Note on Asset Risk and the Opportunity Cost of Capital

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A financial analyst often must estimate the risk of a firm's investment projects, for instance to estimate the opportunity cost of capital. If the analyst is using the Capital Asset Pricing Model to estimate the cost of capital, a natural starting point is the beta of the firm's stock, β_E. But the risk of the firm's *stock* is affected by its capital structure policy as well as the risk of its investment projects, so one must estimate the firm's *asset* beta, β_A. This note addresses how one can reasonably estimate a firm's asset beta. You might see different formulas applied when estimating an asset beta, so you should understand what assumptions underlie these different approaches.

If the firm uses no leverage, then the project cash flows belong entirely to the shareholders, and $\beta_A = \beta_E$. But when the firm utilizes debt, β_E overstates the risk of the firm's projects, so one must "unlever" the equity beta to get an asset beta. In perfect capital markets, this is straightforward. The value of the firm is determined by its investment policy, and different financing policies merely divide these cash flows in different ways (recall Modigliani and Miller's Proposition I). Just as the beta of an investment portfolio is a market-value-weighted average of the component betas, the firm's asset beta is a weighted average of its debt beta and equity beta, where the weights are determined by the market values of debt and equity:

$$\beta_A = \left(\frac{D}{D+E}\right)\beta_D + \left(\frac{E}{D+E}\right)\beta_E . \qquad (1)$$

This is explained quite well in Chapter 9 of Brealey and Myers. In practice, analysts often use this formula, and also often assume that $\beta_D = 0$, so that the asset beta is just the equity beta "deflated" by the factor $\frac{E}{D+E}$, i.e.

$$\beta_A = \left(\frac{E}{D+E}\right)\beta_E . \qquad (2)$$

Assuming that β_D is very close to zero is a good approximation for a firm whose debt is not very risky, but is a poor approximation for a firm with high leverage.

Once we leave the world of perfect capital markets, it is not so clear how one should extract the risk of the firm's assets from the estimated risk of its stock. It is still true that the stock beta of an all-equity firm directly reflects the risk of the firm's projects: $\beta_A = \beta_E$. But now it is possible that the value of the firm is affected by its leverage—because of net tax savings from interest, for example. Therefore, estimating what the firm's risk would be if it were all equity—i.e., its asset beta—is not as simple as taking the weighted average of its debt and equity betas shown earlier.

Let A denote the value of the firm if financed entirely with equity (A is the value of the firm's assets). In a world of imperfect capital markets, using debt financing might create additional value, so let Δ denote the value of the financing benefits: $D + E = A + \Delta$. For example, Δ could represent the present value of the net interest tax shields generated by borrowing. Let β_Δ denote the risk of these cash flows. We are trying to estimate *asset* risk to calculate an opportunity cost of capital, and $A = D + E - \Delta$, so the risk measure we are interested in is

$$\beta_A = \left(\frac{D}{D+E-\Delta}\right)\beta_D + \left(\frac{E}{D+E-\Delta}\right)\beta_E - \left(\frac{\Delta}{D+E-\Delta}\right)\beta_\Delta . \qquad (3)$$

This is the general formula one would like to apply. It requires estimating the risk of the firm's financing benefits (β_Δ), and the net present value of these benefits (Δ). As a practical matter, one will often rely on special cases. Let us consider a few such cases.

Case 1: Perfect capital markets

With perfect capital markets, $\Delta = 0$ by Modigliani and Miller Proposition I, so the formula simplifies to the familiar $\beta_A = \left(\frac{D}{D+E}\right)\beta_D + \left(\frac{E}{D+E}\right)\beta_E$, formula (1) above.

Case 2: Risk of financing benefits equals risk of assets

Consider the situation when interest tax shields are the source of financing benefits and the firm maintains constant leverage in market value terms. Then the amount of future tax deductions is determined by (and varies with) the value of the firm's projects, so the risk of these tax savings is equal to the risk of the assets, $\beta_\Delta = \beta_A$. In this situation, formula (3) simplifies again to the familiar $\beta_A = \left(\frac{D}{D+E}\right)\beta_D + \left(\frac{E}{D+E}\right)\beta_E$.

Case 3: Risk of financing benefits equals risk of debt

It is often assumed that the risk of interest tax shields is approximately equal to the risk of the debt. If interest tax shields are the source of the financing benefits, then this implies $\beta_\Delta = \beta_D$ and $\beta_A = \left(\frac{D-\Delta}{D+E-\Delta}\right)\beta_D + \left(\frac{E}{D+E-\Delta}\right)\beta_E$. If one also assumes that the debt has no systematic risk, $\beta_\Delta = \beta_D = 0$ and $\beta_A = \left(\frac{E}{D+E-\Delta}\right)\beta_E$. This differs from formula (2) in that β_E is not "deflated" as much when estimating β_A. Thus, if one mistakenly ignores the value of interest tax shields, the risk of the firm's projects is understated and the estimated opportunity cost of capital is too small.

Case 4: Financing benefits proportional to amount of debt

Suppose that $\Delta = TD$, so the financing benefits are proportional to the firm's debt. For example, if a firm has a fixed amount of debt in perpetuity, and the net tax savings per dollar of interest are T, then the present value of the net interest tax shields is $\Delta = TD$. If we again assume that the risk of the financing benefits equals the risk of the debt, the general formula simplifies to $\beta_A = \left(\frac{(1-T)D}{(1-T)D+E}\right)\beta_D + \left(\frac{E}{(1-T)D+E}\right)\beta_E$, or $\beta_A = \left(\frac{E}{(1-T)D+E}\right)\beta_E$ if the debt is riskless. This last formula can be written in terms of the debt/equity ratio as $\beta_A = \left(\frac{1}{1+(1-T)(D/E)}\right)\beta_E$. One often sees this formula applied using the corporate tax rate as T.[1] When someone uses the corporate tax rate as T, he or she is implicitly assuming that there are no personal tax benefits of equity mitigating the corporate tax benefits of debt.

When analyzing firms with very little debt in their capital structures, any tax benefits of debt will be a very small portion of total firm value, and the risk of the debt is likely to be very small. Therefore, using the simplest formula to unlever the equity beta, $\beta_A = \left(\frac{E}{D+E}\right)\beta_E$,

[1] For example, see equation (17.4) of Ross, Westerfield and Jaffe, *Corporate Finance*, 4th ed. (1996), Richard D. Irwin.

should give reasonable results. This approach is rigorously correct if the debt has no systematic risk ($\beta_D = 0$) and either:

- Leverage has no valuation effects ($\Delta = 0$), or
- The risk of Δ is equal to asset risk, e.g. if leverage is constant in market value terms.

One should be more careful when debt provides a significant portion of the firm's financing, but the "correct" calculation is not obvious. As is usually the case, using your judgment cannot be avoided!