12	4.88
baa	100.72	99.72	44.07	4.43	56.65	79.01	4.85
ba	223.76	289.59	85.37	60.72	138.40	213.37	5.56
b	329.25	318.72	302.21	207.75	27.05	162.88	0.35
c	1,591.15	698.28	515.54	490.67	1,075.61	235.56	1.32
full	159.95	73.26	69.73	0.38	90.21	59.24	3.13

## Panel B: 10yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		BC Yield	
				mean	med	mean	med
aaa	90	16.84	21.53	5.03	5.06	5.14	4.89
aa	117	13.58	23.25	5.19	5.12	4.97	4.92
a	515	29.33	21.78	5.39	5.31	5.48	5.08
baa	562	29.65	24.59	6.17	6.08	5.58	5.35
ba	203	40.68	21.52	7.33	7.02	6.05	5.60
b	128	51.16	19.79	8.56	8.15	6.93	6.43
c	40	72.67	12.57	12.02	11.37	8.09	7.96
full	1,692	31.70	22.48	6.28	5.74	5.71	5.19

Rating	Observed Spread		BC Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
aaa	53.62	55.68	20.04	0.57	33.57	41.86	2.60
aa	71.34	68.60	8.09	3.47	63.25	62.22	10.86
a	96.04	85.71	55.30	19.55	40.74	61.12	3.26
baa	160.09	147.02	80.03	45.97	80.06	77.53	5.32
ba	271.87	247.23	132.96	83.28	138.91	142.34	6.35
b	394.47	360.82	222.72	158.56	171.75	207.53	4.73
c	748.28	684.33	340.16	325.02	408.12	350.87	2.66
full	175.72	122.39	88.19	30.04	87.54	74.30	7.07

Observations are at the bond-quarter level from 2003 Q1 to 2007 Q4. Leverage, asset volatility, and yields are reported in %. Spreads and the difference in spreads are reported in basis points. T-statistics for the difference use standard errors that are clustered by firm and time.

Table 3: Cross-Sectional Tests

## Panel A: Risk Variables

	Obs Spd	Unexplained Spread			
BC Spd	0.58 [ 8.00]				
Rating				16.77 [ 9.39]	
Mktlev		1.08 [ 2.23]	-0.21 [ -0.52]		
Asset Vol		1.22 [ 1.00]	-1.18 [ -1.23]		
Eq Vol			3.89 [ 6.16]	4.03 [ 4.41]	
IV - Eq Vol				2.57 [ 3.01]	
R-sqd	44.86	3.20	15.50	13.55	29.71
Obs	4,227	4,227	4,227	3,145	4,143

## Panel B: Additional Characteristics

	Unexplained Spread						Rating	
Mktlev	-0.16 [-0.30]	0.28 [0.57]	-0.26 [0.48]	0.16 [0.32]	-0.33 [-0.67]	-0.28 [-0.65]	0.01 [1.24]	0.03 [3.70]
Rating					15.71 [6.21]	15.05 [5.63]		
Eq Vol	3.10 [5.72]	3.09 [5.84]	3.35 [4.05]	3.33 [3.90]	1.97 [2.66]	1.93 [2.59]	0.07 [5.76]	0.08 [5.89]
IV - Eq Vol			2.50 [3.87]	2.53 [3.84]	1.28 [2.43]	1.29 [2.44]	0.05 [4.15]	0.05 [4.30]
ln(Eq Mkt Cap)	-16.18 [-3.84]		-16.00 [-4.12]		0.75 [0.18]		-1.01 [-7.99]	
ln(Firm Value)		-17.08 [-3.93]		-17.39 [-4.03]		-1.96 [-0.41]		-0.96 [-7.96]
Firm Age	0.30 [1.22]	0.31 [1.27]	0.19 [0.76]	0.21 [0.85]	0.32 [1.50]	0.34 [1.59]	-0.00 [-0.70]	-0.00 [-0.70]
ROA	-7.88 [-1.21]	-7.50 [-1.16]	-1.86 [-0.30]	-1.64 [-0.27]	9.98 [1.67]	9.67 [1.63]	-0.63 [-4.15]	-0.62 [-4.09]
Eq Beta	1.02 [0.07]	1.72 [0.12]	4.40 [0.30]	4.59 [0.31]	-2.60 [-0.21]	-1.87 [-0.15]	0.43 [1.68]	0.42 [1.65]
Asset Tangibility	-0.75 [-1.71]	-0.73 [-1.66]	-0.72 [-1.76]	-0.70 [-1.70]	-0.15 [-0.46]	-0.18 [-0.57]	-0.02 [-1.77]	-0.02 [-1.52]
Interest Coverage	-0.38 [-0.83]	-0.21 [-0.48]	-0.39 [-0.86]	-0.22 [-0.52]	0.07 [0.17]	0.14 [0.34]	-0.04 [-3.26]	-0.04 [-2.95]
Hist - Current Lev	0.44 [0.90]	0.42 [0.87]	0.41 [0.77]	0.37 [0.70]	0.30 [0.65]	0.29 [0.63]	0.01 [0.47]	0.00 [0.39]
R-sqd	21.43	21.96	19.01	19.75	29.35	29.38	66.73	66.32
Obs	3,692	3,692	3,021	3,021	2,975	2,975	2,237	2,237

Panel C: Liquidity Variables

	Unexplained Spread					
Rating	14.25	14.20	14.22	14.27	14.29	14.17
	[ 8.02]	[ 8.06]	[ 8.18]	[ 8.10]	[ 8.07]	[ 8.20]
Eq Vol	1.42	1.38	1.36	1.30	1.43	1.38
	[ 2.83]	[ 2.77]	[ 2.72]	[ 2.60]	[ 2.85]	[ 2.76]
Age	2.80	3.14	3.22	3.20	2.67	3.24
	[ 3.07]	[ 3.22]	[ 3.42]	[ 3.35]	[ 2.91]	[ 3.38]
ln(Amt)	-8.79	-11.27	-12.03	-7.91	-7.96	-11.98
	[-3.70]	[-3.74]	[-4.04]	[-3.10]	[-3.15]	[-3.99]
ln(Volume)		3.20				
		[ 1.58]				
ln(Trades)			5.89			
			[ 2.00]			
Turnover				0.32		
				[ 1.84]		
ln(Trd Size)					-3.66	
					[-1.53]	
% Days Trd						0.28
						[ 2.05]
R-sqd	34.08	34.24	34.60	34.41	34.23	34.55
Obs	4,143	4,143	4,143	4,129	4,143	4,143

The data is at the bond-quarter level from 2003 Q1 to 2007 Q4. For each firm, one bond close to four years to maturity and one bond close to ten years to maturity are used. The only exception is in Panel B when ratings are the dependent variable. For those regressions, the data is at the firm-quarter level. The unexplained spread is the predicted residual from a regression of observed yield spreads on BC yield spreads with time fixed-effects. Yield spreads are reported in basis points and are winsorized at 1% of each tail. The dependent variable is the unexplained yield spread unless otherwise labeled. All regressions include time fixed-effects. Ratings are coded as 1 for Aaa and 21 for C with intermediate ratings also coded. Mktlev is the market leverage of a firm, reported in %. Asset volatility is the model-implied asset volatility in %. Equity volatility is the past three months' equity volatility in %. IV - equity volatility is the difference between equity option implied volatility and recently realized equity volatility. Eq Mkt Cap and Firm Value are reported in \$mm. ROA is the mean quarterly return on assets over the last ten years reported in %. Equity Beta is a firm's equity CAPM beta. Asset tangibility is measured using the estimates in Berger, Ofek, and Swary (1996) and is reported in %. Interest coverage is EBIT divided by interest expense. Hist - Current Lev is the difference between a firm's mean leverage from 1993 to 2002 and its current leverage. Age (of a bond) and firm age are reported in years. ln(Amt) is the log face value amount outstanding for a bond in ln(\$mm). ln(Volume) is the volume of trading for a bond reported in ln(\$mm face value). ln(Trd Size) is the log of the average trade size in ln(\$k face value). Turnover is reported in %. % Days Traded is the percentage of days in which a bond was traded at least once. Reported t-stats use standard errors clustered by firm. Reported  $R^2$  values are within-group  $R^2$ s.

Table 4: The Effect of Recent Equity Volatility

BC Quint		Eq Vol Quintile					Q5-Q1	t-stat	std
		1	2	3	4	5			
1 & 2	Eq Vol	13.32	17.08	20.36	24.95	36.25			
	Obs Spd	52.88	55.44	67.71	75.78	114.07	61.19	3.30	68.77
	BC Spd	0.11	0.12	0.12	0.12	0.08	-0.03	-1.56	0.21
3	Eq Vol	14.90	18.72	22.26	26.59	36.73			
	Obs Spd	89.84	98.95	94.38	115.73	140.98	51.14	3.41	73.28
	BC Spd	5.13	4.59	5.31	5.32	6.01	0.87	1.59	3.52
4	Eq Vol	14.81	19.36	23.13	28.54	41.37			
	Obs Spd	116.13	122.41	130.25	150.20	226.00	109.87	4.80	105.39
	BC Spd	39.10	36.92	38.16	39.41	40.34	1.25	0.52	18.69
5	Eq Vol	17.04	23.69	29.76	38.30	59.61			
	Obs Spd	173.09	181.00	262.62	338.32	1,181.89	1,008.80	2.51	2,354.65
	BC Spd	278.44	305.76	229.43	288.36	597.81	319.38	2.76	344.87

The sample is the same as in Table 3. Bonds are sorted into quintiles by Black-Cox yield spreads. Quintiles 1 and 2 are combined and within each group, bonds are then sorted by past three months' equity volatility. Equity volatility is reported in % and yield spreads in basis points. Reported t-stats use standard errors clustered by firm and time. Reported standard deviations are standard deviations within a Black-Cox quintile.

Table 5: Alternative Specifications of Recovery Rates

Panel A: Recovery Rates by Industry					
	Obs Spd	Unexplained Spread			
BC Spd	0.46 [ 7.62]				
Rating					17.15 [ 9.22]
Mktlev		1.24 [ 2.37]	-0.12 [ -0.27]		
Asset Vol		1.44 [ 1.02]	-1.06 [ -0.93]		
Eq Vol			4.04 [ 6.03]	4.22 [ 4.51]	
IV - Eq Vol				2.46 [ 2.77]	
R-sqd	44.57	4.04	16.84	14.57	30.08
Obs	4,034	4,034	4,034	3,009	3,996

Panel B: Recovery Rates by Observed Yield Spread					
	Obs Spd	Unexplained Spread			
BC Spd	0.41 [ 8.46]				
Rating					15.66 [ 9.32]
Mktlev		1.32 [ 3.32]	0.35 [ 1.07]		
Asset Vol		1.88 [ 1.96]	0.08 [ 0.10]		
Eq Vol			2.92 [ 4.72]	3.61 [ 4.13]	
IV - Eq Vol				2.38 [ 3.10]	
R-sqd	50.49	4.85	12.54	12.00	28.80
Obs	4,227	4,227	4,227	3,145	4,143

Variables are defined as in Table 3. The model-implied yield spreads in Panel A are calculated using recovery rates from Altman and Kishore (1996). In Panel B, firms with high average observed yield spreads for their base case average model yield spread decile are assigned a recovery rate of 15.44%, medium average observed yield spreads are assigned a recovery rate of 41%, and low average observed yield spreads are assigned a recovery rate of 66.56%. All regressions include time fixed-effects. Reported t-stats use standard errors clustered by firm. Reported  $R^2$  values are within-group  $R^2$ s.

Table 6: Level of Yield Spreads, Jump Model

## Panel A: 4yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		Jump Yield	
				mean	med	mean	med
aaa	77	20.21	21.84	3.82	4.12	5.26	5.03
aa	182	15.70	22.04	4.01	4.06	5.09	4.94
a	866	24.78	22.16	4.18	4.30	5.31	4.96
baa	677	30.05	23.08	5.01	4.64	5.32	5.20
ba	185	42.11	20.54	6.37	6.54	6.12	5.93
b	125	54.86	20.01	7.42	7.19	8.48	7.66
c	57	67.00	18.99	10.93	10.06	10.60	10.56
full	2,209	29.79	22.06	4.99	4.41	5.70	4.96

Rating	Observed Spread		Jump Spread		mean	Difference	
	mean	med	mean	med		med	t-stat
aaa	32.63	31.31	24.71	9.77	7.92	16.01	0.64
aa	31.94	32.94	14.41	0.26	17.52	29.46	1.55
a	57.38	50.97	33.38	2.13	24.00	40.38	2.67
baa	94.60	93.01	50.72	26.74	43.88	53.98	4.34
ba	210.94	285.63	135.98	101.71	74.96	173.17	2.91
b	307.85	326.21	372.72	277.84	-64.87	90.74	-0.89
c	662.88	628.18	584.54	564.46	78.35	127.39	0.54
full	113.20	63.22	81.16	2.77	32.04	45.11	3.24

## Panel B: 10yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		Jump Yield	
				mean	med	mean	med
aaa	84	16.48	21.00	5.00	5.02	5.23	4.96
aa	105	13.77	22.42	5.16	5.09	5.08	5.02
a	458	29.00	21.25	5.34	5.25	5.69	5.28
baa	478	29.13	23.95	6.09	6.06	5.78	5.69
ba	173	40.42	20.98	7.33	6.92	6.39	6.01
b	98	51.46	18.70	8.39	7.99	7.24	6.63
c	25	73.43	11.39	10.49	11.06	8.60	8.90
full	1,443	30.97	21.83	6.12	5.63	5.89	5.41

Rating	Observed Spread		Jump Spread		mean	Difference	
	mean	med	mean	med		med	t-stat
aaa	51.29	51.25	25.99	2.46	25.30	38.37	1.55
aa	69.28	65.61	14.80	8.24	54.48	54.27	7.97
a	92.22	80.35	72.59	34.01	19.63	42.46	1.34
baa	152.87	142.92	96.52	75.71	56.35	51.99	3.73
ba	270.36	240.21	165.09	119.76	105.27	105.93	4.24
b	375.46	344.54	249.14	173.15	126.32	163.80	3.44
c	585.83	652.88	389.53	414.94	196.29	234.01	2.12
full	160.01	113.44	102.57	47.15	57.44	54.12	4.66

Observations are at the bond-quarter level from 2003 Q1 to 2007 Q4. Leverage, asset volatility, and yields are reported in %. Spreads and the difference in spreads are reported in basis points. T-statistics for the difference use standard errors that are clustered by firm and time.

Table 7: Cross-Section, Jump Model

Panel A: Sorts

BC Quint		Jump Risk Premium Quintile					Q5-Q1	t-stat	std
		1	2	3	4	5			
1 & 2	Jump RP (%)	0.49	2.92	5.24	7.67	15.15			
	Obs Spd	71.79	64.00	62.65	58.80	66.77	-5.02	-0.56	56.87
	BC Spd	0.10	0.09	0.10	0.12	0.09	-0.01	-0.26	0.19
	Jump Spd	0.19	0.34	2.02	4.97	32.22	32.04	2.44	92.05
3	Jump RP (%)	0.56	3.02	5.43	8.02	13.63			
	Obs Spd	101.70	92.58	91.35	95.92	109.91	8.21	0.56	66.53
	BC Spd	4.47	5.08	4.48	4.57	4.88	0.42	1.24	3.13
	Jump Spd	5.91	7.92	10.93	16.58	46.65	40.74	5.90	41.87
4	Jump RP (%)	0.32	2.32	4.84	7.32	14.08			
	Obs Spd	140.43	111.24	139.52	128.97	166.47	26.03	1.99	86.68
	BC Spd	35.52	33.77	37.22	34.27	33.43	-2.09	-1.56	16.73
	Jump Spd	35.85	39.61	53.50	60.44	114.38	78.53	6.83	71.11
5	Jump RP (%)	0.04	0.92	2.91	6.07	19.39			
	Obs Spd	300.17	252.44	250.32	268.59	396.26	96.09	1.34	329.81
	BC Spd	308.05	318.61	345.79	223.15	304.93	-3.12	-0.05	304.18
	Jump Spd	306.17	326.65	378.77	271.42	488.91	182.74	2.24	334.53

Panel B: Regressions

	Full	Full	Full	4yr	10yr	IG
BC Spread	0.54		0.54	0.52	0.55	0.36
	[ 8.29]		[ 8.59]	[ 8.00]	[ 6.85]	[ 7.99]
Jump Spread		0.45				
		[ 10.02]				
Jump Residual			0.24	0.22	0.31	0.16
			[ 4.77]	[ 3.87]	[ 3.71]	[ 2.37]
R-sqd	43.44	43.64	45.29	48.14	41.17	33.73
Obs	3,652	3,652	3,652	2,209	1,443	2,927

The data is at the bond-quarter level from 2003 Q1 to 2007 Q2. The bonds in this table are the subset of bonds from Table 3 where a jump model spread could be calculated. In Panel A, bonds are sorted into quintiles by Black-Cox yield spreads. Quintiles 1 and 2 are combined and within each group, bonds are then sorted by jump risk premia. The jump risk premia are reported in %. Spreads are reported in basis points. Reported t-stats use standard errors clustered by firm and time. Reported standard deviations are standard deviations within a Black-Cox quintile. In Panel B, the Jump residual is constructed as the residual from a regression of the jump model yield spread on the Black-Cox yield spread with time fixed-effects. The dependent variable in the reported regressions is the observed yield spread and all regressions contain time fixed-effects. Yield spreads in this panel are winsorized at 1% of each tail. Reported t-stats use standard errors clustered by firm. Reported  $R^2$  values are within-group  $R^2$ s.

Table 8: Level of Yield Spreads, Stochastic Volatility Model

## Panel A: 4yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		SV Yield	
				mean	med	mean	med
aaa	83	21.21	23.37	3.88	4.17	5.10	4.98
aa	200	14.51	23.91	4.06	4.13	4.92	4.89
a	952	25.35	23.27	4.24	4.37	5.23	4.89
baa	778	32.75	25.16	5.06	4.72	5.72	4.98
ba	214	42.36	24.52	6.49	6.59	7.03	5.77
b	154	55.64	26.41	7.55	7.09	10.15	9.26
c	90	69.94	35.90	12.78	10.71	13.87	13.04
full	2,518	31.62	24.72	5.21	4.52	6.15	4.91

Rating	Observed Spread		SV Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
aaa	35.03	33.70	11.73	8.62	23.30	19.08	4.13
aa	36.09	38.08	0.80	0.14	35.30	37.25	9.27
a	62.33	57.43	28.96	0.55	33.37	50.13	4.09
baa	100.72	99.72	93.61	9.37	7.11	68.80	0.32
ba	223.76	289.59	229.87	90.24	-6.11	188.81	-0.06
b	329.25	318.72	541.89	441.38	-212.63	-71.48	-1.68
c	860.67	694.97	912.21	817.43	-51.53	-6.08	-0.25
full	134.50	73.00	129.33	1.78	5.17	52.75	0.30

## Panel B: 10yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		SV Yield	
				mean	med	mean	med
aaa	90	16.84	21.99	5.03	5.06	5.07	4.89
aa	117	13.58	23.87	5.19	5.12	4.95	4.92
a	515	29.33	22.96	5.39	5.31	5.43	5.08
baa	562	29.65	26.31	6.17	6.08	5.58	5.35
ba	203	40.68	27.20	7.33	7.02	6.37	5.75
b	128	51.16	25.56	8.56	8.15	7.49	7.35
c	40	72.67	25.67	12.02	11.37	9.49	9.67
full	1,692	31.70	24.95	6.28	5.74	5.80	5.20

Rating	Observed Spread		SV Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
aaa	53.62	55.68	13.05	0.40	40.56	42.94	4.87
aa	71.34	68.60	6.07	3.50	65.27	63.98	11.53
a	96.04	85.71	49.53	18.95	46.51	63.68	4.58
baa	160.09	147.02	80.36	46.15	79.73	84.97	5.58
ba	271.87	247.23	165.22	97.99	106.65	132.60	4.09
b	394.47	360.82	278.38	250.26	116.09	131.81	2.80
c	748.28	684.33	480.11	495.77	268.17	212.13	1.27
full	175.72	122.39	97.16	31.08	78.56	74.46	7.30

Panel C: Parameter Estimates, Stochastic Volatility Model

Parameter	Mean	Std Dev	25th	50th	75th
$\kappa_H$	11.67	6.11	7.53	10.68	14.20
$\theta_H$	0.0727	0.0518	0.0443	0.0606	0.0868
$\sigma_H$	0.9436	0.4409	0.6614	0.8464	1.1796
$\rho$	-0.1490	0.2256	-0.2164	-0.1138	-0.0193

In Panels A and B, observations are at the bond-quarter level. Leverage, asset volatility, and yields are reported in %. The reported asset volatility is the square root of the average long-run asset variance,  $\theta_H$ . Spreads and the difference in spreads are reported in basis points. T-statistics for the difference use standard errors that are clustered by firm and time. In Panel C, firm-level parameter estimates are reported for the 286 firms in the sample.

Table 9: Cross-Section, Stochastic Volatility Model

		Panel A: Sorts							
		SV - Mer Quintile					Q5-Q1	t-stat	std
Mer Quint		1	2	3	4	5			
1 & 2	Rho	-0.05	-0.11	-0.11	-0.14	-0.20			
	Obs Spd	60.43	52.86	65.86	71.00	76.77	16.33	1.99	49.44
	Mer Spd	0.13	0.01	0.08	0.34	0.49	0.36	5.67	0.37
	SV Spd	0.11	0.02	0.16	0.67	8.64	8.53	3.15	18.69
3	Rho	0.02	-0.08	-0.12	-0.24	-0.23			
	Obs Spd	143.01	97.63	97.20	95.37	92.19	-50.82	-2.90	67.05
	Mer Spd	8.39	6.24	6.99	7.75	8.97	0.59	0.92	4.79
	SV Spd	7.43	6.67	8.32	10.55	43.02	35.59	3.46	36.74
4	Rho	-0.04	-0.08	-0.17	-0.20	-0.27			
	Obs Spd	188.05	129.93	149.09	154.94	151.78	-36.27	-1.33	100.02
	Mer Spd	52.14	37.88	42.25	47.02	54.45	2.31	0.41	22.88
	SV Spd	46.85	38.47	45.06	52.94	100.24	53.39	4.50	40.46
5	Rho	-0.40	-0.18	-0.18	-0.24	-0.21			
	Obs Spd	508.40	439.38	363.30	320.52	196.34	-312.05	-3.50	482.40
	Mer Spd	1,058.01	446.09	321.34	318.61	376.22	-681.79	-3.24	564.72
	SV Spd	910.44	430.94	323.75	336.33	475.67	-434.77	-2.66	465.34

Panel B: Regressions

	Full	Full	Full	4yr	10yr	IG
Merton Spread	0.38		0.38	0.35	0.61	0.24
	[ 7.00]		[ 7.00]	[ 6.54]	[ 8.14]	[ 4.26]
SV Spread		0.39				
		[ 7.05]				
SV Residual			0.04	0.05	-0.05	-0.11
			[ 0.27]	[ 0.42]	[ -0.13]	[ -1.40]
R-sqd	46.01	45.11	46.02	50.04	53.12	23.64
Obs	4,186	4,186	4,186	2,499	1,687	3,273

The data is at the bond-quarter level from 2003 Q1 to 2007 Q4. For each firm, one bond close to four years to maturity and one bond close to ten years to maturity are used. In Panel A, bonds are sorted into quintiles by Merton yield spreads. Quintiles 1 and 2 are combined and within each group, bonds are then sorted by the difference between stochastic volatility yield spreads and Merton yield spreads. Rho is the estimated correlation between asset return and asset variance shocks. Spreads are reported in basis points. Reported t-stats use standard errors clustered by firm and time. Reported standard deviations are standard deviations within a Merton quintile. In Panel B, the SV residual is constructed as the residual from a regression of the stochastic volatility yield spread on the Merton yield spread with time fixed-effects. The dependent variable in the reported regressions is the observed yield spread and all regressions contain time fixed-effects. Yield spreads are winsorized at 1% of each tail. Reported t-stats use standard errors clustered by firm. Reported  $R^2$  values are within-group  $R^2$ s.

Table 10: Alternative Specifications

Panel A: Stochastic Volatility with Slower Mean-Reversion

	Full	Full	Full	4yr	10yr	IG
Merton Spread	0.38		0.38	0.35	0.61	0.24
	[ 7.00]		[ 7.13]	[ 6.63]	[ 8.85]	[ 4.11]
SV Spread		0.46				
		[ 6.09]				
SV Residual			-0.12	-0.07	-0.41	0.17
			[ -0.88]	[ -0.54]	[ -1.87]	[ 1.49]
R-sqdl	46.01	41.80	46.18	50.10	54.24	24.11
Obs	4,186	4,186	4,186	2,499	1,687	3,273

Panel B: Black-Cox with Recent Equity Volatility

	Full	Full	Full	4yr	10yr	IG	Unexp Spd
BC Spread	0.58		0.58	0.58	0.58	0.32	
	[ 8.02]		[ 7.98]	[ 6.90]	[ 6.66]	[ 8.10]	
BC Spread (period)		0.70					
		[ 8.84]					
BC Spread (period) Residual			0.44	0.45	0.42	0.25	
			[ 5.90]	[ 4.83]	[ 4.72]	[ 4.23]	
Eq Vol							1.71
							[ 3.23]
R-sqdl	44.92	48.12	52.95	55.05	49.50	32.97	3.32
Obs	4,238	4,238	4,238	2,546	1,692	3,324	4,238

In Panel A, the SV residual is constructed as the residual from a regression of the (alternative) stochastic volatility model yield spread on the Merton yield spread with time fixed-effects. In Panel B, the BC (period) residual is constructed as the residual from a regression of the BC (period) yield spread on the Black-Cox yield spread with time fixed-effects. In both panels, the dependent variable is the observed yield spread with the exception of the last column of Panel B where the dependent variable is the residual from a regression of the observed yield spread on BC (period) yield spread. All yield spreads are winsorized at 1% of each tail and all regressions contain time fixed-effects. Reported t-stats use standard errors clustered by firm. Reported  $R^2$  values are within-group  $R^2$ s.

Table 11: Credit Default Swaps

Panel A: Levels of CDS Premia

Rating	Obs	Observed CDS Spread	Model CDS Spread	Difference		t-stat
				Mean	Med	
Aaa & Aa	195	12.42	4.66	7.76	9.97	2.50
A	538	26.51	10.23	16.28	18.98	3.48
Baa	741	63.91	31.98	31.92	32.65	3.76
Junk	407	337.48	229.23	108.25	118.90	2.01
Full	1,905	107.61	65.23	42.38	25.64	3.24

Panel B: Cross-Sectional Regressions

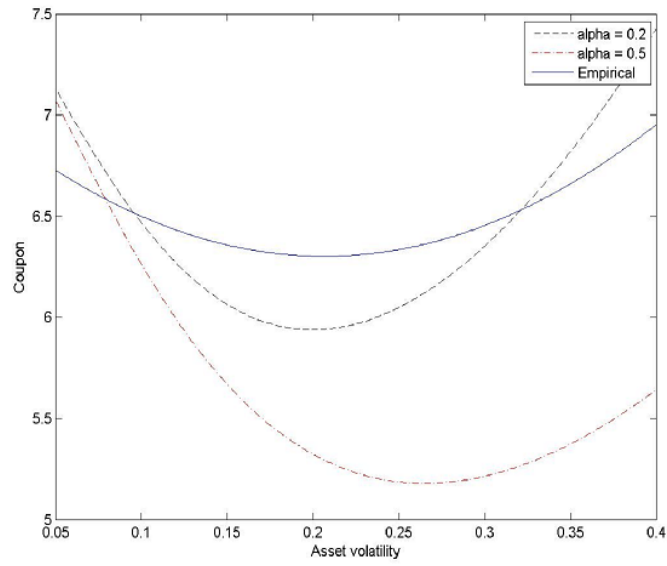
	Obs Spd	Unexplained Spread					
BC Spread	0.59 [3.94]						
Rating				17.80 [6.66]			
Mktlev		2.15 [2.51]	0.40 [0.46]		0.70 [0.74]	0.95 [1.01]	
Asset Vol		1.08 [0.74]	-1.50 [-1.28]				
Eq Vol			5.60 [5.27]	6.41 [5.06]	5.68 [5.30]	5.67 [5.45]	
IV - Eq Vol				4.57 [4.09]	3.71 [5.13]	3.72 [5.08]	
ln(Eq Mkt Cap)					-10.88 [-1.77]		
ln(Firm Value)						-12.40 [-2.28]	
Firm Age					0.34 [1.47]	0.37 [1.55]	
ROA					6.35 [0.69]	6.61 [0.72]	
Eq Beta					18.68 [1.10]	18.88 [1.10]	
Asset Tangibility					-1.57 [-3.30]	-1.58 [-3.27]	
Interest Coverage					0.08 [0.12]	0.19 [0.34]	
Hist - Current Lev					0.28 [0.52]	0.27 [0.50]	
R-sq <sub>d</sub>	36.58	8.91	22.66	22.42	23.27	29.59	29.93
Obs	1,905	1,905	1,905	1,405	1,881	1,358	1,358

Panel C: Commonality

	CDS Spread	$\hat{\epsilon}_{CDS,1}$	$\hat{\epsilon}_{CDS,2}$
Bond Spread	1.05 [23.61]		
$\hat{\epsilon}_{Bond,1}$		0.98 [14.78]	
$\hat{\epsilon}_{Bond,2}$			0.87 [8.33]
R-sqd	70.16	49.62	37.67
Obs	1,914	1,914	1,876

The sample is a panel of quarterly five-year CDS with a matching panel of bonds that represent the bond closest to five years to maturity for each issuer. Data is from 2004 Q4 to 2007 Q4. CDS spreads are reported in basis points. Ratings are the average of the ratings of the corporate bonds for the issuer. In Panel A, t-stats use standard deviations clustered by time and firm. In Panel B, the dependent variable is the residual from a regression of observed CDS spreads on model CDS spreads with the exception of the first column for which the dependent variable is the observed CDS spread. In Panel B, CDS spreads and model CDS spreads are winsorized at 1% of each tail and all regressions contain time fixed-effects and reported t-stats use standard errors clustered by firm. Dependent variables are as defined in Tables 1 and 3. In Panel C,  $\hat{\epsilon}_{Bond,1}$  and  $\hat{\epsilon}_{CDS,1}$  have the Black-Cox model spreads partialled out while  $\hat{\epsilon}_{Bond,2}$  and  $\hat{\epsilon}_{CDS,2}$  also have recent equity volatility and ratings partialled out. All regressions contain time fixed-effects. T-statistics in Panels B and C are clustered by firm. Reported  $R^2$  values are within-group  $R^2$ s.

Figure 1: Relation Between Coupons and Asset Volatility



The empirical relation between coupons and asset volatility is plotted along with the theoretical relation from Leland (1994).  $\alpha$  represents the loss given default.