

An EBIT-Based Model of Dynamic Capital Structure ¹

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Formerly, “Endogenous Bankruptcy, Endogenous Restructuring, and Dynamic Capital Structure”

Abstract

A model of dynamic capital structure is proposed. Even though the optimal strategy is implemented over an arbitrarily large number of restructuring-periods, a scaling feature inherent in the framework permits simple closed-form expressions to be obtained for equity and debt prices. When a firm has the option to increase future debt levels, tax advantages to debt increase significantly, and both the optimal leverage ratio range and predicted credit spreads are more in line with what is observed in practice.

Most capital structure models assume that the decision of how much debt to issue is a static choice. In practice, however, firms adjust outstanding debt levels in response to changes in firm value. In this paper, we solve for the optimal dynamic capital strategy of a firm, and investigate the implications for optimal leverage ratios and the magnitude of the tax benefits to debt.

Below, we consider only the option to increase future debt levels. While in theory management can both increase and decrease future debt levels, Gilson (1997) finds that transactions costs discourage debt reductions outside of Chapter 11. In addition, equity's ability to force concessions when the firm is in distress (Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997)) may further reduce equity's incentive to repurchase outstanding debt.¹

Compared to an otherwise identical firm that is constrained to a static capital structure decision, a firm's option to increase future debt levels has two immediate consequences. First, management will choose to issue a smaller amount of debt initially. This might explain why most static models predict optimal leverage ratios that are well above what is observed in practice. Second, for a given level of initial debt, bonds issued from such a firm are riskier, since the bankruptcy threshold rises with the level of outstanding debt. This might explain why most static models predict yield spreads that are too low.² While covenants are often in place to protect debt holders, in practice, firms typically have the option to issue additional debt in the future without recalling the outstanding debt issues. Furthermore, in the event of bankruptcy, it is typical that all unsecured debt receive the same recovery rate, regardless of the issuance date.³ Clearly, such debt is riskier than that modeled in both the 'traditional' static capital structure models (e.g., Brennan and Schwartz (1978), Leland (1994), Leland and Toft (1996)), and most corporate bond pricing models (e.g., Longstaff and Schwartz (1995)), where it is assumed that the bankruptcy threshold remains constant over time. Interestingly, within our framework, when a firm has the right to issue additional debt at a specified upper boundary V_U , and it is assumed that all debt will receive the same recovery rate in the event

¹In addition, the fact that equity prices tend to trend upward, and that a typical maturity structure is a quickly decaying function of time (see Barclay and Smith (1985)) makes the option to issue additional debt a more important feature to account for than the option to repurchase outstanding debt.

²The strategic debt service literature of Anderson and Sundaresan (1992), and Mella-Barral and Perraudin (MP 1997) also help explain these puzzles. However, if the maturity structure of debt were modeled as it is observed in practice, that is, a quickly decaying maturity structure (Barclay and Smith (1995)), with an average maturity of about seven years (Stohs and Mauer (1996)), rather than as a perpetuity, as in MP (1997), then the strategic debt argument by itself is not sufficient to explain the observed yield spreads. See Goldstein and Pirinsky (1998).

³Longstaff and Schwartz (1995) report that in December 1992, GMAC had 53 outstanding long-term debt issues listed in Moody's Bank and Finance Manual, and that all would likely receive the same recovery rate in the event of bankruptcy.

of bankruptcy, then the present value of the current debt issue is the same as if it were callable at par when firm value reaches V_U . In this sense, it is clear that such debt is not as valuable as debt issued by a firm constrained to never issue additional debt. This lower price in turn implies a higher yield spread.

Models of dynamic capital structure choice have been proposed by Kane, Marcus, and McDonald (KMM 1984, 1985) and Fischer Heinkel and Zechner (FHZ 1989). Concerned with the possibility of arbitrage inherent in the ‘traditional frameworks’, FHZ base their model on the following assumptions:

- 1) The value of an optimally levered firm can only exceed its unlevered value by the amount of transactions costs incurred in order to lever it up.

- 2) A firm that follows an optimal financing policy offers a fair risk-adjusted rate of return. Therefore, if leverage is advantageous, then it follows that unlevered firms offer a below-fair expected rate of return.

Their first assumption is intended to eliminate the following arbitrage strategy: purchase the firm cheaply, lever it up, then sell for a riskless profit. Their second assumption implies that tax benefits to debt are to be measured as a flow, not as an increase in value. However, we argue that these assumptions do not hold in practice. First, consider a firm whose (well-entrenched) management refuses to take on debt (e.g., Microsoft). It would be irrational for agents to push the value of the firm up to its optimally levered value, since shareholders are currently not receiving any tax benefits. Furthermore, the above-mentioned ‘arbitrage strategy’ is not operational: usually, a bidder must pay a large premium in order to gain control of management to force an optimal capital structure policy to be followed. This premium will typically eliminate any profit from such a strategy. Note that this first assumption predicts that no portion of the premium paid during a managed buyout is due to increased tax benefits, since all benefits are assumed to be embedded into the price before the buyout. This prediction is in conflict with the findings of both Kaplan (1989) and Leland (1989).

While the second assumption might be applicable to an all-equity homeowner who forgoes tax benefits that are inherent in a mortgage on a house,⁴ we argue that it is not applicable for publicly traded assets. Rather, rational agents in the economy will push down current prices so that fair expected return is obtained on any traded asset *regardless* of the policies followed by management.

⁴This example was given by KMM (1985) to motivate their assumption.

KMM (1984, 1985) and FHZ (1989) obtain relatively simple solutions for the optimal dynamic strategy because their assumptions effectively reduce the analysis to a one-period framework.⁵ Indeed, the boundary conditions satisfied by the levered firm value (Equations 11 and 12) in FHZ (1989) are exactly the boundary conditions satisfied by a firm which wishes to be leveraged for just one single period. As a consequence of these assumptions, FHZ (1989) predict that the tax benefits to debt are mostly negligible, and that tax benefits to debt are an increasing function of firm value volatility. Both of these results are in contradiction to the more direct tax-benefit measurements of Graham (1998), and the results obtained below.

In this paper, we determine the optimal capital structure strategy of a firm when it has the option to increase debt levels in the future. In particular, we investigate the contingent cash flows for arbitrary capital structure strategies, and allow management to choose the one that maximizes current shareholder wealth, subject to limited liability.

A scaling feature inherent in our framework permits simple closed-form expressions to be obtained for the security prices even though the optimal strategy is implemented over an arbitrarily large number of restructurings. The intuition is as follows.⁶ Define period-zero as the time interval for which firm value remains between the bankruptcy threshold V_B^0 and the restructuring threshold V_U^0 . The value of the firm when a capital structure decision is made is V_0^0 . If V_B^0 is reached, the firm declares bankruptcy, and all future cash flows are zero. However, if V_U^0 is reached, the firm increases its debt level, and period-1 begins. Define the claim to period-0 cash flows as CF_0 . The scaling feature implies that the present value of period-n cash flows equal $CF_0 * \beta^n$, where the endogenously obtained constant β accounts for both the fact that each period's cash flows scale upward by a factor $\gamma \equiv \frac{V_U^0}{V_0^0}$, and that these cash flows will be discounted for both time and risk of default. This scaling feature implies that the present value of the claim to all period's cash flows is

$$\sum_{n=0}^{\infty} CF_0 * \beta^n = \frac{CF_0}{1 - \beta}.$$

Hence, it is the factor $\left(\frac{1}{1-\beta}\right)$ that captures the dynamic features of our framework.

We note that the 'traditional framework' for these models is to endow the unlevered equity with log-normal dynamics, and take it to be the state variable of the model. While the model proposed below can be set in this traditional framework,⁷ we propose a framework similar to

⁵KMM (1985) acknowledge so much themselves.

⁶For more details see Appendix B.

⁷In an earlier version, we used the traditional framework to measure the present value of tax benefits and bankruptcy costs of all periods in order to determine optimal strategy. Results similar to those presented below were obtained.

that used by Mello and Parsons (1992), Mella-Barral and Peraudin (1997), and others that we feel offers several advantages. But before introducing this proposed framework, we review the traditional framework. The state variable in the traditional framework is the unlevered equity value, which is presumed to have the following risk-neutral dynamics

$$\frac{dV}{V} = \left(r - \frac{\delta}{V}\right) dt + \sigma dz^Q.$$

Here, V is the unlevered equity value, r is the (pre-tax) risk-free rate, $\frac{\delta}{V}$ is the payout rate, and σ is the unlevered firm volatility. The present value of both the tax-benefit claim and the bankruptcy cost claim are then determined for a given level of debt issuance in order to estimate the net tax benefits to debt. However, the risk neutral drift being set to $(r - \frac{\delta}{V})$ raises the ‘delicate issue’ of whether the unlevered firm remains a traded asset after debt has been issued.⁸

Another difficulty with these models is that they treat cash flows to government (via taxes) in a fundamentally different manner than they treat cash flows to equity and debt. Indeed, they model the ‘tax benefit’ as an inflow of funds, rather than as a reduction of outflow of funds. Hence, these models characterize firm payout as the sum of dividends and coupon payments, less this tax benefit inflow, even though the assets which generate the cash flows for dividend and interest payments are the same assets which generate the cash flows for tax payments. This leads to several problems. First, this causes comparative statics analysis to predict that equity value is an increasing function of the tax rate, since the ‘tax benefit’ paid to equity increases with this rate. From a normative standpoint, this prediction is in conflict with a discounted cash flow analysis of equity pricing, unless an increase in the tax rate leads either to an increase in the growth rate of future cash flows, or to a decrease in the risk of the cash flow streams – both of which seem unlikely. Furthermore, Lang and Shackelford (1998) provide strong evidence against this prediction by investigating stock price reactions during the week that the May 1997 budget accord was signed.⁹ In contrast, our model predicts equity prices are a decreasing function of the tax rate.

Second, the traditional framework may significantly overestimate the risk-neutral drift $(r - \frac{\delta}{V})$, which in turn will cause the probability of bankruptcy to be biased downward, *ceteris paribus*. In contrast, in our framework, the appropriate risk-free rate is the after-tax rate,

⁸Criticism of this approach is noted in footnote 11 of Leland (1994). Eliminating this ‘delicate issue’ is one of the motivating factors behind the frameworks of KMM (1984, 1985) and FHZ (1989). Hence, it is reassuring that this criticism can be circumvented without needing to accept the previously-mentioned assumptions of their models.

⁹The budget accord reduced the long-term capital gains from 28 percent to 20 percent.

and the appropriate payout ratio includes payouts to government via taxes.¹⁰ Empirically we find that our framework implies a significantly lower risk-neutral drift than the traditional framework, in turn predicting higher probability of bankruptcy, thus leading to a lower optimal leverage ratio. Indeed, to compensate for this bias, models using the traditional framework typically need to assume unrealistically large bankruptcy costs in order to obtain yield spreads consistent with empirical evidence. Such large bankruptcy costs are in conflict with the empirical evidence of Andrade and Kaplan (1998), and also force the predicted recovery rates to be considerably lower than what is observed.

As a third difficulty, the traditional models cannot justify their assumption that the payout ratio $\frac{\delta}{V}$ is a constant, independent of V . Indeed, since the interest payments and related tax benefits are presumed to be constant, these models inherently assume that the equity dividend ratio changes dramatically with changes in firm value, in conflict with empirical findings. In contrast, in the model proposed below, where the cash flow to government is treated in the same fashion as cash flows to equity and debt, an increase in firm value leads to an increase in tax payments, making more reasonable the assumption that the payout ratio is constant.

To overcome these difficulties, we propose a different framework. Even before Black and Scholes (1973) it was realized that it is better to model the dynamics of (equity + debt) as log-normal, rather than equity alone.¹¹ Here, we extend this logic and model the dynamics of the claim to EBIT as log-normal. The implication is that all contingent claimants (equity, debt, government, etc.) to future EBIT flows are treated in a consistent fashion.¹² Below, we assume that EBIT is invariant to changes in capital structure. This follows from the usual assumption of separation of investment and financing policy. Such an approach has intuitive appeal: the EBIT-generating machine, which is the source of firm value, runs independently of how the EBIT flow is distributed among its claimants. In particular, an extra dollar paid out to either taxes, interest payments, or dividends all affect the firm in the same way. One advantage of this framework is that, as in practice, levered and unlevered equity never exist simultaneously. Another advantage is that, in contrast to the unlevered firm value, which ceases to exist upon issuance of debt, the claim to future EBIT flow should in practice be reasonably insensitive to changes in capital structure. Hence, the claim to EBIT flow is an appropriate choice for state variable. Indeed, this invariance feature makes our framework ideal for investigating multiple capital structure changes, and hence, optimal dynamic capital

¹⁰Note that in our framework both δ and V will be larger, since now V includes the government claim. Still, our estimates using Compustat find the risk neutral drift to be significantly lower in our model than in the traditional model.

¹¹For empirical support, see Toft and Prucyk (1997).

¹²A similar argument has been suggested recently by Galai (1998).

structure strategy. As demonstrated below, when a firm has the ability to increase its leverage in the future, predicted optimal leverage ratios are reasonably consistent with observed ratios.

Our framework has some similarity to that proposed by Graham (1998), who models EBIT flow to obtain an estimate of the tax advantage to debt. Since it is numerically based, his framework can accurately account for tax carry-forward and tax carry-back effects. His model first obtains an estimate of the marginal tax rate associated with an extra dollar of debt, and then subtracts a ‘personal tax penalty’ that exists due to the tax rate on interest being higher than the tax rate on capital gains. Note, however, that the framework requires the equity-holder to pay this personal tax penalty no matter how poorly the firm performs in the future. That is, this model does not permit equity-holders to choose bankruptcy. Besides affecting the estimated tax advantage to debt, such a framework precludes an estimate of the spread paid on the risky debt, since the debt is presumed to be riskless. Still, we find estimates of the optimal tax advantage to debt to be very similar to those obtained by Graham (1998).

The rest of the paper is as follows. Section 1 investigates a one-time capital structure decision of management whose objective is to maximize current shareholder wealth. Results are compared to Leland (1994). We also determine the current value of unlevered equity when a (static) capital structure change may occur in the future. In Section 2, we determine optimal dynamic capital structure when management is able to increase its outstanding debt in the future. We conclude in Section 3.

1 Optimal Static Capital Structure

Here we consider a simple model of firm dynamics. To emphasize that the proposed framework is self-consistent, we demonstrate that it can be supported within a rational-expectations general equilibrium framework. Consider a special case of Goldstein and Zapatero (1996). The single technology of a (pure-exchange) economy produces a payout flow that is specified by the process

$$\frac{d\delta}{\delta} = \mu_P dt + \sigma dz, \quad (1)$$

where μ_P and σ are constants. It can be shown that if the representative agent has a power-function utility, then both the endogenously obtained risk premium θ and risk-free rate r supported by this economy are constants. It is well known that any asset of the economy can then be priced by discounting expected cash flows under the risk-neutral measure. In particular, the value of the claim to the entire payout flow is

$$V(t) = E_t^Q \left[\int_t^\infty ds \delta_s e^{-rs} \right]$$

$$= \frac{\delta_t}{r - \mu}, \quad (2)$$

where $\mu = (\mu_P - \theta \sigma)$ is the risk-neutral drift of the payout flow rate:

$$\frac{d\delta}{\delta} = \mu dt + \sigma dz^Q.$$

Since r and μ are constants, both V and δ share the same dynamics:

$$\frac{dV}{V} = \mu dt + \sigma dz^Q. \quad (3)$$

It follows that

$$\frac{dV + \delta dt}{V} = r dt + \sigma dz^Q. \quad (4)$$

Thus, the total risk-neutral expected return on the claim is the risk-free rate, as necessary. The existence of an equivalent martingale measure guarantees that every contingent claim of this economy will receive ‘fair expected return’ for the risk borne. Note that this is in contrast to the assumptions of KMM (1984, 1985) and FHZ (1989).

We assume a simple tax structure that includes personal as well as corporate taxes. Interest payments to investors are taxed at a personal rate τ_i , ‘effective’ dividends are taxed at τ_d ,¹³ and corporate profits are taxed at τ_c , with full loss offset provisions.

First, consider a debtless firm with current value V_0 . Note that in the present framework there is no such thing as an all-equity firm. Rather, both equity and government have a claim to the firm’s payouts. Assume that the current management refuses to take on any debt, and that no takeover is likely (e.g., Microsoft). Then, the current firm value is divided between equity and government as

$$E = (1 - \tau_{\text{eff}}) V_0. \quad (5)$$

$$G = \tau_{\text{eff}} V_0, \quad (6)$$

where the effective tax rate is

$$(1 - \tau_{\text{eff}}) = (1 - \tau_c)(1 - \tau_d).$$

Now, consider an otherwise identical firm whose management decides to choose a (static) debt level that will maximize the wealth of current equity holders.¹⁴ It will issue a consol

¹³We shall be interpreting effective dividends to account for the fact that much of the double taxation occurs at a significantly lower, deferred-capital gains rate.

¹⁴We emphasize that, although the actual shares of stock are not recalled after such a debt issuance takes place, there no longer exists a claim to the contingent cash flows of the unlevered equity claim. Hence, the ‘delicate issue’ referred to previously is circumvented in this framework.

bond, promising a constant coupon payment C to debt holders as long as the firm remains solvent. Due to the issuance of a perpetuity, the threshold at which the firm chooses to default is time-independent. We define this threshold as V_B . If firm value reaches V_B , then an amount αV_B will be lost to bankruptcy costs.

In general, any claim must satisfy the partial differential equation (pde)

$$\mu V F_V + \frac{\sigma^2}{2} V^2 F_{VV} + F_t + P = rF, \quad (7)$$

where P is the payout flow. Due to the issuance of perpetual debt, all claims will be time-independent. Thus the pde reduces to an ordinary differential equation:

$$0 = \mu V F_V + \frac{\sigma^2}{2} V^2 F_{VV} + P - rF, \quad (8)$$

The general solution to

$$0 = \mu V F_V + \frac{\sigma^2}{2} V^2 F_{VV} - rF \quad (9)$$

is

$$F_{GS} = A_1 V^{-y} + A_2 V^{-x}, \quad (10)$$

where

$$x = \frac{1}{\sigma^2} \left[\left(\mu - \frac{\sigma^2}{2} \right) + \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right], \quad y = \frac{1}{\sigma^2} \left[\left(\mu - \frac{\sigma^2}{2} \right) - \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right],$$

and A_1, A_2 are constants, determined by boundary conditions. Note that x is positive, while y is negative, so the first term explodes as V becomes large. Hence, in this section, A_1 equals zero for all claims of interest.

The general solution does not account for intertemporal cash flows. Rather, these are accounted for by the particular solution. For example, if the relevant cash flow is the entire payout, $P = \delta = V(r - \mu)$, then the particular solution to Equation 8 is

$$F_{PS}^\delta = V. \quad (11)$$

Whereas, if the relevant payout is the coupon payment, $P = C$, then the particular solution to Equation 8 is

$$F_{PS}^C = \frac{C}{r}. \quad (12)$$

Before proceeding, it is convenient to define $p_B(V)$ as the present value of a claim that pays \$1 contingent upon firm value reaching V_B . Since such a claim receives no intermediate cash flows, we know from Equation 10 that $p_B(V)$ will be of the form:

$$p_B(V) = A_1 V^{-y} + A_2 V^{-x}. \quad (13)$$

Taking into account the boundary conditions

$$\lim_{V \rightarrow \infty} p_B(V) = 0, \quad \lim_{V \rightarrow V_B} p_B(V) = 1,$$

we find

$$p_B(V) = \left(\frac{V}{V_B} \right)^{-x}. \quad (14)$$

While the firm is solvent, equity, government, and debt share the payout δ through dividends, taxes, and coupon payments, respectively. Now, if all three claims were held by a single owner, she would be entitled to the entire payout δ as long as firm value remained above V_B . We define the value of this claim as $V_{solv}(V)$. From Eqs. 10 and 11, we know V_{solv} is of the form

$$V_{solv} = V + A_1 V^{-y} + A_2 V^{-x}. \quad (15)$$

For $V \gg V_B$, this claim must approach total firm value V . This implies that $A_1 = 0$. For $V = V_B$, the value of this claim vanishes. This constraint determines A_2 , giving

$$V_{solv} = V - V_B p_B(V). \quad (16)$$

The value of the claim to interest payments during continued operations, V_{int} , can be obtained in a similar fashion. As long as the firm remains solvent, the coupon payment is C . Thus, from Eqs. 10 and 12, V_{int} is of the form

$$V_{int} = \frac{C}{r} + A_1 V^{-y} + A_2 V^{-x}. \quad (17)$$

Again, for $V \gg V_B$, $D_c \rightarrow \frac{C}{r}$, implying $A_1 = 0$. Accounting for the fact that this claim vanishes at $V = V_B$, we obtain

$$V_{int} = \frac{C}{r} [1 - p_B(V)]. \quad (18)$$

Separating the value of continuing operations between equity, debt, and government, we find

$$E_{solv}(V) = (1 - \tau_{\text{eff}}) [V_{solv} - V_{int}] \quad (19)$$

$$G_{solv}(V) = \tau_{\text{eff}} [V_{solv} - V_{int}] + \tau_i V_{int} \quad (20)$$

$$D_{solv}(V) = (1 - \tau_i) V_{int}. \quad (21)$$

Note that the sum of these three claims add up to V_{solv} .

Equation 19 is straightforward to interpret from a cash flow standpoint. From the theorem of Feynman and Kac, it is well known that the value of $V_{solv}(V)$ in Eq. 16 can be obtained

from the risk-neutral expectation

$$V_{solv}(V_0) = E_0^Q \left[\int_0^{\tilde{T}} ds e^{-rs} \delta_s \right], \quad (22)$$

where \tilde{T} is the (random) bankruptcy time. Similarly, $V_{int}(V)$ in Eq. 18 can be obtained from the risk-neutral expectation

$$V_{int}(V_0) = E_0^Q \left[\int_0^{\tilde{T}} ds e^{-rs} C \right]. \quad (23)$$

Thus, Equation 19 can be written in the form

$$E_{solv}(V_0) = (1 - \tau_{\text{eff}}) E_0^Q \left[\int_0^{\tilde{T}} ds e^{-rs} (\delta_s - C) \right]. \quad (24)$$

Eq. 24 implies that, at each instant s , equity has a claim to $(1 - \tau_{\text{eff}}) (\delta_s - C) \mathbf{1}_{(\tilde{T} > s)}$. That is, after the coupon payment is made, what remains is divided between equity and government according to the tax code. Similar interpretations hold for the other claims.

Due to protective covenants, no part of the EBIT flow machine may be sold off by shareholders. When the payout level falls below promised interest payments, equity has the right to infuse payments to avoid bankruptcy. However, for a sufficiently poor state of the firm, shareholders will optimally choose to default. At this point, the remaining firm value is divided up between debt, government, and bankruptcy costs.¹⁵

The present value of the default claim $V_{def}(V)$ can be written as

$$V_{def} = A_1 V^{-y} + A_2 V^{-x}. \quad (25)$$

For $V \gg V_B$, the value of this claim must vanish, so again $A_1 = 0$. The boundary condition $V_{def}(V = V_B) = V_B$ implies

$$V_{def}(V) = V_B p_B(V). \quad (26)$$

Note that the sum of the present value of the claim to funds during solvency (Eq. 16) and the present value of the claim to funds in bankruptcy (Eq. 26) is equal to the present value of the claim to EBIT, V . Hence, value is neither created or destroyed by changes in the capital structure. Rather, it is only redistributed among the claimants. This invariance result is reminiscent of the ‘pie’ model of Modigliani and Miller (1958), except that in this framework the claims of government and bankruptcy costs are also part of that pie.

¹⁵We assume that in the event of bankruptcy, the remaining portion of the EBIT-generating machine not lost to bankruptcy costs is sold to competitors. Since they will have to pay taxes on future EBIT, government indeed has a claim in bankruptcy.

The default claim is divided up between debt, government, and bankruptcy costs as follows:

$$D_{def}(V) = (1 - \alpha)(1 - \tau_{\text{eff}})V_{def}(V) \quad (27)$$

$$G_{def}(V) = (1 - \alpha)\tau_{\text{eff}}V_{def}(V) \quad (28)$$

$$BC_{def}(V) = \alpha V_{def}(V). \quad (29)$$

We also account for restructuring costs, which are deducted from the proceeds of the debt issuance before distribution to equity occurs. These costs are assumed to be proportional to the initial value of the debt issuance:¹⁶

$$RC(V_0) = q(D_{\text{solv}}(V_0) + D_{\text{def}}(V_0)). \quad (30)$$

1.1 Optimal Default Level

We assume that management chooses the coupon level C and the bankruptcy level V_B to maximize equity wealth, subject to limited liability.¹⁷ The optimal bankruptcy level V_B is obtained by invoking the smooth-pasting condition¹⁸

$$\left. \frac{\partial E}{\partial V} \right|_{V=V_B} = 0. \quad (31)$$

Solving, we find

$$V_B^* = \lambda \frac{C^*}{r}, \quad (32)$$

where

$$\lambda \equiv \left(\frac{x}{x+1} \right). \quad (33)$$

1.2 Optimal Coupon

If one assumes that the payout ratio is independent of the chosen coupon level, then by using Equation 32, the optimal coupon level can be obtained in closed-form.

The objective of management is to maximize shareholder wealth:

$$\max_C \left[(1 - q)D(V_0, C, V_B(C)) + E(V_0, C, V_B(C)) \right]. \quad (34)$$

¹⁶If one wishes to model restructuring costs as tax-deductible, then qD should be interpreted as equity's portion of the restructuring costs. Any portion of the restructuring costs 'paid' by government (via decreased tax cash flows) has no effect on optimal capital structure choice. This is clear, since equity is maximizing its portion of debt plus equity, which is identical to minimizing the claims to $(G + BC + RC)$. Thus, any transfers of wealth between government, bankruptcy costs, and restructuring costs will have no effect on capital structure decisions.

¹⁷See Leland (1994). This assumes that equity cannot pre-commit to a particular V_B .

¹⁸See Dixit (1991), Dumas (1991).

That is, current equity holders receive fair value for the debt claim sold, minus their portion of the restructuring costs.

By differentiating Equation 34 with respect to C and setting the equation to zero, we find that the optimal coupon level, chosen when firm value is V_0 , is

$$C^* = V_0 \left(\frac{r}{\lambda} \right) \left[\left(\frac{1}{1+x} \right) \left(\frac{A}{A+B} \right) \right]^{\frac{1}{x}}, \quad (35)$$

where

$$\begin{aligned} B &= \lambda(1 - \tau_{\text{eff}})(1 - (1 - q)(1 - \alpha)), \\ A &= (1 - q)(1 - \tau_i) - (1 - \tau_{\text{eff}}). \end{aligned}$$

For there to be a tax advantage to debt, A must be positive. This can be interpreted as follows: if restructuring costs q were zero, the requirement that A be positive is equivalent to the requirement that $(\tau_{\text{eff}} > \tau_i)$. This result is reminiscent of the work of Miller (1977). For $q > 0$, the effective tax rate must be sufficiently high to cover both the taxes paid by the debt holders, and the restructuring costs in order for there to be a tax benefit to debt.

1.3 Tax Advantage to Debt

Plugging Eq. 32 and Eq. 35 into Eq. 34, we find that the value of the equity claim just before the debt issuance is

$$E(V_{0-}) = \left[(1 - q) D(V_0, C^*, V_B(C^*)) + E(V_0, C^*, V_B(C^*)) \right] \quad (36)$$

$$= V_0 [(1 - \tau_{\text{eff}}) + A Q], \quad (37)$$

where

$$Q \equiv \left[\left(\frac{A}{A+B} \right) \left(\frac{x}{1+x} \right) \right]^{\frac{1}{x}}. \quad (38)$$

Note that we can rewrite Equation 36 as

$$E(V_{0-}) + RC = \left[D(V_0, C^*, V_B(C^*)) + E(V_0, C^*, V_B(C^*)) \right], \quad (39)$$

where the restructuring costs are $RC = q D(V_0, C^*, V_B(C^*))$. In this form, we see that Eq. 36 is the ‘no-arbitrage’ condition of FHZ (1989). In their language, the difference in value between the levered firm (the RHS of Equation 39) and the unlevered firm, $E(V_{0-})$, is the cost to leverage the firm, RC . It is at this point that FHZ (1989) begin their analysis, but in this framework it is clear that all tax advantages to debt have already been incorporated into

$E(V_0^-)$. Indeed, from Eq. 5 and Eq. 37, we find the percentage increase in the value of equity to be

$$\% \text{ increase} = 100 \times \frac{AQ}{(1 - \tau_{\text{eff}})}. \quad (40)$$

This statistic captures the tax advantage to debt in our (static) model.

1.4 Coupon-Dependent Payout

Above, we assumed that the payout ratio is independent of the coupon level. In practice however, larger coupon payments are typically associated with larger payout ratios. As a typical example, consider a debtless firm with current EBIT = \$100, and a ‘price-earnings ratio’ of 20. Hence, total claims to EBIT = \$2000. If $\tau_c = .35$, and $\tau_d = .2$, then the current equity value is $2000(1 - .35)(1 - .20) = \1040 . With EBIT = \$100, and $\tau_c = .35$, \$35 is paid out to taxes. Of the remaining \$65, assume \$35 is paid out as dividend. Note that this implies a reasonable equity dividend payout ratio of $\frac{35}{1040} = 3.37\%$. Also note that this implies that the firm as a whole has a payout ratio of $\frac{35+35}{2000} = .035$. The implication is that even debtless firms have a relatively small drift $\mu = r - \frac{\delta}{V}$ in this framework.

Now, consider a firm which pays a coupon C . In this case, earnings before taxes falls to $(100 - C)$. The corporate tax payout to government is then $\delta_{govt} = (.35)(100 - C)$. Assuming the same dividend payment of 35, the total payout is

$$\delta = [C + (100 - C)(.35) + 35] = 70 + .65C.$$

Hence, the initial payout ratio is

$$\frac{\delta}{V_0} = .035 + .65 \frac{C}{V_0}. \quad (41)$$

As in previous models of optimal capital structure, we assume the payout ratio remains constant. That is, we assume

$$\frac{\delta}{V} = .035 + .65 \frac{C}{V_0}. \quad (42)$$

for all values of V . This is a reasonable assumption in our model because the payout to government in the form of taxes moves in the same direction as the value of the claim to EBIT. Thus, in contrast to previous frameworks, only minor changes need be made to the equity dividend ratio for this assumption to hold.

Using data from Compustat, we test Equation 41 to see whether firms with large interest payments tend to lower their dividend payments in an attempt to reduce the total payout ratio. That is, we test whether the coefficient of $\frac{C}{V}$ is less than $(1 - \tau_i) \approx .65$. In fact, we find

$$\frac{\delta}{V} = .017 + .764 \frac{C}{V}, \quad (43)$$

with t-statistics above 15 for both coefficients. While admittedly such a cross-sectional analysis may not be an appropriate measure of how a capital structure decision of an individual firm affects the payout ratio, it does seem to support the notion that a firm's total payout increases with interest payments.

1.5 Loss of Tax-Shelter

Recall from Eq. 24 that the equity claim can be written as the risk-neutral expectation

$$E_{solv}(V_0) = (1 - \tau_{\text{eff}}) E_0^Q \left[\int_0^{\tilde{T}} ds e^{-rs} (\delta_s - C) \right]. \quad (44)$$

However, this simple formula obtains only because we previously assumed that the firm receives full loss offset. In practice, when a firm is not profitable, it loses part of its tax shelter, receiving only tax-carry forward benefits. To account for loss of tax shields, we now assume that, for current firm value V_s above some specified value V_* , the instantaneous claim of equity remains

$$P_{eq}(V_s > V_*) = (1 - \tau_{\text{eff}}) (\delta_s - C) \mathbf{1}_{(\tilde{T} > s)}. \quad (45)$$

But now, when firm value drops below V_* , some of the tax benefits are lost, exposing equity to a larger proportion of the debt payment:¹⁹

$$P_{eq}(V_s < V_*) = [(1 - \tau_{\text{eff}}) \delta_s - (1 - \epsilon \tau_{\text{eff}}) C] \mathbf{1}_{(\tilde{T} > s)}. \quad (46)$$

Here, ϵ is constrained to the region $0 \leq \epsilon \leq 1$. If $\epsilon = 1$, we have the full-loss offset model above. However, if we set $\epsilon = 0$, then equity loses all tax shields when the claim to EBIT falls below V_* . Due to tax-loss carry-forwards and carry-backs, the best estimate lies somewhere in the middle. The solution to this model is similar to that in Leland and Toft (1996) and is derived in Appendix A.

1.6 Comparative Statics

Our base case parameters are $\tau_c = .35$, $\tau_i = .35$, $\tau_d = .2$, $r = .045$, $\sigma = .25$, $\alpha = .05$, $\epsilon = .5$, $q = .01$, a firm P:E ratio of 17, and $\frac{\delta}{V} = .035 + .65 \frac{C}{V_0}$. Note that $r = .045$ is an after-tax risk-free rate, since we are dealing with after-tax dollars. Hence, $\mu = (r - \frac{\delta}{V_0}) = .01 - .65 \frac{C}{V_0}$. Since $V = 17 \cdot \text{EBIT}$, the firm will begin to lose tax benefits when firm value falls to $V_* = 17C$. It is interesting to note that, whereas previous frameworks needed to assume unreasonable

¹⁹This model can be generalized by assuming that the amount of lost tax shield is an increasing function of $(V_* - V)$. Such a model is probably more realistic, since the lower V drops below V_* , the lower the probability that the firm will return to profitability in order to use the tax carry-forward benefits.

1	2	3	4	5	6	7
	Optimal Coupon $\left(\frac{C}{V_0}\right)$ $\times 100$	Bankruptcy Level $\left(\frac{V_B}{V_0}\right)$ $\times 100$	Optimal Leverage $\left(\frac{D_0}{D_0+E_0}\right)$ $\times 100$	Credit Spread $\left(\frac{C}{D_0} - \frac{r}{1-\tau_i}\right)$ $\times 10^4$	Recovery Rate $\left(\frac{D_{def}}{D_0}\right)$ $\times 100$	Tax Advantage to Debt $\left(\frac{E(V_{0-}) - (1-\tau_{eff})V_0}{(1-\tau_{eff})V_0}\right)$ $\times 100$
Base	2.52	29.4	49.8	221	52.9	6.3
$\alpha = 0.03$	2.62	30.6	51.3	228	54.2	6.5
$\alpha = 0.10$	2.29	26.9	46.3	207	49.6	5.7
$\tau_c = 0.33$	2.42	28.1	47.8	205	53.2	5.1
$\tau_c = 0.37$	2.60	30.6	51.6	237	52.5	7.5
$\sigma = 0.23$	2.55	31.3	51.5	199	54.1	6.8
$\sigma = 0.27$	2.48	27.7	48.1	245	51.6	5.9
$r = 0.040$	2.46	30.2	52.2	235	51.6	6.6
$r = 0.050$	2.56	28.65	47.6	207	54.0	6.0
$\epsilon = 0.3$	2.36	29.1	47.7	206	54.8	5.9
$\epsilon = 0.7$	2.80	30.4	53.5	250	50.6	6.9

Table 1: Comparative statics for select parameter values (static model).

bankruptcy costs to obtain reasonable estimates for optimal leverage ratios and credit spreads, in this framework we can set α to a value that is more consistent with bankruptcy and recovery rates that are observed in practice.²⁰ The results are collected in Table 1.

Most comparative statics results are similar to Leland (1994). However, in contrast to Leland (1994), equity is a decreasing function of τ_{eff} .²¹ This is because, in the present framework, an increase in τ_{eff} increases the government claim at the expense of equity, while in Leland (1994), an increase in τ increases the tax benefit (i.e., inflow of cash). Of course, in Leland (1994), the comparative static is performed while holding unlevered firm value constant, which is unlikely to be true when taxes are increased. Indeed, Lang and Shackelford (1998) find a strong price-reaction to the signing of the 1995 budget accord. The advantage of treating all claimants in a systematic basis allows a precise prediction for the effect that changes in tax code has on equity.

As with most static models, the predicted optimal initial leverage ratio is well above what

²⁰Warner (1977) finds direct bankruptcy costs to be about 1%. However, we also account for indirect costs in choosing 5% for α in the base-case.

²¹We emphasize that, even though the tax benefit is an increasing function of τ_c , equity is a decreasing function of τ_c . For example, when $\tau_c = .35$, equity rises 6.3% from 52.0 to 55.3 when management decides to issue debt. However, when $\tau_c = .33$, equity rises 5.1% from 53.6 to 56.3 when management decides to issue debt. Although tax benefits close the gap, our model predicts equity is still better off with lower taxes.

is observed in practice. As we shall see in the next section, accounting for the fact that equity has the option to increase future debt levels reduces this ratio considerably.

1.7 Equity Pricing with Future (Static) Capital Structure Changes

In practice, management may decide to operate for many years without taking on any debt. In such a case, equity holders must take management's decision as given, and price equity shares appropriately. Here, we demonstrate to what extent the claim of KMM (1984), that the value of unlevered assets reflects the potential to optimally lever them in the future, is valid. We determine the value of equity at time t for an unlevered firm whose management has promised to leverage optimally at a future date T . Of course, a more realistic scenario is that equity holders account for the possibility that at some point in the future a firm may either decide to take on some debt, or be bought out at the levered price, in which case T is a random variable. We deal with this more realistic case below.

Consider a debtless firm, at time t , which commits to leveraging optimally at some future time T . Hence, during the time interval s , ($t < s < T$), equity holders have a claim to after-tax cash flows of $(1 - \tau_{\text{eff}}) \tilde{\delta}_s$, and at time T , they will have a claim worth $E(\widetilde{V}_{T-})$, as defined in Eq. 36. The present value of these claims is

$$E(t) = (1 - \tau_{\text{eff}}) V_t + e^{-(r-\mu)(T-t)} V_t A Q. \quad (47)$$

Consider $E(t)$ as a function of T . For $T \gg t$, the tax advantages to debt will not be realized for a long time, and thus the value of equity will be only slightly larger than that of a firm which refuses to ever take on debt (Eq. 5). As T approaches t , almost all tax benefits to debt are reflected in the equity price (Eq. 36). Note that only when $T = t^+$ does the FHZ (1989) 'no-arbitrage' condition hold. That is, only moments before the restructuring is to occur does the unlevered equity value equal the levered value, minus restructuring costs.

Above, we took T to be a predetermined number. In practice, the time at which management decides to take on debt, or the time at which an outside firm attempts to purchase the unlevered firm, is random.²² If we assume that \tilde{T} , at time t , has an exponential distribution, independent of EBIT value dynamics,

$$\pi(\tilde{T} = T) \Big| \mathcal{F}_t = \left(1/\overline{T}\right) e^{-(1/\overline{T})(T-t)} \mathbf{1}_{(T>t)}, \quad (48)$$

where \overline{T} is the expected time of restructuring, then the current equity has a simple closed-form

²²We assume here that the outside firm would have to pay leveraged-firm value in order for the takeover to be successful. This assumption seems to hold in practice.

solution:

$$E(t) = (1 - \tau_{\text{eff}}) V_t + \left(\frac{1}{1 + \bar{T}(r - \mu)} \right) V_t A Q. \quad (49)$$

Again, for large \bar{T} , equity value is only slightly larger than the value of equity for a firm that refuses to ever take on debt, whereas for $\bar{T} \rightarrow 0^+$, almost all tax benefits are already embedded in equity value.

2 Optimal Upward Dynamic Capital Structure Strategy

In this section, we determine the optimal capital structure strategy for a firm which has the option to increase its debt level in the future. As in FHZ (1989), we find that there will be a range of debt ratios for which management will maintain its current debt level. Similar to Section 1, there will be a threshold V_B where the firm will optimally choose bankruptcy. But now, there will also be a threshold V_U where management will call the outstanding debt issue, and sell a new, larger issue.

The tractability of our model stems from a scaling feature inherent in modeling the EBIT claim dynamics as log-normal (or proportional). Indeed, some implications of this scaling feature are already apparent in the static model of the previous section. For example, note that from Eq. 32 that the optimal bankruptcy level V_B is proportional to the optimal coupon, and that from Eq. 35, the optimal coupon is proportional to V_0 . This implies that if two firms A and B are identical (i.e., same volatility σ and payout ratio $\frac{\delta}{V}$) except that their initial values differ by a factor $V_0^B = \gamma V_0^A$, then the optimal coupon $C^B = \gamma C^A$, the optimal bankruptcy level $V_B^B = \gamma V_B^A$, and every claim will be larger by the same factor γ . Note that this same argument also holds for a single firm that first issues debt at firm value V_0 , and later finds its value has increased to $V_U = \gamma V_0$, at which time it calls all outstanding debt, and then, as an unlevered firm, takes on an optimal level of debt. For our dynamic model, the scaling feature implies that since the period-1 initial value, $V_0^1 = V_U^0 \equiv \gamma V_0^0$ will be a factor γ larger than its time-0 initial value, it will be optimal to choose $C^1 = \gamma C^0$, $V_B^1 = \gamma V_B^0$, $V_U^1 = \gamma V_U^0$, and all period-1 claims will scale upward by the same factor γ . This behavior is captured in Figure 1.

We assume that the debt is callable at par to simplify the analysis.²³ However, we claim

²³Two changes must be made if the debt were not modeled as callable. First, the restructuring costs need to have both a fixed component, proportional to current firm value, and a variable component, proportional to the amount of new debt issued. The fixed component prevents infinitesimal increases in debt from being an optimal strategy. The second change is more difficult. In our analysis below, every period looks the same, because after the debt is called, the firm is again all-equity, and simply larger by a factor γ from the previous period. However, if debt is not callable, then the first period, where debt goes from zero to some finite amount, is different than all other periods, where the debt jumps by a factor γ . Treating the first period differently than all the others complicates the analysis without providing any additional insights.

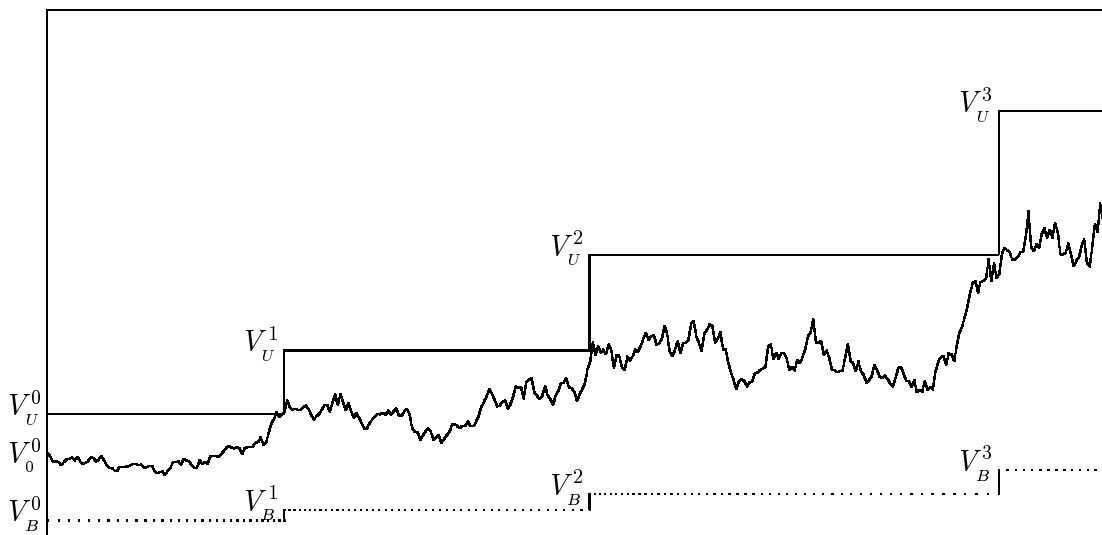


Figure 1. A typical sample path of firm value with log-normal dynamics. Initially, firm value is V_0^0 . Period-0 ends either by firm value reaching V_B^0 , at which point the firm declares bankruptcy, or by firm value reaching V_U^0 , at which point the debt is recalled, and the firm again chooses an optimal capital structure. Note that the initial firm value at the beginning of period-1 is $V_0^1 = V_U^0 \equiv \gamma V_0^0$. Due to log-normal firm dynamics, it will be optimal to choose $V_U^n = \gamma^n V_U^0$, $V_B^n = \gamma^n V_B^0$.

that, taking management's strategy as given, the value of the debt issue would be the same even if it were not callable, as long as we assume that all debt issues receive the same recovery rate in the event of bankruptcy.²⁴ To prove this claim, we must show that when firm value reaches V_U that the value of the previously issued debt is identical to its original value, *as if* it were called at par. Due to the scaling feature inherent in our model, the bankruptcy threshold will rise by the same factor $\gamma = \frac{V_U}{V_0}$ that firm value has risen. The implication is that the probability of bankruptcy at the beginning of each period will be the same: simply, all claims will have scaled up by the same factor γ . Since the old debt still has the same promised coupon flows, and since it faces the same risk of (and recovery rate in the event of) bankruptcy as it had at issuance, it follows that its price must equal its initial par value. Note that, in contrast to the static models, this framework implies that debt is not a monotonically increasing function of firm value.

We develop the model as follows. First, we determine the value of the period-0 claims. We

²⁴We can actually account for a type of seniority, where older issues receive a higher recovery rate in the event of bankruptcy, and still obtain closed-form solutions. Basically, the recovery rate for a debt issue sold n periods in the past receives in bankruptcy a recovery rate $(1 - \rho^n)$, where $\rho (< 1)$ is determined so that the total payout to all debt issues sums to the asset value remaining after bankruptcy costs are paid. Note that debt issued long ago ($n \rightarrow \infty$) is nearly riskless in this model.

then point out that *if* it is optimal for each period's coupon payments, default-threshold, and bankruptcy threshold to increase by the same factor γ that the 'initial' EBIT value increases by each period, then all claims will also scale up by this same factor. Then, taking this strategy to be optimal, we determine the present value of all claims. In Appendix B we use a backward induction scheme to demonstrate that such a strategy is in fact optimal.

2.1 Present Value of Period-0 Claims

In order to determine the value of the total claims, it is convenient to first evaluate the claims to cash-flows during period-0, that is, the value of cash flows before firm value reaches V_U .²⁵ We then use the scaling argument to determine future-period claim values in terms of the period-0 claim values.

It is convenient to define $p_U(V)$ as the present value of a claim that pays \$1 contingent upon firm value reaching V_U (before reaching V_B). Since such a claim receives no dividend, we know from Equation 10 that $p_U(V)$ will be of the form

$$p_U(V) = A_1 V^{-y} + A_2 V^{-x}, \quad (50)$$

and will satisfy the boundary conditions

$$p_U(V_B) = 0, \quad p_U(V_U) = 1. \quad (51)$$

Plugging in, we find

$$p_U(V) = -\frac{V_B^{-x}}{\Sigma} V^{-y} + \frac{V_B^{-y}}{\Sigma} V^{-x}, \quad (52)$$

where we have defined

$$\Sigma \equiv V_B^{-y} V_U^{-x} - V_U^{-y} V_B^{-x}. \quad (53)$$

Similarly, it is convenient to define $p_B(V)$ as the present value of a claim that pays \$1 contingent upon firm value reaching V_B (before reaching V_U). We find

$$p_B(V) = \frac{V_U^{-x}}{\Sigma} V^{-y} - \frac{V_U^{-y}}{\Sigma} V^{-x}. \quad (54)$$

All period-0 claims can be written compactly in terms of $p_U(V)$ and $p_B(V)$. For example, consider the claim, V_{solv}^0 , to the payout δ for as long as current firm value remains between V_U and V_B . From Equation 15, $V_{solv}^0(V)$ will be of the form

$$V_{solv}^0(V) = V + A_1 V^{-y} + A_2 V^{-x}, \quad (55)$$

²⁵It is more appropriate to define the variables as V_B^0 , V_U^0 , since the boundaries will scale upward in the next period. However, the superscript notation is too clumsy when these factors are written to some power. Therefore, we simplify the notation at the expense of conciseness.

and will satisfy the boundary conditions

$$V_{solv}^0(V_B) = 0, \quad V_{solv}^0(V_U) = 0. \quad (56)$$

Plugging in, we find

$$V_{solv}^0(V) = V - p_U(V) V_U - p_B(V) V_B. \quad (57)$$

Similarly, the claim to interest rate payments during period-0 is

$$V_{int}^0(V) = \frac{C^0}{r} \left(1 - p_U(V) - p_B(V) \right), \quad (58)$$

and the claim to all EBIT at the moment firm value reaches V_B and defaults is

$$V_{def}^0(V) = p_B(V) V_B. \quad (59)$$

As before, the restructuring costs are a proportion q of the value of the debt issue.

The claim to all EBIT at the moment firm value reaches V_U and restructures is

$$V_{res}^0(V) = p_U(V) V_U. \quad (60)$$

Below, we use the scaling inherent in the model to determine what portion of this claim belongs to the different claimants. Note that the sum

$$V_{solv}^0(V) + V_{def}^0(V) + V_{res}^0(V) = V,$$

so again EBIT value is invariant to capital structure choice – only distribution of assets is affected by capital structure decisions.

Assuming full-loss offset, the period-0 claim values, after the restructuring occurs, are

$$d^0(V) = (1 - \tau_i) V_{int}^0(V) + (1 - \alpha)(1 - \tau_{\text{eff}}) V_{def}^0(V) \quad (61)$$

$$e^0(V) = (1 - \tau_{\text{eff}}) \left[V_{solv}^0(V) - V_{int}^0(V) \right] \quad (62)$$

$$g^0(V) = \tau_{\text{eff}} \left[V_{solv}^0(V) - V_{int}^0(V) \right] + \tau_i V_{int}^0(V) + (1 - \alpha) \tau_{\text{eff}} V_{def}^0(V). \quad (63)$$

$$bc^0(V) = \alpha V_{def}^0(V). \quad (64)$$

The case where tax benefits are lost is considered in Appendix A.

Note that the claims described in Eqs. 61-64 sum to $V_{solv}^0(V) + V_{def}^0(V)$, which is equivalent to $V - p_U(V) V_U$. The intuition is that we have yet to ‘dole out’ the value of claims to cash flows in future periods $V_{res}^0 = p_U(V) V_U$.

2.2 Present Value of Future Period Claims

It is convenient to ‘reset time’ back to zero at each instant the firm restructures. Hence, the period-1 initial firm value equals the upper boundary value of period-0. Varying slightly from the above notation,

$$V_0^1 = V_U^0 \equiv \gamma V_0^0.$$

Note that if both the lower and upper boundary also scale by the same factor:

$$V_U^1 = \gamma V_U^0, \quad V_B^1 = \gamma V_B^0,$$

then it follows from their definitions that

$$p_B^1(V_0^1) = p_B^0(V_0^0), \quad p_U^1(V_0^1) = p_U^0(V_0^0).$$

If the optimal coupon C^* also scales upward by a factor γ each period, then necessarily so will each claim on EBIT, as is clear from Eqs. 57 - 59. Due to this scaling feature, the firm will look identical at every restructuring date, except that all factors will be scaled up by a factor $\gamma = (\frac{V_U}{V_0})$. Hence, the ratios of the different claims must remain invariant for all periods. Thus, all remaining firm value not ‘doled out’ in the period-0 claims must be divided among the claimants in the same proportions as in the period-0 claims. Now, define $e(V)$ as the equity claim to intertemporal EBIT flows for all periods, with similar definitions for $d(V)$, $g(V)$, $bc(V)$. We emphasize that $e(V)$ is not the total equity claim, since there are also cash transfers between the claimants at each restructuring date.²⁶ We obtain the total equity claim value, $E(V)$, below.

The scaling feature inherent in our model implies

$$\frac{e(V_0)}{e^0(V_0)} = \frac{d(V_0)}{d^0(V_0)} = \frac{g(V_0)}{g^0(V_0)} = \frac{V_0}{V_0 - V_U^0(V_0) p_U(V_0)} = \frac{1}{1 - \gamma p_u(V_0)} \quad (65)$$

Thus, for example, the equity claim to intertemporal cash flows can be written

$$e(V_0) = \frac{e^0(V_0)}{1 - \gamma p_u(V_0)}. \quad (66)$$

The relation between the period-0 claim and the claim to cash-flows for all periods has a simple intuitive explanation. One can rewrite Equation 66 as

$$\begin{aligned} \frac{e^0(V_0)}{1 - \gamma p_u(V_0)} &= e^0(V_0) \left(1 + \gamma p_u(V_0) + (\gamma p_u(V_0))^2 + \dots \right) \\ &= e^0(V_0) \left(\sum_{j=0}^{\infty} (\gamma p_u(V_0))^j \right) \end{aligned}$$

²⁶This is because the claim to future debt issues are currently owned by equity and restructuring costs. Future debt-holders will have to pay fair price in the future for these claims.

In particular, at time-0, the present value of the claim to the period- j cash flow is a factor $(\gamma p_u(V_0))^j$ times the present value of the period-zero cash flow. The factor γ expresses the increase in size of the future cash flows, and the factor $p_u(V_0)$ captures the discount for time and risk.

2.3 Cash Flow Transfers at Restructuring Dates

In order to determine the portion of the future ‘debt claims’ that currently belong to equity and restructuring costs, we first determine the value of the initial debt issue. We assume that the debt issue is called at par. Hence, the initial value of the debt claim equals the sum of the claim to period-0 cash flows, $d^0(V)$, plus the present value of the call value:

$$D^0(V_0) = d^0(V_0) + p_U(V_0) D^0(V_0), \quad (67)$$

which can be written as

$$D^0(V_0) = \frac{d^0(V_0)}{1 - p_U(V_0)}. \quad (68)$$

In order to receive this claim, debt holders must pay fair value, namely, $D^0(V_0)$, at the time of issuance. This inflow of cash is divided between equity and restructuring costs in the proportion $(1 - q)$ and q , respectively. Using the scaling argument, the total restructuring claim is

$$RC(V_{0-}) = q D^0(V_0) [1 + \gamma p_U(V_0) + \dots] \quad (69)$$

$$= q D^0(V_0) \frac{1}{1 - \gamma p_U(V_0)} \quad (70)$$

The total equity claim, just before the initial debt issuance, is the sum of the intertemporal claims of debt and equity, less the restructuring costs claim:

$$E(V_{0-}) = \frac{e^0(V_0) + d^0(V_0) - q D^0(V_0)}{1 - \gamma p_U(V_0)}. \quad (71)$$

Anytime after the initial debt issuance, the total equity claim for arbitrary V during period-0 is

$$E(V) = \gamma p_U(V) E(V_{0-}) + e^0(V) - p_U(V) D^0(V_0). \quad (72)$$

This reduces to

$$E(V_{0+}) = E(V_{0-}) - (1 - q) D^0(V_0) \quad (73)$$

immediately after the debt issuance, as necessary to preclude arbitrage. It also reduces to

$$E(V_{U-}) = \gamma E(V_{0-}) - D^0(V_0), \quad (74)$$

just before the next issuance, demonstrating that the scaling feature obtains after equity pays the call price of the period-0 bond.

2.4 Optimal Dynamic Capital Structure

Management's objective is to choose C , $\gamma \equiv \frac{V_U}{V_0}$ and $\psi \equiv \frac{V_B}{V_0}$ in order to maximize the wealth of the equity holder, subject to limited liability.²⁷ As before, V_B is determined by the smooth-pasting condition. The results are tabulated in Table 2. Note that the optimal initial leverage ratio is substantially lower than that obtained in section 1, and reasonably consistent with leverage ratios observed in practice. The intuition of this result is straightforward – since the firm has the option to increase its leverage in the future, it will wait until firm value rises to the point where it becomes optimal to exercise this option. Also note that the tax advantage to debt has increased significantly. This is intuitive, because the static model implies that the long-run expected leverage ratio drops to zero, as (expected) firm value increases exponentially. Hence, the long-run expected tax-benefits become negligible in the static model. Finally, note that V_B has dropped significantly in the dynamic case. This can be understood in that a firm which has the option to adjust its capital structure is more valuable, and hence will have incentive to avert bankruptcy at EBIT-value levels for which a firm that is constrained to its previously chosen debt level will choose to default.

3 Conclusion

A model of dynamic capital structure is proposed. We find that when a firm has the option to increase debt levels, our model predicts optimal debt levels and credit spreads similar to those observed in practice. Further, the tax-benefits to debt increase significantly over the static-case predictions.

We argued above why accounting for upward-restructures may in practice be more important than accounting for downward-restructures. Still, since downward restructuring does occur sometimes in practice, our model will bias the optimal leverage ratio downward. We note that our framework can incorporate downward restructuring also, and the model is available upon request from the authors. However, it neglects issues such as asset substitution, asymmetric information, equity's ability to force concessions, Chapter 11 protection,²⁸ and many other important features that would affect optimal strategy at a lower boundary. Thus we are

²⁷Since the debt is callable, we have assumed that management can pre-commit to a particular V_U in the bond indenture. It is straightforward to also consider the case where V_U cannot be pre-committed to.

²⁸See, for example, Francois and Morellec, 1998.

1	2	3	4	5	6	7	8
	Optimal Coupon $\left(\frac{C}{V_0}\right)$ $\times 100$	Bankruptcy Level $\left(\frac{V_B}{V_0}\right)$ $\times 100$	Restructure Level $\left(\frac{V_U}{V_0}\right)$ $\times 100$	Optimal Leverage $\left(\frac{D_0}{D_0+E_0}\right)$ $\times 100$	Credit Spread $\left(\frac{C}{D_0} - \frac{r}{1-\tau_i}\right)$ $\times 10^4$	Recovery Rate $\left(\frac{D_{def}}{D_0}\right)$ $\times 100$	Tax Advantage to Debt $\left(\frac{E(V_{0-}) - (1-\tau_{eff})V_0}{(1-\tau_{eff})V_0}\right)$ $\times 100$
Base	1.85	21.78	169.74	37.14	193.55	51.43	8.31
$\alpha = .03$	1.92	22.55	169.08	38.24	198.43	52.67	8.59
$\alpha = .10$	1.70	20.05	171.30	34.63	182.72	48.39	7.69
$\tau_c = .33$	1.80	21.07	176.30	36.07	180.38	51.97	6.76
$\tau_c = .37$	1.89	22.38	164.48	38.04	205.87	50.81	9.97
$\sigma = .23$	1.93	23.80	165.35	39.40	173.86	52.82	8.65
$\sigma = .27$	1.78	19.95	174.08	35.04	214.13	50.07	8.00
$r = .040$	1.75	21.59	170.61	37.84	202.11	49.75	8.98
$r = .050$	1.94	21.78	168.91	36.28	183.69	52.92	7.74
$\epsilon = .3$	1.74	21.55	170.82	35.55	180.98	53.36	7.90
$\epsilon = .7$	2.06	22.50	168.19	39.93	216.01	49.10	9.01

Table 2: Comparative statics for select parameter values for upward restructuring model.

skeptical about the results that obtain when extending our model to account for downward restructures.

Appendix A

Following the approach of Leland and Toft (1996), we derive the value of equity assuming that it loses some of its tax shield when EBIT value V drops below some specified level V_* . As mentioned in the text, we assume that the payout ratio is a constant:

$$\frac{\delta}{V} = .035 + .65 \frac{C}{V_0}. \quad (75)$$

Also, we assume that $V_* = 17C$. Hence, for a given coupon rate, both $\frac{\delta}{V}$ and V_* are constants. We derive the equity value for all cases considered in the text.

Static Case

It was demonstrated in Eq. 24 that the equity claim can be written as

$$E_{sol}(V_t) = (1 - \tau_{\text{eff}}) E_t^Q \left[\int_t^{\tilde{T}} ds e^{-r(s-t)} (\delta_s - C) \right], \quad (76)$$

where \tilde{T} is the (random) time of bankruptcy. That is, equity could be priced *as if* it had an instantaneous payout

$$P_{Eq}(s) = (1 - \tau_{\text{eff}}) (\delta_s - C) \mathbf{1}_{(s < \tilde{T})}. \quad (77)$$

To account for the loss of tax shields, we now assume that the ‘effective payout’ takes the form

$$P_{Eq}(s) = \begin{cases} K (\delta_s - C) \mathbf{1}_{(s < \tilde{T})} & \text{if } V > V_* \\ (K \delta_s - H C) \mathbf{1}_{(s < \tilde{T})} & \text{if } V < V_*, \end{cases} \quad (78)$$

where

$$K \equiv (1 - \tau_{\text{eff}}), \quad H \equiv (1 - \epsilon \tau_{\text{eff}}),$$

where ϵ is a parameter in the range $0 < \epsilon < 1$. If $\epsilon = 1$, then $H = K$, and the model reduces to the full-offset case. When $\epsilon = 0$, then equity loses all tax benefits to debt when $V < V_*$. Due to loss carry-forwards, the best estimate of ϵ is some intermediate value.

Using arguments of Section 1, we know equity will be of the form

$$E(V) = \begin{cases} A_1 V^{-y} + A_2 V^{-x} + K \left(V - \frac{C}{r} \right) & \text{if } (V > V_*) \\ B_1 V^{-y} + B_2 V^{-x} + \left(KV - H \frac{C}{r} \right) & \text{if } (V < V_*). \end{cases} \quad (79)$$

Due to the boundary condition $E(V \Rightarrow \infty) \Rightarrow K \left(V - \frac{C}{r} \right)$, we know $A_1 = 0$. The other three boundary conditions are:

$$E(V = V_B) = 0, \quad (80)$$

$$E(V = V_{*\uparrow}) = E(V = V_{*\downarrow}), \quad (81)$$

$$E_V(V = V_{*\uparrow}) = E_V(V = V_{*\downarrow}). \quad (82)$$

Solving, we find

$$B_1 = \frac{x(H - K)CV_*^y}{r(x - y)}, \quad (83)$$

$$B_2 = (HC/r - KV_B)V_B^x - B_1 \frac{V_B^x}{V_B^y}, \quad (84)$$

$$A_2 = B_2 + yB_1V_*^x/(xV_*^y). \quad (85)$$

Note that these equations hold for any given V_B . Equity chooses V_B using the smooth pasting condition

$$E_V(V = V_B) = 0. \quad (86)$$

This implies

$$0 = K - (yB_1V_B^{-y} + xB_2V_B^{-x})/V_B. \quad (87)$$

Upward Restructuring

From previous arguments, we know that the period-0 equity claim has the form

$$e^0(V) = \begin{cases} A_1V^{-y} + A_2V^{-x} + K \left(V - \frac{C}{r} \right) & \text{if } (V > V_*) \\ B_1V^{-y} + B_2V^{-x} + \left(KV - H\frac{C}{r} \right) & \text{if } (V < V_*) . \end{cases} \quad (88)$$

By definition, the period-0 claim must vanish at $V = V_U$. Hence, the four boundary conditions are:

$$e^0(V = V_U) = 0, \quad (89)$$

$$e^0(V = V_B) = 0, \quad (90)$$

$$e^0(V = V_{*\uparrow}) = e^0(V = V_{*\downarrow}), \quad (91)$$

$$e_V^0(V = V_{*\uparrow}) = e_V^0(V = V_{*\downarrow}). \quad (92)$$

Define the constants

$$c_1 = V_B^{-y}, \quad c_2 = V_B^{-x},$$

$$c_3 = V_U^{-y}, \quad c_4 = V_U^{-x},$$

$$c_5 = V_*^{-y}, \quad c_6 = V_*^{-x},$$

$$\Sigma = c_1c_4 - c_2c_3.$$

Then we find

$$\begin{aligned} B_1 &= (HC/r - KV_B)c_4/\Sigma + K(V_U - C/r)c_2/\Sigma + \\ &\quad (K - H)C/rc_2(xc_3/c_5 - yc_4/c_6)/((x - y)\Sigma), \end{aligned} \tag{93}$$

$$B_2 = (HC/r - KV_B)/c_2 - B_1c_1/c_2, \tag{94}$$

$$A_1 = B_1 - x(K - H)C/r/((y - x)c_5), \tag{95}$$

$$A_2 = B_2 - y(K - H)C/r/((x - y)c_6). \tag{96}$$

Appendix B

Here we demonstrate that it will be optimal to increase C^* , V_B^* , and V_U^* each period by the same factor $\gamma \equiv \frac{V_U}{V_0}$ that the ‘initial firm value’ scales by. But before we offer a mathematical backward-induction argument, we first give the following intuitive argument.

Suppose the optimal strategy for the first period is to issue debt with total coupon C and call back the bond at V_U . Suppose the dollar is the monetary unit used initially. That is, the firm value at the beginning of the first period is V_0 dollars and the bond will be called when the firm value rises to V_U dollars. Now, for the next period, consider a change in numeraire from dollars to γ -dollars, where we define $\gamma \equiv \frac{V_U}{V_0}$. In terms of the new numeraire, the firm value at the beginning of the second period is V_0 . Because of the log-normal process, the drift and diffusion are unaffected by the change in numeraire:

$$\frac{dV_U}{V_U} = \frac{\gamma dV_0}{\gamma V_0} = \frac{dV_0}{V_0} = \mu dt + \sigma dz^Q.$$

Therefore the firm at the beginning of the next period in terms of the new numeraire is *identical* to the firm at the beginning of the current period in terms of dollars. Hence, if the optimal strategy at the beginning of the current period is to issue debt with total coupon C^0 dollars per year, and call back the debt when the firm value rises to V_U dollars, then the optimal strategy at the beginning of the next period must be to issue a bond with coupon C^0 measured in new numeraire, which equals γC^0 dollars, and call it back when firm value reaches V_U measured in the new numeraire, which equals γV_U dollars.

We now demonstrate this scaling feature more rigorously by use of a backward-induction argument. Note that for models with a *finite* number of periods N , the backward-induction approach first determines the optimal strategy for the next-to-last period $N-1$, and then works backward period-by-period to determine the entire optimal strategy. However, for infinite-period ‘games’, this is clearly not possible. Still, the optimality is guaranteed by the following argument.

The present value of equity is equal to the sum of the present value of each period’s cash flows to equity:

$$\begin{aligned} E &= \sum_{j=0}^{\infty} e_j \\ &= \sum_{j=0}^{N-1} e_j + \sum_{j=N}^{\infty} e_j \quad \forall N. \end{aligned} \tag{97}$$

Note that the total equity value is strictly bounded above by the claim to EBIT, V . Hence,

for arbitrarily large N , it must be that

$$\lim_{N \rightarrow \infty} \left(\sum_{j=N}^{\infty} e_j \right) \rightarrow 0. \quad (98)$$

This result is perfectly intuitive: most of the equity value is due to cash flows the holders will receive over the next century, say. The implication is that, as we increase $N \rightarrow \infty$, the strategy chosen for all periods greater than N has an increasingly negligible effect on the present value of equity. Hence, if we impose a given strategy on the firm for all periods greater than some arbitrarily large number N , such a constraint becomes increasingly unimportant. Yet, if we constrain the firm to take the strategy we have proposed in the text for all periods greater than N , we can show that the firm will optimally choose to never stray from this proposed strategy in the previous periods. This is done by showing it is optimal in period $N - 1$ to follow the proposed strategy. By repeating the same inductive argument, we find it is optimal to follow this strategy for all periods prior to N .

We note that *if* our scaling assumption is valid, then C^* and γ^* are obtained from first-order conditions of Eq. 71:

$$\frac{\partial E(V_{0-})}{\partial C} = 0 \quad \frac{\partial E(V_{0-})}{\partial \gamma} = 0 \quad (99)$$

The smooth-pasting condition applied to Eq. 72 determines the optimal bankruptcy threshold:

$$\left. \frac{\partial E(V)}{\partial V} \right|_{V=V_B} = 0 \quad (100)$$

Here we demonstrate that the proposed solution is in fact optimal. We constrain the firm to take the strategy implied from Equations 99-100 for all periods greater than N , but allow it to choose any strategy for period $(N - 1)$. With this strategy, the firms equity before the start of period $(N - 1)$ takes the form

$$E(V_{0-}^{N-1}) = \gamma p_U(V_0^{N-1}) E(\gamma^*, \psi^*, C^*) + e(V_0^{N-1}) + D(V_0^{N-1}) (1 - q - p_U(V_0^{N-1})). \quad (101)$$

Let us interpret this equation. Effectively, the equity-holder has a claim to future equity flows, whose future initial value, γE^* , is discounted by $p_U(V_0^{N-1})$. In addition, she receives dividend flow during the period currently valued at $e(V_0^{N-1})$, and payment from new debt-holders of magnitude $D(V_0^{N-1})(1 - q)$ to begin the period. However, she also has an obligation to pay the call price of the debt, with present value $D(V_0^{N-1}) p_U(V_0^{N-1})$ to ‘finish’ the current period. Using similar arguments, the equity claim after the period begins can be shown to be

$$E(V^{N-1}) = \gamma p_U(V^{N-1}) E(\gamma^*, \psi^*, C^*) + e(V^{N-1}) - p_U(V^{N-1}) D(V_0^{N-1}). \quad (102)$$

Taking the first order conditions and smooth pasting condition for this model produces the same optimal choice variables as in Eqs. 99-100

We note that the determination of an optimal control strategy for an infinite-period game has been investigated previously in many other settings. For example, Taksar, Klass, and Assaf (1988) investigate the optimal trading strategy for an individual in an economy with a risky and risk-free asset who is free to adjust her portfolio at will, but faces a transactions cost. More generally, Harrison (1985, pg. 86) investigates the properties of models with a ‘regenerative structure’.

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